In the transcendental exposition of the concept of space in the “Space” section of the Transcendental Aesthetic Kant argues that “geometry is a science which determines the properties of space synthetically and yet a priori”\textsuperscript{1}. Together with the claims from the metaphysical exposition in the same section that space is not derived from any outer experience but it is a pure intuition and necessary a priori representation that is given as infinite magnitude, this builds up the general framework of the relation between space and geometry in Critique of Pure Reason. For Kant there exists only one geometry and this is the Euclidean geometry. On this basis runs what Friedman calls “the standard modern complaint against Kant”, namely, that he did not make the crucial distinction between pure geometry and applied geometry. Since pure geometry makes no appeal to spatial intuition or other experience and since the truth of the axioms of the applied geometry depends upon an interpretation in the physical world the question about which axiomatic system, the pure or the applied one, is true is settled only by empirical investigation\textsuperscript{2}. This directly contradicts Kant’s fundamental claim that we can know the proposition of the Euclidean geometry a priori.

Major part in this complaint is played by appeal to the non-Euclidean geometries. Historically, this was initiated by Helmholtz who argued that Kant’s theory of space is untenable in the light of the discovery of the non-Euclidean geometries\textsuperscript{3}. His line was later forcefully supported by Paton, Russel, Carnap, Schlick and probably at most Reichenbach, who famously criticized Kant’s conception of space on the basis of a complex analysis of the visual a priori which he took to underlie Kant’s doctrine of geometry\textsuperscript{4}. Recently, Parsons refers to this line as “the most common objections to Kant’s theory of space”\textsuperscript{5} and concedes that Kantian could still accept some more primitive geometrical properties (than those provided by the 5\textsuperscript{th} postulate of Euclid’s Elements, for example) to be known a priori even if he abandons the claim that in specific propositions of the Euclidean geometry can be known a priori. Though this is an attempt to salvage some part of the geometry doctrine I do not think that this is in the spirit of the Transcendental Aesthetic and also, I believe that it would be insufficient for Kant’s purposes.

My aim in this paper will be defend the view that Kant’s doctrine of geometry can survive criticism based on appeal to the non-Euclidean geometries. I will argue that we can still make sense of Kant’s claim that it is the Euclidean geometry that determines the properties of space and that it does it a priori provided that we have proper understanding of his space conception as a pure form of the intuition. I will try to show that Kant’s appeal to basic propositions of the Euclidean geometry as necessary true and intuitively certain could still be defended. I will argue, partly in the sense of Friedman, that we do not have to accept the pure – applied geometry distinction but my justification will not be based on his interpretation of the connection between Kant’s logic and philosophy of mathematics. I would rather argue that such distinction is inappropriately applied ad hoc to the doctrine of space alone but its claim is of much bigger scope and as such should be met by frontal debate against Kant’s system of transcendental idealism. Such debate, however, often is either not offered by the critics or dissolves the strength of the criticism upon much broader field.
If the distinction is dispensed in such way, most critiques based on appeal to the non-Euclidean geometries will be reduced to the principle claim that the propositions of the Euclidean geometry are not the only candidates for true with certainty any more but the propositions of the non-Euclidean geometry are competitors as well. The common reading of this claim is that the non-Euclidean propositions are also intuitable or visualizable in our intuition, as such they could (alone or also) be true of it and therefore the Euclidean ones are not apodeictic, necessary true and intuitively certain. I will claim that the non-Euclidean propositions cannot be true of the space of the intuition since they lack a set of substantial properties that are necessary for the space of the intuition and that are actually found there upon introspection. I will argue that only the Euclidean geometry provides these properties and as such it is indispensable for the space of the intuition not only in the Kant’s but in a broader sense.

PURE – APPLIED GEOMETRY DISTINCTION

Among the recent studies of the topic probably Friedman formulated most clearly this objection, when he says that “Kant fails to make the crucial distinction between pure and applied geometry”6. Interestingly, this approach has been used in both directions, as criticism of Kant’s theory of space (Russell, Reichenbach, and Hopkins) but also as a way to save the doctrine (Ewing, Strawson). The division gave birth to different interpretations that multiplied the possible options among which we could choose from. Thus Schiller divided the space to perceptual and conceptual7, Craig proposed three readings of what the geometrical axioms (the Euclidean ones) could be true of, the space of sense-impressions, the space of mental images and the space of the way things look8. Lucas distinguished between four different senses of use of the word “space”: as a term of the pure mathematics, as a term of the physics, as a space of our ordinary experience and as a possibility for existence (construction) of objects (geometrical and physical)9. Satisfactory comment on all these readings is beyond the scope of the present discussion but it is important to point that although all they have merits of their own yet some of them fail to account properly for one but crucial aspect of space’s meaning in the Critique of Pure Reason, namely that space is pure intuition10 and was never meant to apply to things in themselves. Traditionally, this is the sense in which applied geometry is interpreted and at present day most physicist regard the rules of geometry, whatever they are as applied to the realm of the things as they are by themselves. Pure intuition, however, is what provides for Kant immediate representation to objects for the subject11 and which not only precedes the actual appearances of the objects but in fact makes them possible12. The term “pure” is clarified (in the transcendental sense) as “there is nothing that belongs to sensation”.13 Together with the claim that “space does not represent any property of things in themselves, nor does it represent them in their relation to one another”14 this implies that on the one hand the space notion Kant builds is not determined by the experience but on the opposite, makes it possible and on the other hand that has no claim whatsoever about the world of the things in themselves, what at present day is meant under the term “physical world”.

Proper understanding should recognize that the lack of epistemic access to the world of the things in themselves is not simply due to the character of the space notion but it is a fundamental feature of Kant’s epistemic and ontological model of the world. Taking this into account any criticism against the non-applicability of the space to the...
physical world rush ahead and neglect the fact that such ambitious criticism is to be targeted against the complete model instead. Since it is not that much of a criticism against space that it is not applied to the physical world, the responsibility for the application of space is dependent on the general model. In this sense it is not a deficiency of Kant’s doctrine of space and geometry in particular whether it is or it could be true of the physical world. The space notion and the role of geometry is to be understood properly only within Kant’s system and within the system the notion of space is consistent and even necessary for the lack of appeal to the physical world as such (an sich). Any criticism against the big model, however, was rarely if at all presented as accompanying talks about space and geometry. In this sense, any objection of the simplified form “Kant claims that the propositions of the Euclidean geometry are the only true ones as describing space, our space is the physical space and not a subjective one, there is physical geometry that actually describes the space truly and this geometry is non-Euclidean, therefore Kant is wrong when he says that we do not know the things in themselves but only as they appear to us. Such criticism, however, becomes much more complicated than the relatively simple discussion about space and geometry and since it would position itself within centuries of unresolved philosophical debates it is much less obvious than the ad hoc objections about geometry. It is not surprising that most critics prefer to attack the concrete issue and avoid more fundamental debate. However, no such approach could be entirely successful since it does not explain why we should prefer, for example, direct realism about the external world instead a representational theory of the kind proposed by Kant. I believe that mere appeal for skeptical implausibility about external world as feature of Kant’s general model will not do the job, at least with respect to space and geometry. Without successful final of frontal criticism against the general model there is no justification for introducing “applied geometry” as an option of a distinction as it is meant to apply to the world of the things in themselves at all. This would, however, undermine the reasons for introducing the distinction with respect to Kant’s theory of space and geometry.

As an illustration, often the objection about the non-applicability of geometry to the physical world takes the following form, the example here being taken from Russell:

On the other hand there is geometry as a branch of physics, as it appears, for example, in the general theory of relativity; this is an empirical science, in which the axioms are inferred from measurements and are found to differ from Euclid’s. Thus … it is synthetic but not a priori.

There are two difficulties for Kant here: one about the a posteriori character of the axioms and another (an implicit one) that the Euclidean geometry is not true as applied to the physical world. Response to this could be the following: even if the large scale of the universe is properly described according to the general theory of relativity by a spherical geometry the scale of the ordinary human experience is still almost complete approximation of the Euclidean geometry. The differences are so minute that they are practically undistinguishable. In this sense the Euclidean geometry is still true when applied to some scales and definitely true when applied to the human scale ordinary experience. Further, for Kant the question would not be that much if the geometry is applied to the world of the things in themselves since this option is ruled out in general but whether this geometry is true of how the world appears to us. Here, the supposition
that this is a non-Euclidean geometry is not obvious at all. Later in the paper I will argue that practically the geometry of the space of the intuition is Euclidean and cannot be non-Euclidean. Nevertheless all this is more or less irrelevant for Kant’s space doctrine since the physical geometry and the pure one would coincide in his system. There will be only one geometry in and this would be the geometry that reigns the space of the intuition. This is Strawson’s point in *The Bounds of Sense* when he comments on the traditional criticism:

He thought that the geometry of the physical space had to be identical with the geometry of the phenomenal space. And this mystery does invite the suggestion that the geometry of the phenomenal space embodies, as it were, conditions under which alone things can count as things in space, as physical objects, for us. Especially does it invite this suggestion if we think of something’s counting as a physical body for us in terms of it’s appearing to us, presenting to a phenomenal figure …

This geometry is not related to the world of the things in themselves at all, it prescribes (geometrical) predicates only as far as the objects appear to us. Also, even if this geometry happens to be non-Euclidean one this still does not mean that it is a posteriori since, as Jones points

Nor did Gauss, Lobachevski, or Bolyai follow such (empirical, *my note*) procedure in developing non-Euclidean geometry. They all carried out their work without recourse to experience, and thus a priori. Just what their criterions of truth need not be considered here, but they certainly were not empirical …

So, after the claim that geometry must describe the physical world as such is dismissed the argument against apriority of the axioms could be met by simple appeal to the history of the development of the non-Euclidean geometries and the foundations of their axioms. A weaker variation of criticism based on non-Euclidean geometries says that since Kant affirmed that only one geometry is true of the intuitive space the very discovery of the alternative geometries proved him wrong. This could be met by pointing that Kant actually anticipated such possibilities and that the question is reduced to the next one whether the geometries are and could be intuitively true. As I mentioned above, this question will be discussed further in the paper.

To sum up, an attempt to criticize Kant’s theory of space on the basis that he did not distinguished between pure and applied geometry could not be successful since the question of what does the space apply to with Kant is resolved by appeal to the world as it appears to us. Whatever this world looks like, certain geometry is applied to it and the question why this geometry is not applied to the world of the things in themselves is meaningless for Kant since the only thing we can know about such world is that it exists and no geometrical predicate can be applied to it or to its objects and the relations between them. The further question about the status of the non-Euclidean geometries is thus reduced to the question whether they apply to the space of the intuition and how. In addition, an important explanatory remark in this respect is the point made by Friedman who argues convincingly that the distinction between pure and applied geometry goes together with certain understanding of logic that was not available to Kant since appears with Gottlob’s Frege *Begriffsschrift* in 1879. The importance of this relation is, as Friedman stresses, also supported by Hintikka and Parsons.
ESSENTIAL PROPERTIES OF THE EUCLIDEAN GEOMETRY

If criticism based on lack of pure – applied geometry distinction is shown to be to a large extent irrelevant for the true significance of the concept of space within Kant’s system we must turn to the further argument, namely, that after the discovery of the non-Euclidean geometries the candidates to describe the domain of the space as pure form of the intuition are not reduced solely to the Euclidean propositions. This can be read as the claim that the Euclidean geometry is not the science that determines exclusively the properties of space and this endangers the status of its propositions as a priori, universal, necessary true and intuitively certain. It is remarkable how the positions on both sides of the debate differ in this respect. Among the defenders of Kant’s doctrine of space Strawson argues that the Euclidean geometry is at least true of everything that can be spatially intuited, he adopts the term “phenomenal geometry”:

Consider the proposition that not more than one straight line can be drawn between two points. The natural way to satisfy ourselves of the truth of this axiom of phenomenal geometry is to consider an actual or imaginable figure. When we do this it becomes evident that we cannot, either in the imagination or on paper, give ourselves a picture such that we are prepared to say of it both that it shows two distinct straight lines, and that it shows these lines as drawn between the same two points. 23

Similar is the view of Frege who maintains that “the truths of geometry govern all that is spatially intuible” 24 where under “geometry” he means “Euclidean geometry”. Among the critics, somewhat surprisingly we find that Bennett concedes with this as well. 25 Again among the critics, but on the other side we have Craig, who claims that “… the answer will be that they would look just as they were, namely non-Euclidean” 26, Hopkins, who argues in favor of a kind of indeterminacy of the space of the visual geometry that would allow for both Euclidean and non-Euclidean intuition 27 and Reichenbach, who argues in favor of the visualization of the non-Euclidean geometries, though in a more complex manner. 28 The critics usually look for support in constructing examples that show either that we could have non-Euclidean governed intuition or at least that Euclidean intuition is not necessary. Similar is the appeal to intuitive solution in the approach of the proponents of the Euclidean intuition, both Strawson and Frege make their case with resort to the unimaginability of the opposite of the Euclidean propositions. Though I believe the latter to be plausible I think that there exist other, probably even stronger reasons why we should regard our intuition as Euclidean. Because mere appeal to one intuition does not simply resolve the problem, as we see a lot of other intuitions are at least claimed to be possible.

Kant’s examples of Euclidean propositions could be divided in two general types:

- **concrete examples** - “space has only three dimensions” 29, “there should be only one straight line between two points” 30, “in a triangle two sides together are greater than the third” 31,

- **explicative examples** – clarification of geometrical procedures as “All proofs of the complete congruence of two given figures (where the one can in every respect be substituted for the other) ultimately come down to the fact that they may be made to coincide” 32, “we can require a line to be drawn to infinity” 33 or the incongruent counterparts example with the left and right hands 34.

The first type is the traditional appeal to the intuition. The second type is slightly different in purpose though – it aims not to *demonstrate* but to *explicate* the appeal to the
geometrical propositions. This could be interpreted as an attempt to clarify deeper than the intuitively certain one layer of meaning. In the light of subsequent to Kant modern discoveries in geometry the role of the Euclidean geometry for the intuition becomes more clear and deeper. Such layer could provide different reasons for the rules of geometry being Euclidean than the reasons of intuitive certainty. From the examples given above we can read off the crucial appeal to the notion of congruence and the notion of infinite extension. It is important to point out that these notions, along with some others play two-fold role – if not available they could not just disable the appeal to intuitive certainty but could deprive the space of the intuition of the very capability to host or to generate and construct objects in itself. Such and other notions of the same primitive importance are available only because some primitive properties of the geometry presuppose them. These geometrical properties I will regard as essential for the intuition.

The appeal to the certainty of the geometrical has important role in this sense:

The apodeictic certainty of all geometrical propositions and the possibility of their a priori construction is grounded in this a priori necessity of space. And

Since the propositions of geometry are synthetic a priori, and are known with apodeictic certainty, I raise the question, whence do you obtain such propositions, and upon what does the understanding rely in its endeavour to achieve such absolutely necessary and universally valid truths?

The certainty here and in the concrete examples grants at least two things: the discernibility of the objects of intuition’s properties and the truth of the geometrical propositions. The certainty in question is not certainty derived from concepts but is the intuitive certainty:

in a triangle two sides together are greater than the third, can never be derived from the general concepts of line and triangle, but only from intuition, and this indeed a priori, with apodeictic certainty.

Regarding the objects of the intuition we can discern two functions of certainty: to provide certain objects and to provide certain relations between them. Only together both functions could grant two of the components of the geometric propositions: meaning and truth. Intuitive certainty has crucial role for epistemic claim that we know the propositions of Euclidean geometry, for the semantic claim that we know them to be true a priori and for the modal claim that we know them to be necessary.

The only way for the objects to be certain is to be well defined in terms of the intuition. Well defined objects in this sense are intuitively well discernible objects – we have object $x$ that is discerned in the intuition as having some properties, say being 1 dimensional (line), being extendable ad infinitum, being straight or being curved. This allows the object $x$ to be intuited on the one hand as being $x$ but on the other as not being non-$x$. Thus it is provided a way not to misintuit different object as being some other object, a way to intuit the qualitative (as well as the numerical) identity of the objects. Well defined intuitively objects are granted the possibility to exist at a certain position and, with the help of the other pure form of the intuition, the time, to exist at a position at a certain time moment $t$. The qualitative identity of the object involves intuitive
discernibility of its having a certain *shape* and being of certain *size*. Identity of the object over time provides means to keep track of the objects *changing* or *staying the same* over time. More important, both identities allow for the object to change while *stays the same object*.

Further, when qualitative identity and identity over time of the objects of intuition is available we can compare different objects. We can find whether two objects of intuition are *congruent* or not. Congruency of objects is necessary condition for the intuition to draw substantial knowledge about the content of the geometrical propositions. If the principle possibility to discern (intuit) relatively determined congruence of objects was not available to the intuition the knowledge that the intuition would be able to dress up in the form of geometrical propositions would be very poor and not very informative. The reason for that is that the intuition would not be able to discern *what is going on with which exactly object* and *which properties pertain to which exactly object*. This consequence however, is highly problematic as claim about intuition for several reasons: First, we do not find such uncertainty upon introspection of the faculty of intuition; second, the intuition (not only in Kant terms) as pure form will be completely helpless with the task of ordering the appearances in the appropriate relations:

but that which so determines the manifold of appearance that it allows of being ordered in certain relations, I term the *form* of appearance. That in which alone the sensations can be posited and ordered in a certain form, cannot itself be sensation; and therefore, while the matter of all appearance is given to us *a posteriori* only, its form must lie ready for the sensations *a priori* in the mind, and so must allow of being considered apart from all sensation. 38

and this is not the case. No matter what does Kant claim about the intuition he is consistent with the interpretation that whatever we intuit is the same geometrical content as what a realist will call “content of perceptual space” or “content of the propositions of the physical (applied) geometry”. Third, but not least, on any epistemic access account, idealism or realism this would be completely weird: whether Euclidean or non-Euclidean or whatever geometry determines the way things look to us we *do have* congruence ability between the intuited (perceived) objects. The possible objection that we can mistakenly attribute congruence between spatially intuited objects would only confirm the existence of this ability, since only its presence would provide contrast for the mistake to take place. This is what allows us to cope with either the world as the things appear to us or the world of the things in themselves.

We come up to an important assumption, tacitly assumed by most criticisms that claim that a) non-Euclidean geometries are visualizable and b) their propositions are true of the space of the intuition. The assumption is that

The congruence ability of the intuition will still be granted in the non-Euclidean case

Since nobody would like to claim that we are or we could be deprived of our, intuitive or not ability to pronounce two triangles or other geometrical figures as congruent or that by mere change in the size of a figure we will receive a figure of different shape. This will be if not complete lack of congruence capability then sufficient amount of congruence uncertainty that would prevent any intuition of its practical possibility to render knowledge with meaningful content about such features of the objects of intuition as
“same shape thought different size” and identity of objects based on their shape. As far as we speak of the space as a pure form of the intuition, whose properties are described exclusively by certain geometry, shape is essential for it. Because surely we do find such property in the geometrical objects of our intuition and we certainly do operate with it all the time.

The point however, is that it is exactly this lack of congruence that is encoded in the non-Euclidean geometries. Because the change of size of a geometrical figure in space with either spherical or pseudo-spherical characteristics will necessarily lead to a change in the figure’s shape. The possible qualification of the non-Euclidean critic that we can still have good enough approximation as a matter of degree of the Euclidean one and the differences in shape are so minute that are negligible does not help much since the very idea of change in the geometrical shape properties only because the size or the position of the figure become different is highly counterintuitive. This is simply not the way our spatial intuition and experience work. We will be never certain anymore that, given, for example, triangle \( x \) did not happen to undergo some change in size as intuited since we cannot with complete accuracy construct and maintain the same size of the figure over time in our intuition. Also, we would not be sure any more that the triangle \( x \) did not change geometrically by mere intuited it as displaced from a position with coordinates \([p_1, p'_1]\) in our space of intuition to position with different coordinates \([p_2, p'_2]\). If we cannot be certain about the size and other geometrical properties of the intuited figure how can we be sure that our next appeal to it will be appeal to the same figure? We will lose any certainty about any content of a geometrical proposition that involves the shape notion whatsoever. Fortunately, this is just not the case with our spatial intuition. We are secure that when we transport an intuited triangle from one position in our intuited space to another it will not change in shape – we simply see this, we are certain of it and we make good use of it.

The main reason for this is the primitive properties our space of intuition has. As even Schiller, a proponent of the non-Euclidean criticism stresses, though in a slightly different context, the crucial difference between the Euclidean and the other spaces are differences with respect to primitive properties. Euclidean space is

a) one, empty, homogenous, continuous, infinite, infinitely divisible, identical, invariable

The competitors are

b) many, filled, heterogeneous, possibly finite, not infinitely divisible, variable,
have such properties shows that they are redundant, unnecessary and clearly do not have some important intuitive job to do.

We are intuitively certain for some set of geometrical propositions and all these are based on the crucial notion of congruence, identity and difference on a basis of shape. All this should be sufficient to show that the intuitive space is consistent with the geometrical picture as provided by the Euclidean space and inconsistent with the picture provided by any of the rivals. This is enough for Kant’s purposes in the Transcendental Aesthetic, and even the further claim that this picture is true of any space understood as intuited space governed by certain geometrical rules seems to be supported as well.

All of the above analysis of the space of intuition as described by geometrical system(s) is done in accordance to the gist of Felix Klein’s *Erlangen Program*. He took the questions of the content and the task of geometry as questions of utmost priority. Result of this direction of research is the mathematical Theory of Groups. The group, by the words of Hermann Weyl, is perhaps the most characteristic mathematical concept of the period.\(^{42}\) Klein questioned the problem of the legitimacy of mathematical operations and argued that “Every geometry in its general concept and aim is a theory of invariants with respect to a certain group, and the special nature of each depends upon the choice of the group. Not every observation of a special object and not every proof bearing on it, has the characteristics of a *geometrical* proposition.”\(^{43}\) Thus,

> “Only properties that are characterized by an invariance with respect to a certain transformations can be called “geometrical”\(^{44}\)."

This “invariance” is in fact the one sought in the intuitive space. In the *explicative examples* Kant aims to point on the importance of such primitive properties of the space that will still allow for invariance of the geometrical properties of the object of intuitive space. We find such invariance upon introspection, we make use of it and we need it. Among the many possibilities for invariance plausible group candidates as most important ones are the most simple and primitive ones, as Lukas points, based on the operations *displacement*, *rotation* and *reflection*.\(^{45}\) He argues convincingly that without geometrical properties remaining invariant upon these three basic operations we would not have been able to form the concept of a material object (or in Kant’s terms – a concept of an material object as it appears to us) and this is something that we definitely do and need to be able to do. Lukas stresses that even if we could define non-Euclidean geometries in terms of other groups, they will be less simple and less fundamental that the Euclidean one. Putting aside these purely formal considerations the important consequence that matters the most for the present purpose is that in the Euclidean case we do keep the invariant we find in our intuitive space and in the non-Euclidean case we do not.

Logical result of all of the above is the contention that if we find and if we need the notion of primitive invariance of geometrical properties in our spatial intuition we cannot resort to any of the non-Euclidean geometries. The only source of such invariance is provided by the Euclidean geometry and hence any criticism basing itself to attributing non-Euclidean geometries to the realm of the intuition must fail. And, I believe, even Kant’s critics would admit the need of such primitive invariance.
CONCLUSION

My defense to the “the standard modern complaint against Kant” discussed two main claims of the criticism: that Kant failed to distinguish between pure and applied geometry and that space could be described by the propositions of the different non-Euclidean geometries. The former claim argues that the propositions of geometry as truly describing the world (as such or as it appears to us) could not be Euclidean since there are two meanings of “space” - in the pure (mathematical sense) all consistent geometrical systems could be valid and in the physical sense the geometry that describes the space is non-Euclidean, consequently, Kant’s theory of space and geometry is wrong. The latter claim is the core of the traditional criticism against Kant’s theory of space and geometry; it argues that Euclidean geometry does not determine exclusively the properties of space or that it does not determine them with apodeictic certainty and a priori. In different forms this attacks the Euclidean propositions as a priori, universal, necessary true and intuitively certain. If successful both lines of criticism would destroy the fundamentals of Kant’s aprioristic doctrine of space and Euclidean geometry.

With respect to the first one I argued that it is not necessary to accept this modern distinction. I tried to show that the attempts to criticize Kant’s theory of space on the basis that he did not distinguished between pure and applied geometry could not be successful because the world of the things as themselves for Kant is given and exists only in the epistemic dressing that we could not know anything about it as it is by itself. The opposite, however, is what is presupposed by the label “physical world” in the traditional criticism. In particular, the traditional view accepts for granted that we can know geometrical predicates and propositions true of objects in themselves. Yet whatever the world of the things in themselves looks like for Kant it is knowable only as far as it appears to us. The propositions of geometry are applied to it solely to this extent. The question why this geometry is not applied to the world of the things in themselves is meaningless for Kant since the only thing we can know about such world is that it exists and no geometrical predicate can be applied to it or to its objects.

With respect to the second one I tried to show that the propositions of the non-Euclidean geometries cannot be true of the space of the intuition because they lack a set of substantial properties that are necessary for it and which are actually found upon introspection. Namely, these are the properties that provide the primitive geometrical invariance that allows us to make use of such notions as congruence and identity (including identity over time) of geometrical objects based on the concept of geometrical shape. I argued, in the spirit of Klein’s Erlangen Program and the Theory of Groups, that only the axioms of the Euclidean geometry provide these properties and as such they are indispensable for the space of the intuition not only in Kant’s but in a broader sense. Hence any criticism basing itself to attributing non-Euclidean geometries to the space of the intuition cannot be successful.

I believe that most of the energy behind the traditional criticism against Kant’s theory of space comes from unwillingness to accept the general framework of the Critique. I do not think, however, that we are justified to accept criticism which suffers from various deficiencies both with respect to the analysis approach towards Kant’s model and with respect to necessary features of the geometrical rules of our spatial intuition.
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