(Con)fusing the un(con)fusable

István Aranyosi

In a recent article, David H. Sanford (2003) offers an argument for the need to revise one of the basic axioms of mereology proposed by David Lewis (1991: 74). The axioms proposed by Lewis are these:

> Transitivity: If *x* is a part of some part of *y*, then *x* is a part of *y*.
> Unrestricted Composition: Whenever there are some things, then there exists a fusion of those things.

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> Uniqueness of Composition: It never happens that the same *things* have two different fusions. (My emphasis)

Sanford argues that the last one of these axioms has to be revised. He considers a house made of Tinkertoys and two fusions:

(1) The fusion of the roof, the four walls, and the floor.
(2) The fusion of all the Tinkertoys.

He says that neither of these fusions is identical to the house and they are also not identical to each other. This is because (2) can survive an arbitrary disassembly of the Tinkertoys, while (1) can be destroyed without destroying any Tinkertoy, but by disassembly. How does this show the falsity of Uniqueness of Composition? Sanford appeals to the notion of a Set of Parts (McTaggart 1921: § 124), about which he says that it 'is very similar to the notion of a fusion or mereological sum'. A set of parts, as the name suggests, is a set. McTaggart's point is an intuitive one: one and the same individual x can fuse many different sets of its parts at the same time. For example, a certain cat c fuses infinitely many sets of its parts, like {its halves}, {its left eye, the rest of the cat}, {its first two legs, its eyes, the rest of the cat}, and so on. More generally:

> x fuses set A iff every element a of A is a part of x and there is no part of x that is not an element of A.

Of course, there will be many sets S such that x fuses S. But Sanford thinks, wrongly, that this intuitive point was 'overlooked' among others by Lewis, when he put forward the aforementioned basic axiom. I think Lewis's axiom and McTaggart's point are perfectly compatible, and I suspect that Sanford misunderstood Lewis's axiom because he confuses two predicates: 'to have a fusion' and 'to be a fusion'. The confusion can already be detected in Sanford's first question about Lewis's axiom: 'What does it mean to say that *a thing* has a fusion?' As you can see, I italicized two expressions, one in Lewis's axiom, 'things', which is a plural, and one just above, 'a thing', which is a singular. Lewis's axiom can be more explicitly stated as follows:

(*) It never happens that (α) individuals x and y are non-identical and (β) x fuses set $\{a_1, \ldots, a_n\}$ and (γ) y fuses set $\{b_1, \ldots, b_n\}$, such that $a_1 = b_1$ and $a_2 = b_2$, and, … , and $a_n = b_n$.

So *having a fusion* is an asymmetric relation that many things collectively bear to one individual, this is why Lewis uses the plural. When there exists an x that fuses set $\{a_1, \ldots, a_n\}$, we say that $a_1, \ldots, a_n$ have a fusion, their fusion being x. So it is (normally) many things that are said to have a fusion.[1] The property of *being a fusion* is also implicit here: x is a fusion

[1] See also Lewis's definition of 'fusion' (1991: 73), where this point is very clear.

when there exist $a_1, \ldots, a_n$ such that x fuses set $\{a_1, \ldots, a_n\}$. So *being a fusion of* is an asymmetric relation that one individual bears to many individuals. As I have said, Sanford seems to confuse these two notions, having a fusion and being a fusion (of), because only in this way could someone think that Lewis's axiom 'overlooks' McTaggart's point about the multiplicity of sets of parts a whole can fuse. This is because McTaggart's notion of 'one whole having many sets of parts' is the *converse* (not the equivalent) of 'many sets of parts having a fusion'.[2] So it seems to me that Sanford interprets Lewis's axiom like this:

(**) It never happens that (α) individuals x and y are identical and (β) x fuses set $\{a_1, \ldots, a_n\}$ and (γ) y fuses set $\{b_1, \ldots, b_n\}$, such that $a_1 \neq b_1$ and $a_2 \neq b_2$, and, … , and $a_n \neq b_n$.

Of course, in the light of what we learn from McTaggart's point, this principle is false. Counterexamples to it are countless, one of them being the aforementioned cat c, which fuses many distinct sets of parts simultaneously. But this principle is not one of the uniqueness of composition but one of the *uniqueness of decomposition*, something neither Lewis nor anyone else has ever claimed to be true. A decomposition of fusion x is a set A such that x fuses A. A decomposition A is unique iff x fuses A and there is no set B, different from A, such that x fuses B. McTaggart's point is that, generally, an individual that is a fusion does not have a unique decomposition.

So I think there is so far no reason to revise Uniqueness of Composition. But there is another part to Sanford's argument. He says that Lewis's basic axioms plus the Principle of Extensionality, which says that for any individuals, x and y, $x = y$ provided that x and y have the same proper parts, imply that ordinary things like cats last forever if no part of them is destroyed. That is, if cats are fusions of all their parts, then by disassembling a cat no parts of it would be destroyed, and therefore the cat would survive. But this is not true, since the disassembly of a cat results in the destruction of it. Sanford's solution (besides revising Uniqueness of Composition) is that we should accept that cats are not fusions, except in the limiting case in which they are fusions of their improper parts, that is of themselves, otherwise we would be committed to the view that cats last forever. But if cats are not fusions, they are simples. Since simples are by definition partless individuals, if we say that cats are not fusions, then we are committed to saying that cats have no (proper) parts. As far as I can see, this is false: our cat c has many proper parts, its tail, its eyes, etc.

[2] In other words, it is nonsense to say that a whole can have many fusions, but it is perfectly fine to say that more than one set of parts can have the same fusion.

But what about the difference in persistence conditions between cats and cat-parts? Sanford relies heavily on this in his argument. He attacks another statement by Lewis (1991: 2):

> Since all and only overlappers of cats are overlappers of cat-parts, the fusion of all cats is the same as the fusion of all cat-parts.

He says that what Lewis claims is true only if cat-parts are parts of actual cats. But this is not necessary:

> An unexpected widespread enthusiasm for eating cats, plus a fatal pandemic kitten disease, causes the extinction of cats. Yet cat-parts remain, some in jars of formaldehyde, and some wrapped in shrink-wrap and forgotten at the back of supermarket meat lockers. (Sanford 2003: 3)

I think the answer to this problem, as to the others I have discussed so far, is again to be found in Lewis's system. Cats are fusions, since they have parts and by this they fuse all their parts. But cats are special. The fusion of all cats does not have a unique decomposition into proper parts, but it has one in cats. Cats are then said to be 'nice parts'[3] of the cat-fusion. So nice parts have the function of ensuring unique decomposition for a given fusion. Cats are nice parts not only for the cat-fusion, but also for the fusion of all cat-parts. When the cats exist, the fusion of all cat-parts has as nice parts, cats – given some definition of 'nice part'. What about the situation described by Sanford? Here cats are dismembered, so there are no cats. But this does not show that the fusion of all cat-parts is not identical to the fusion of all cats. It only shows that niceness is not a necessary property of parts of fusions. So, I would say that the fusion of all cats is (necessarily) identical with the fusion of all cat-parts, but the fusion of cat-parts has cats as nice parts only contingently, namely when cats exist. This is to say that the fusion of all cats is identical with the fusion of all cat-parts, *regardless of whether all cat-parts are parts of cats or not*. Hence, we can maintain all the basic axioms of mereology and the claim that cats are fusions without being committed to cats' lasting forever. In what follows, I will try to show why this is so, by appealing to Lewis's idea of nice parts.

> **Proposition (1).** The fusion of all cats is just the fusion of all the nice parts – according to a specific definition of 'nice part' – of the fusion of all cat-parts, regardless of whether all cat-parts are parts of cats or not.

Proof. Suppose the definition of 'nice part' yields all and only cats as nice parts of the fusion of all cat-parts. Case 1: all cat-parts are parts of cats.

[3] Lewis (1991: 22) puts forward the notion of a nice part as a mereological emulator of the set-theoretical notion of a member.

Then it follows that the fusion of all cat-parts will have a unique decomposition into cats, which further means that the fusion of all cats is the fusion of all the nice parts of the fusion of all cat-parts. Case 2: no cat-part is a part of any cat. Then it follows that the fusion of all cat-parts will have no unique decomposition into parts, which further means that the set of nice parts of the fusion of all cat-parts is empty. Since in this case also the set of cats is empty (since there are no cats), then the fusion of all the cats is again just the fusion of all the nice parts of the fusion of all cat-parts, which is the individual *x* that fuses the empty set, i.e. *x* does not exist. Case 3: not all cat-parts are parts of cats. Then a part of the fusion of all cat-parts will have a unique decomposition into cats, that is, again, the fusion of all cats is the fusion of all the nice parts of the fusion of all cat-parts. **QED.**

> **Proposition (2).** There is a definition of 'cat part' that yields cats as nice parts and such that every part of the fusion of all nice parts of the fusion of all cat-parts overlaps some cat-part and there is no part of any cat-part that does not overlap some nice part (i.e. the fusion of all nice parts of the fusion of all cat-parts is identical with the fusion of all cat-parts), regardless of whether all cat-parts are parts of cats or not.

Proof. Let us adopt a definition chosen by Lewis (1991: 22, 1998: 188), according to which nice parts are maximal spatio-temporally connected parts, that is fusions all of whose parts are arbitrarily spatio-temporally close to each other and which are not parts of anything else for which this is true. Case 1: all cat-parts are parts of cats. In this case, all and only cats are maximal connected parts and nice thereby, hence the fusion of cats overlaps all cat-parts and there is no part of any cat-part that does not overlap some cat. So, every part of the fusion of all nice parts of the fusion of all cat-parts overlaps some cat-part. Case 2: no cat-part is part of any cat. Here only cat-parts are maximal connected parts and nice thereby, hence every part of the fusion of all nice parts of the fusion of all cat-parts overlaps some cat-part and there is no part of any cat-part that does not overlap some nice part. Case 3: not all cat-parts are parts of cats. Here both cats and maximal connected cat-parts that are not parts of cats are nice, but still every part of the fusion of all nice parts of the fusion of all cat-parts overlaps some cat-part and there is no part of any cat-part that does not overlap some nice part. **QED.**

From propositions (1) and (2) we get: there is a definition D of 'nice part', such that the fusion of all cats is identical to the fusion of all cat-parts regardless of whether all cat-parts are parts of cats or not. Since (a) according to proposition (1), the fusion of all cats is identical with the fusion of all nice parts of the fusion of all cat-parts, regardless of whether all cat-parts are parts of cats, and (b) according to proposition (2), there is a

definition D of 'nice part' such that the fusion of all nice parts of the fusion of all cat-parts is identical with the fusion of all cat-parts, regardless of whether all cat-parts are parts of cats or not, then (c) according to D, the fusion of all cats is identical with the fusion of all cat-parts, *regardless of whether all cat-parts are parts of cats or not*.

All that can be shown by Sanford's story about the dismembered cats is not the falsity of Uniqueness of Composition, but the falsity of an alleged principle of uniqueness of decomposition, like the following:

> Uniqueness of Decomposition: whenever there is a fusion x, it has a unique decomposition into parts.

This principle is, of course, shown to be false, but, as I said, nobody has ever claimed its truth. The fusion of cats has no unique decomposition into parts, it can be decomposed into cats, halves of cats, quarters of cats, etc. But this is consistent with saying that the fusion x of all the cats is identical with the fusion y of all cat-halves, regardless of whether cat-halves are actually parts of cats or not. At the same time, something like the uniqueness of decomposition into nice parts is true (by the definition of the function nice parts are required to have). But this again does not affect the identity of the fusions that are decomposed: fix on some definition of 'nice parts', then if cat-parts $p_1, \ldots, p_n$ are parts of cats, the fusion x of all cat-parts will have a unique decomposition into cats $c_1, \ldots, c_n$, each of them being a nice part according to the definition. When cat-parts $p_1, \ldots, p_n$ are not parts of cats but each of them is a nice part according to the definition, the fusion y of all cat-parts has a unique decomposition into $p_1, \ldots, p_n$. All this is consistent with x being identical to y.

Central European University
1051 Budapest, Zrínyi u. 14, Hungary
fphari01@phd.ceu.hu

References

Lewis, D. 1991. *Parts of Classes*. Oxford: Blackwell.

Lewis, D. 1998. Nominalistic set theory. In his *Papers in Philosophical Logic*, 186–202. Cambridge: Cambridge University Press.

McTaggart, J. McT. E. 1921. *The Nature of Existence*, Volume I. Cambridge: Cambridge University Press.

Sanford, D. H. 2003. Fusion confusion. *Analysis* 63: 1–4.