Endogenous Market Power in an Emissions Trading Scheme with Auctioning

by

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Abstract

This paper contributes to the literature in market power for emissions permits, modeling an emissions trading scheme in which polluters differ only in their business-as-usual emissions. They play a two-stage static complete information game in which their market power arises endogenously from the business-as-usual emissions. In the first stage the polluters bid in an auction for the distribution of the fixed supply of permits and in the second stage they trade these permits in a secondary market. For compliance, they can also engage in abatement activity at a quadratic cost. In equilibrium all polluters are successful in the auction. In the secondary market the low emitters are net sellers and the high emitters are net buyers. Moreover, the secondary market price is unambiguously above the auction clearing price. The welfare analysis shows that the aggregate compliance cost when polluters act strategically increases in the heterogeneity of their business-as-usual emissions. Furthermore, there exists a threshold of the fixed supply of permits above which strategic behavior is socially preferable. Finally, for certain distributions of the business-as-usual emissions, strategic behavior is cost-effective, regardless of the level of the available supply of permits.

Keywords: auction, market power, emissions trading

JEL classification: L13, Q52

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1 Introduction

During the last decades emissions trading schemes have increased in popularity as policy tools for emissions reductions. The European Union (EU) commenced such a scheme in 2005, namely the EU ETS, which is currently the biggest emissions trading scheme (ETS) in the world. However, smaller scale emissions trading schemes under the Clean Air Act had been in effect in the United States ten years before the EU implemented its own scheme, and recently the state of California has also begun an in-state scheme, with enforcement starting in 2013. Therefore, understanding the functioning of emissions permits markets as tools for fighting climate change is becoming important in order to inform the policy makers responsible for assuring that these schemes achieve their goal with the least cost for the society.

Several authors have raised concerns about the exercise of market power in the spot markets for emissions permits even in the context of big emissions trading schemes like the European Union Emissions Trading Scheme (EU ETS) (see, for example, Klingenberg (2007)). Such concerns have been expressed both with regard to the trading of the emissions permits in the secondary market and to the initial allocation of permits via an auction.

For example, in a paper assessing the effectiveness of the UK Emissions Trading Scheme, Smith & Swierzbinski (2007) put forth the possibility of the exercise of market power as an explanation for the substantial difference between the auction clearing price and the market price of the emissions permits when traded in the secondary market. Indeed, using a stylized model of a monopoly (a group of firms which coordinate their actions in the auction) and a competitive fringe, they are able to reproduce the market price of a permit in the first year of the scheme following the auction.

In addition, as it was also pointed out by Ellerman et al. (2010), although the EU scheme covers more than 11,000 installations, many installations are owned by the same firm. Hence, it is conceivable that they obey the same trading and bidding strategies, especially since the auction regulations in these schemes allow firms to act in business groups. Based on the Community Independent Transaction Log (CITL) data and ownership information, Trotignon & Delbosc (2008) found that that in Phase I of the EU ETS, 74% of the emissions cap was assigned to only 100 firms across the EU, which creates the conditions for exercising market power in the secondary market. Moreover, using CITL data, Schleicher (2012) shows the uneven distribution of the emissions across the EU ETS: 84% of the installations in the EU ETS accounted for only 10% of the emissions generated within the scheme in 2011, which indicates a relatively concentrated market.

Regarding the markets for the initial allocation, recent evidence from the advance auctions held by the EU supports the assumption of thin markets when permits are initially distributed by the regulator. In the two consecutive auctions held by the EU in December 2012 for the distribution of Phase III allowances, there were only 11 and 13 bidders, respectively, with 9 successful bidders in each case. This is surprisingly low participation in the primary auction compared to the total number of the EU covered installations. Another example in this direction comes from the newly established Californian cap-and-trade scheme. In November 2012, the Air Resources Board organized the first two auctions for selling not only current allowances for the 2013 vintage, but also allowances from the 2015 vintage. The auction had only about 70 participants. Moreover,
the 2015 vintage auction has a Herfindahl - Hirschman Index equal to 1485, which indicates a moderately concentrated market.

Motivated by the above-mentioned anecdotal evidence as well as by a gap in the theoretical literature on market power in ETSs, the aim of this paper is to understand the consequences of the exercise of market power in a cap-and-trade system in which polluters act strategically both in the auction, where permits are initially distributed and in the secondary market, where the polluters trade the permits among themselves. To this end, I augment the model of Malueg & Yates (2009a), including a new decision stage. In my model the emitters firstly bid in an auction for the initial distribution of permits and then they trade these permits in a secondary market. Thus, unlike in Malueg & Yates (2009a) and Malueg & Yates (2009b), where the initial allocation is grandfathering, in this paper the initial allocation is via an auction. The auction format is that of a uniform price sealed-bid auction in which the participants, simultaneously and independently, submit bidding schedules to the regulatory agency who issues the permits. The latter clears the auction by equating the aggregate demand with the fixed supply of permits and distributes the permits to the polluters according to their bids and the market clearing price. Therefore, in the two-stage static complete information model of emissions trading developed below, the initial allocation is not exogenous. This allows to assess the effect triggered by the anticipated exercise of market power in the secondary market over the bidding strategy at the initial allocation stage. To the best of my knowledge, this is the first analysis of this type found in the literature. Although market power in emissions permits markets has received some attention in the past two decades (e.g. Hahn (1984), Maeda (2003), Montero (2009), Malueg & Yates (2009a), Hintermann (2011), Lange (2012)), theoretical results on market power when permits are distributed via an auction are missing.

The contribution of this paper is twofold. First, most papers examining market power in emissions trading, starting with the seminal paper by Hahn (1984), rely on the crucial assumption that one or two dominant firms have the ability to influence the price, while the rest of the firms, the fringe, act as price takers. Hence, the market power is exogenously assigned and it typically comes from the initial permits endowment. Therefore, the discussion about inefficiency revolves around the initial allocation of the permits. Unlike this literature, my model does not a priori assign market power to any of the polluters, but it rather allows all polluters to exercise market power to the extent that this is allowed by their individual characteristics, i.e. their parameters. Second, as it was already suggested, in my model the initial allocation is not exogenous (grandfathering), but via an auction.

The main question to be answered in this paper is how the (anticipated) exercise of market power affects the overall effectiveness of the ETS. For this, I first solve the model for the benchmark case in which the polluters act in a competitive manner, i.e. the price-taking case. Due to the exercise of market power inefficiency is expected from the outset. Thus, the overall aim of this exercise is to conduct a welfare analysis and to understand the direction of the inefficiencies in this kind of setting. More specifically, from the policy perspective it is relevant to understand who are the "winners" and the "losers" in an ETS where market power arises endogenously and the initial allocation of permits is via an auction.

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3The term "polluters" will be used interchangeably with the term "emitters" to denote the members of the ETS as players of the emissions markets.
As it can be anticipated, the polluters shade their bids in the auctioning stage and decrease the price relative to the price-taking case. In fact, I show that strategic behavior in both markets results in an unambiguously lower auction clearing price relative to the secondary market price of emissions permits. It will be seen later that this is one reason for which the high emitters are hurt when everybody acts strategically. As they receive fewer permits in the action as compared to the price-taking case, they must buy the deficit for a higher price in the secondary market as compared to the price of the primary auction. In addition, they also engage in more abatement relative to the price-taking behavior. In sum, at individual level I find that strategic behavior favors the low emitters, who are net sellers in the secondary market, but also some who become net buyers in the secondary market and whose business-as-usual emissions are low enough.

At aggregate level I find that, depending on the distribution of the business-as-usual emissions in the scheme, it is possible that strategic behavior results in a lower compliance cost than the price-taking behavior, regardless of the fixed supply of permits chosen by the regulator. In terms of policy, if the regulator intends to induce price-taking behavior using the supply of permits as its instrument, she only needs to know the number of polluters, the business-as-usual emissions of the smallest emitter and the first two moments of the distribution of the business-as-usual emissions in the scheme.

The paper is organized as follows. Section 2 briefly outlines the related literature. Section 3 sets up the model and discusses the underlying assumptions. In Section 4 I solve the model under the benchmark case in which polluters act as price-takers. Further, Section 5 solves the model and discusses the results for the case in which the polluters act strategically in both markets for permits. Section 6 compares the two types of behavior and conducts the welfare analysis. Finally, Section 7 concludes.

2 Related Literature

This paper contributes to two strands of literature. The first strand of studies models permits markets by exogenously assigning market power to one of the firms in the scheme, designated as the dominant firm. This direction started with the seminal paper by Hahn (1984), who found that the cost effective solution is achieved only if the firm with market power is initially endowed with the number of permits that she holds in equilibrium. In addition, Hahn (1984) finds that if an interior solution to the cost minimization problem exists, then the permit price increases in the permit allocation of the dominant firm. Further, Westskog (1996) extends the model of Hahn by assuming a group of Cournot players (countries) as leaders in a leader-follower game with a group of fringe price-taking countries, recovering, in principle, the implications delivered by the seminal model. A more recent paper by Maeda (2003) models an emissions permits market in which two emitters are assigned to be price makers, one a net buyer and one a net seller, and it derives conditions under which the two emitters can exercise effective market power. While the two price makers play a Nash game, the fringe emitters, who are present in an infinite number, are price takers. The paper finds that only the market maker with excess initial allocation of permits (the net seller) can influence the price above the competitive level, and its only source of market power is the excess of permits. However, the market maker with deficit of permits (the net buyer) has no ability to exercise effective market power.
Hintermann (2011) brings into the discussion the effect of the free permit allocation on price manipulation when market power exists both in the emissions and the product market. Again, the ability to exercise market power is exogenously assigned to one dominant firm, which enjoys market power both in the permits and the product market. The main result of his paper is that the dominant firm may find it profitable to manipulate the permit price upwards even if she is a net buyer of permits, provided that her initial allocation is sufficiently large and that she can pass on the carbon cost to the consumers of her final product.

Apart from the exogenously assigned market power to one or two firms, a common element of this group of papers is that the source of market power of the dominant firm(s) is the initial allocation of permits. Moreover, this initial allocation is granted for free to firms and, thus, exogenous. However, surveying the theoretical literature on market power in emissions permits markets, Montero (2009) brings into attention the case in which the permits are auctioned off instead of being freely allocated to firms in a framework where a single large firm is exercising market power. He conducts a graphical analysis of the dominant firm’s strategy under the assumption of a sealed-bid uniform price auction and concludes that the optimal strategy for the large firm is to bid an empty schedule and buy the permits from the fringe in the after-auction market. Nevertheless, nothing is said about the after-market price.

The second group of papers is related to the endogenous market power. Under this approach of modeling market power it is assumed that all players, buyers and sellers, have the potential to exercise it through the manipulation of the demand and supply schedules submitted to the market maker. Along this line, Weretka (2011) provides a general model of endogenous market power in bilateral trading, in an exchange economy with consumption. He sets up a model with two types of traders, consumers and producers characterized by utility and cost functions, respectively. The result of his model is two-fold. First, the trading volume in the case of the exercise of endogenous market power is lower than the one obtained in the perfect competition case, leading to Pareto inefficiency. Second, the sign on the price bias relative to the competitive case depends on the convexity of the utility and cost functions of the traders.

In the same vein, Lange (2012), assuming endogenous market power in an emissions trading market, recovers the efficiency condition from Hahn’s seminal paper: efficiency is reached if the players with market power (in this case all players) receive an initial allocation of permits equal to their efficient emissions level. Note that this paper assumes free allocation of the initial permit endowment.

Another paper which relaxes the assumption of exogenous market power is that of Malueg & Yates (2009a). In their model both the buyers and the sellers exercise market power in a so-called bilateral oligopoly framework. Their analysis is based on the supply function equilibria à la Klemperer & Meyer (1989) and the strategies of the players are the net-trade functions, i.e. the supply and the demand functions, respectively, boiling down to the choice of the intercept of the marginal abatement cost function. The one-dimensional strategy and the linearity of the net-trade function assures a closed-form solution for the equilibrium trading. They analyze both the case of full information and private information with respect to the intercept of the abatement cost function and find that private information attenuates the effect of the inefficiency driven by the exercise of market power as compared to the case of full information.

In these papers the endogenous market power arises from the initial allocation of permits, in
that firms with a high initial allocation have the ability to manipulate the price. My paper differs from the above-mentioned literature through the fact that the initial allocation is via auctioning. This approach raises the question of the propagation of the bidding strategies on the exercise of market power in the secondary market as well as the effect of the anticipation of this behavior on the auction clearing price. Under this set-up it is clear that the initial endowment cannot be the soul source of market power, since it is in itself endogenous. Therefore, in my model, the source of market power in both markets is the business-as-usual emissions level, which can be further interpreted as polluter’s technology.

Hence, the contribution of my paper is that of filling the gap between the two streams of literature discussed above. On the one hand, I relax the assumption of exogenously assigned market power and, on the other hand, I consider auctioning rather than grandfathering as the method of permits allocation in the primary market.

3 The Model

Consider an emissions trading scheme with \( N > 2 \) emitters indexed by \( i = 1, \ldots, N \). They are required to comply with the environmental regulations, such that the total emissions in the scheme should not exceed a fixed emissions target. This target is equivalent to a fixed supply of emissions permits, \( \bar{E} \), which is exogenously determined by the regulatory authority. The regulator distributes the permits to the polluters via a uniform price sealed-bid auction. Let \( D_i \geq 0 \) denote the number of permits earned by polluter \( i \) in the auction, such that \( \sum_{i=1}^{N} D_i = \bar{E} \). For compliance, the emitters can trade permits in a secondary market, and they can abate emissions at a quadratic cost, \(^4 c(r_i) = \theta r_i^2 \), where \( r_i \) is the amount of emissions reductions from the business-as-usual emissions level. In this model the heterogeneity of the polluters consists of their business-as-usual emissions level, \( e_i > 0 \). Without any loss of generality, let \( e_1 \geq e_2 \geq \cdots \geq e_N \) and \( \bar{e}_k = \frac{1}{k} \sum_{i=1}^{k} e_i \) be the average level of business-as-usual emissions of the \( k \) largest emitter in the economy. Further, \( e_i \) can be interpreted as the level of production, represented by the corresponding emissions level, which polluter \( i \) would produce as a solution to her profit maximization problem, given her technology and the market price of her final output, in the absence of the regulation. For the purpose of this paper I assume that this level is exogenously given.\(^5 \) Endogenizing \( e_i \) as a result of a production decision does not change the qualitative results of the following analysis, as long as the polluters are assumed to be price-takers in their output markets. Therefore, for the sake of mathematical neatness I maintain the business-as-usual emissions as an exogenous parameter.

The timing of the game is the following. In the first stage the polluters decide their bids in the uniform price sealed bid auction, choosing their bidding function \( D_i(p_1) \), where \( p_1 \) is the auction price. In the second stage they decide how much to trade in the secondary market, choosing the supply/demand schedule \( t_i(p_2) \), where \( p_2 \) is the secondary market price. The decision criterium

\(^4\)The choice of a particular functional form for the abatement cost function is imposed by the desire of having a complete close form solution of the game.

\(^5\)For example, Quirion (2005) define the abatement cost as a function of the final emissions rate and the exogenous business-as-usual emissions.
of polluter $i$ is the minimization of the emissions cost,\footnote{Note that $r_i \leq e_i$ should also hold. However, this is omitted, as it becomes redundant through the second and the third constraint.}

$$C_i = \theta r_i^2 + p_2 t_i + p_1 D_i, \quad (1)$$

under the constraints:

$$r_i \geq 0 \quad \quad \quad t_i \geq -D_i \quad \quad \quad e_i = D_i + r_i + t_i, \quad (2)$$

where $t_i$ is the net purchase ($t_i > 0$) or net sale ($t_i < 0$) of permits in the market following the initial distribution. The first constraint in (2) assures that the amount is not negative, i.e. there is no possibility of disinvestment in emissions reductions, while the second constraint says that the polluter cannot sell in the secondary market more permits than it owns. The last constraint in (2) is the emissions compliance constraint. Note that due to this constraint, polluter’s trading decision trivially determines her abatement decision, making the abatement decision stage redundant in the set-up of this game.

Since the emissions constraint is binding, it can be used to determine the level of abatement, which can be then substituted in (1) and (2), producing the following constrained decision criterion:

$$\min_{D_i, t_i} C_i = \theta (e_i - D_i - t_i)^2 + p_2 t_i + p_1 D_i,$$

$$e_i - D_i - t_i \geq 0 \quad \quad \quad t_i \geq -D_i \quad \quad \quad (3)$$

Before proceeding to finding the equilibrium, two important assumptions are in place in order to ensure that the game has solution. First, Assumption 3.1 introduces the scarcity of permits, which assures that there is a price for permits. In particular, this assumption guarantees that the first constraint in problem (3) is satisfied.

**Assumption 3.1** There is a market for permits, i.e. the total supply of permits is lower than the total business-as-usual emissions in the scheme: $E < \sum_{i=1}^{N} e_i$.

Second, due to the additive nature of the emissions constraint, the following assumption is sufficient in order for the second constraint in problem (3) to hold for all polluters in the scheme.

**Assumption 3.2** For any polluter $i$, $e_i \geq \bar{e}_N - \frac{E}{N}$.

The interpretation of Assumption 3.2 is that the small emitters (i.e. $e_i < \bar{e}_N$) are not too small. In other words, their business-as-usual emissions are not too far from the average business-as-usual emissions in the scheme. Precisely, the distance from the average emissions of the small emitters ($i$ such that $e_i < \bar{e}_N$) is bounded by the average number of permits in the scheme, i.e. $\bar{E}/N$. However, the big emitters ($i$ such that $e_i \geq \bar{e}_N$) do not obey any restriction regarding how far from the average business-as-usual emissions they can be, since the inequality in Assumption 3.2 holds true for any such emitter. Therefore, Assumption 3.2 puts a restriction only on the
small emitters, such that the variance of the business-as-usual emissions in the scheme can be driven down only by the small emitters. In essence, recalling the ordering of the business-as-usual emissions, for Assumption 3.2 to hold, it is enough to say that the distance between the business-as-usual emissions of the smallest emitter and the average emissions in the scheme, is bounded by the average number of permits available in the scheme, or that the total abatement in the scheme is lower than $N$ times the business-as-usual emissions of the smallest emitter.

While this assumption may appear to be restrictive, it has a plausible realistic interpretation. For example, in the EU ETS, the definition of the installations covered by the scheme, which can be found in Annex 1 of the EU Directive, involves a threshold of minimum capacity, which implicitly defines a threshold for emissions (European Commission 2003). Similarly, in the cap-and-trade scheme of the state of California, an entity becomes regulated if its emissions exceeded 25,000 metric tonnes in any year from 2008 to 2011.

If Assumption 3.2 does not hold for a certain emitter, that would be the case of a polluter which short sells permits, i.e. sells more permits in the secondary market than it currently owns. However, since in this model there is only one round of trading, this case cannot be accommodated because polluters have to close their position. Specifically, for the case of efficient initial allocation, which means that no secondary market trading is needed, the violation of Assumption 3.2 amounts to the polluter being endowed with a negative number of permits.

Moreover, as it will be seen further for the case of strategic behavior, the violation of Assumption 3.2 leads to the situation in which the polluter abates more than her business-as-usual emissions. This would be equivalent with the emitter selling abatement. This case has a plausible interpretation in the context of the EU ETS where the emitters can gain credits (also called offsets) through Joint Implementation and Clean Development Mechanism projects, which they can use for compliance against their own emission or sell on a secondary market. For these credits there is a separate market on which they are traded. Bringing this possibility into the discussion would imply that the market for these credits should be included into the model. However, the analysis of this mechanism is out of the scope of this paper.

With Assumptions 3.1 and 3.2 at hand, I further focus on the interior equilibrium of the game. (The corner solutions obtained from the Kuhn-Tucker conditions for the constrained problem (3) in the case of price-taking behavior are discussed in Appendix A.) Thus, the cost function with constraints from (3) can now be written as an unconstrained cost function:

$$C_i = \theta(e_i - D_i - t_i)^2 + p_2t_i + p_1D_i, \quad (4)$$

This set-up allows for the identification of the actual abatement levels, which is not clear from the set-up of Malueg & Yates (2009a). Although they identify the non-cost effectiveness in abatement decisions coming from the strategic behavior of the polluters, they do not make any analyses concerning the direction of the distribution of the burden of the emissions reductions, as they model the abatement cost as a function of emissions rather than as a function of the abatement.

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7These credits are called Emission Reduction Units (ERUs) and Certified Emission Reduction units (CERs), respectively, and they are two of the three flexible mechanisms of the Kyoto Protocol, together with Emissions Trading, designed to help the so-called Annex I countries of this protocol to meet their greenhouse gas emissions targets.

8In the following sections I will use superscript $c$ and $s$ to refer to the price-taking and strategic behavior variables, respectively.

8
level itself. In addition, my model incorporates the auction as the method of distribution of the initial endowment of permits, allowing thus to assess the propagation of the anticipated strategies on the secondary market to the auction outcome.

This model builds on the fact that there is no reason to \textit{a priori} assume that one emitter or the other has market power, but rather that all participating agents recognize their ability to influence prices. The equilibrium concept used for both markets is the supply function equilibria (SFE). Notice the bilateral nature of the trade in the secondary market, where there are both sellers and buyers among the polluters, as compared to the unilateral nature of the primary market. The former market structure was coined by Hendricks & McAfee (2010) as ”bilateral oligopoly”, while the latter is the dual problem encountered in the context of auctions in electricity markets à la Green (1999), Rudkevich (2005) and Rudkevich (1999).

As this is a sequential game, the solution method is backward induction, starting from the trading stage. For reference and as a benchmark, the price-taking solution will be considered first.

## 4 Price-taking Behavior

In this case, all polluters choose their trading volume and auction bids, taking prices as given.

### 4.1 Secondary market trade

Minimizing

\[
C_i^c = \theta(e_i - D_i^c - t_i^c)^2 + p_i^c t_i^c + p_i^c D_i^c,
\]

with respect to \(t_i^c\), the net-trade function of polluter \(i\) is given by:

\[
t_i^c(p_2^c) = e_i - D_i^c - \frac{1}{2\theta} p_2^c.
\]

Equation (6) gives the net supply (demand) of polluter \(i\) as a function of the market price. The market mechanism chooses \(p_2^c\) such that the excess demand is zero:

\[
\sum_{i=1}^{N} t_i^c(p_2^c) = 0.
\]

Assuming that the whole supply of permits is distributed in the auction, i.e. \(\sum_{i=1}^{N} D_i = \bar{E}\), this provides the secondary market price:

\[
p_2^c = 2\theta \left( \bar{e}_N - \frac{\bar{E}}{N} \right),
\]

which essentially says that the price of permits equals the marginal cost of abatement in the scheme. Assumption 3.1 assures that \(p_2^c\) is positive. Substituting (7) in (6), the equilibrium trading volume is obtained:\footnote{Note the distinction between the trade function \(t_i^c(p_2)\) given by (6) and the equilibrium trading volume \(t_i^c\) given by (8).}

\[
t_i^c = e_i - D_i^c - \bar{e}_N + \frac{\bar{E}}{N}.
\]
Equation (8) easily identifies the abatement level of each polluter $i$, that is

$$r_i^c = \bar{e}_N - \frac{E}{N} = \frac{p_i^c}{2\theta}. \quad (9)$$

Again, by Assumption (3.1), the amount of abatement is positive for any polluter $i$. Note that the burden of abatement is equally split among the polluters. Moreover, this is the usual cost effective solution, i.e. the marginal abatement costs are equal among themselves and to the permit price. Given that the polluters have the same abatement cost, the fact that they abate in equal amounts is not surprising.

### 4.2 Initial allocation

At this stage the initial allocation $D_i^c$ is determined. I assume that the initial distribution takes place via a uniform price sealed-bid auction where the total number, $E$, of permits is distributed to the members of the emissions trading scheme. Thus, participants submit demand schedules $D_i^c(p_i^c)$ to a market mechanism which finds the clearing price at the point where the horizontal summation of these demand schedules equates the total supply of permits.

Substituting (6) and (7) in the cost function (4) and minimizing the latter with respect to $D_i^c$, it obtains that the prices of the two markets should be equal, i.e. $p_1^c = p_2^c$. That means that the polluters are indifferent between buying the permits in the auction and buying them in the secondary market. Note that this is a result that hinges on the risk neutrality assumed in this model.

One way of initial distribution of the permits is by random assignment. In this case, any reallocation which takes place through the secondary market does not involve any wealth re-distribution from one polluter to the other. Alternatively, the permits can be distributed proportionally to the business-as-usual emissions, i.e. $D_i^c = e_i$. Another distribution method is such that the net-trade for each polluter is zero, i.e. $t_i^c = 0$. This yields the initial distribution:

$$D_i^c = \frac{E}{N} + (e_i - \bar{e}_N), \quad (10)$$

which is positive by Assumption 3.2. This allocation coincides with the efficient allocation, since it does not need any re-allocation via the secondary market. In other words, this rule of initial allocation leads to $t_i^c = 0, \forall i$. The last two terms in (10) reflects polluters’ asymmetry with respect to their business-as-usual emissions. Note that if the polluters were symmetric, i.e. $e_i = e_j, \forall i \neq j$ the efficient distribution has the total number of permits being evenly distributed among the $N$ polluters. In what follows I shall assume that equation (10) is the allocation rule for the price-taking case.

### 5 Strategic Behavior

In the strategic case the polluters recognize and use their ability to influence the prices for permits, both in the primary (the auction) and the secondary market. However, the strategic behavior is exercised differently in the two markets. In the auction, which is the first stage of the game, all
polluters are buyers and the supply of permits is fixed. Thus, all polluters form strategic demand schedules and the market power is unilateral, since the regulator is not an active player in this model. In the secondary market, which is the second stage of the game, the polluters act both on the supply and on the demand side of the market for permits. This makes the market power bilateral.

For tractability and keeping the comparability with the price-taking case, I will further focus on the family of linear strategies in both markets.

Specifically, in the primary market, polluters’ strategies consist of the bids they submit to the regulator in a uniform price sealed-bid auction. The polluters decide on their linear bidding schedules by choosing the price, acting as monopsonists on the residual supply of permits. The role of the regulator is that of a market mechanism, which clears the market by equating the fixed supply of permits with the total demand resulted from the aggregation of the individual demand schedules submitted by the polluters.

Regarding the secondary market, the competitive trade function in (6) shows that the polluters are heterogeneous only with respect to the intercept of this function, as a result of the assumption of symmetry in the abatement cost. Therefore, for the strategic behavior case, I will assume that polluters exercise market power by choosing the intercept of their net-trade function. One could also assume that the polluters use both the intercept and the slope of this function as their strategy. However, due to the symmetry in the abatement cost, the slopes will be equal among polluters. Thus, allowing for ”full” market power would affect the results only quantitatively, without adding any qualitative insights. Moreover, the secondary market price will be the same, regardless of assuming that the polluters choose both the intercept and the slope, or only the intercept (see Appendix B for details).

The solution method for finding the strategic bids in the auction and the strategic net-trade functions in the secondary market is the standard one used in the supply-function equilibria literature (Klemperer & Meyer 1989, Rudkevich 2005, Green 1999, Baldick et al. 2000)). Again, the model is solved by backward induction. However, before turning to the solution of the strategic behavior, one additional assumption is required in order to ensure that the first constraint in problem (3) is satisfied. For this, it suffices to tighten Assumption 3.1 as follows:

**Assumption 5.1** The total supply of permits is lower than one fraction of the total business-as-usual emissions in the scheme: $\bar{E} \leq \frac{N(N-1)}{N(N-1) + 1} \left( \sum_{i=1}^{N} e_i \right)$.

Note that this assumption encompasses Assumption 3.1, since $\frac{N(N-1)}{N(N-1) + 1} < 1$. Hence, strategic behavior under the setup of this paper is feasible if the emissions cap is tight enough. However, when $N$ becomes large, $\frac{N(N-1)}{N(N-1) + 1}$ converges to 1 such that this assumption becomes equivalent with Assumption 3.1. Therefore, Assumption 5.1 is not very restrictive.

### 5.1 Secondary market trade

As already explained, for consistency and comparability with the trade function in (6), let

$$t^*_i(p^*_2) = a_i - \frac{1}{2\theta}p^*_2,$$  \hspace{1cm} (11)
be the net trade function of polluter $i$, where the intercept $a_i$ is her strategy, chosen such that to minimize the emissions cost function:

$$C_s^i = \theta(e_i - D_s^i - t_s^i)^2 + p_s^2 t_s^i + p_s^1 D_s^i,$$

(12)

where $t_s^i$ is given by equation (11) and $p_s^2$ is given by the market clearing condition $\sum_{i=1}^{N} t_s^i = 0$ as:

$$p_s^2 = \frac{2\theta}{N} \sum_{i=1}^{N} a_i.$$

(13)

Substituting (13) and (11) into (12) and writing the first order condition with respect to $a_i$, it gives the reaction function of polluter $i$:

$$a_i = \frac{1}{N^2 - 1} a_{-i} + \frac{N}{N + 1} (e_i - D_s^i)$$

(14)

where $a_{-i} = \sum_{j=1, j\neq i}^{N} a_j$. In Appendix C it is shown that

$$a_i = \frac{N - 1}{N} (e_i - D_s^i) + \frac{1}{N^2} \left( \sum_{i=1}^{N} e_i - \bar{E} \right)$$

(15)

Therefore, the equilibrium secondary market price for the strategic case is given by

$$p_s^2 = 2\theta \left( \bar{e}_N - \frac{\bar{E}}{N} \right)$$

(16)

and the net trade function is

$$t_s^i(p_s^2) = \frac{N - 1}{N} (e_i - D_s^i) + \frac{1}{N} \left( \bar{e}_N - \frac{\bar{E}}{N} \right) - \frac{1}{2\theta} p_s^2$$

(17)

with its equilibrium value given by

$$t_s^e = \frac{N - 1}{N} \left( e_i - D_s^e - \bar{e}_N + \frac{\bar{E}}{N} \right).$$

(18)

Hence, the secondary market equilibrium price is the same as in the perfect competition case (compare (16) with (7)) and it is not affected by the initial distribution of permits. As it has been pointed out by Matsukawa (n.d.), Lange (2012) and Weretka (2011), this result is a consequence of the form of the abatement cost function and of the symmetry assumption with respect to the slope of this cost function. Intuitively, this is due to the fact that the strategic behavior on the side of the buyers and the sellers cancel out, i.e. the buyers and the sellers have the same price impact. While the buyers pretend to be needing fewer permits for the same price in order to drive down the equilibrium price, the sellers decrease their supply in the attempt to drive up the price. Due to the symmetry in the abatement costs, the polluters only shift their intercepts relative to the price-taking case; moreover, they do it proportionally. Thus, the market clears and the price remains unchanged. However, as Weretka (2011) shows, this is not the case when the buyers' and

10 Although this model assumes a specific functional form for the abatement cost function, Weretka (2011) shows that this result holds for any payoff function with constant second derivative.
the sellers’ utility functions have different convexities. Precisely, the market would clear the favor of the polluter with a smaller convexity of the utility function, or a flatter marginal utility.

Further, the abatement level of polluter \( i \) can also be calculated:

\[
 r_i^* = \frac{1}{N} \left( e_i - D_i^* + (N-1) \left( \bar{c}_N - \frac{\overline{E}}{N} \right) \right) = \frac{1}{N} (e_i - D_i^*) + \frac{N-1}{N} r_i^c \tag{19}
\]

Unlike the perfect competition case where the abatement level was split evenly across the participants, in the strategic behavior case the abatement level is individualized. More specifically, it depends both on the business-as-usual emissions level and on the initial endowment of permits.

### 5.2 Initial allocation

Taking as given the equilibrium values for \( t_i^* \) and \( p_i^* \), the objective function of a polluter is to minimize a quadratic function in \( D_i^* \):

\[
 C_i^*(D_i^*) = \frac{\theta}{N^2} (D_i^*)^2 - (\alpha_i - p_i)D_i^* - \beta_i, \tag{20}
\]

with \( \alpha_i = \frac{2\theta}{N^2} e_i + \frac{2\theta(\sum_{j=1}^N e_j - \overline{E}) (N^2 - 1)}{N^3} \) and \( \beta_i = -\frac{\theta}{N^2} e_i^2 + \frac{\theta(\sum_{j=1}^N e_j - \overline{E}) (N^2 - 1) (-2N e_i + \sum_{j=1}^N e_j - \overline{E})}{N^4} \).

Note that the cost functions \( C_i^* \) differ only with respect to the coefficient of the linear term \( \alpha_i \) and the intercept \( \beta_i \). Since the intercept of the cost function, \( \beta_i \), is irrelevant for the optimal bidding strategies of the polluters, I omit the discussion of this parameter. However, the coefficients of the linear term in the \( C_i^* \) functions, \( \alpha_i \)'s, only differ with respect to \( e_i \). This implies that the equilibrium bids are asymmetric and the asymmetry depends only on the differences in \( e_i \).

Consequently, polluters’ valuations for permits only differ with respect to \( e_i \), and the relationship is positive: \( \partial \alpha_i / \partial e_i > 0 \). Thus, \( e_i \), which is the only dimension of heterogeneity, is responsible for bidders’ aggressiveness in the auctioning stage of the game.

In order to find the strategic bids, the SFE method is applied. That means that each bidder will act as a monopsonist, choosing the price on the residual supply. Hence, bidder \( i \) will solve the following problem:

\[
 \min_{p_i^*} \left( \frac{\theta}{N^2} D_i^*(p_i^*)^2 - (\alpha_i - p_i^*)D_i^*(p_i^*) \right) \text{ such that } D_i^*(p_i^*) = \overline{E} - D_{-i}^*(p_i^*), \tag{21}
\]

where \( D_{-i}^*(p_i^*) = \sum_{j \neq i} D_j^*(p_i^*) \).

This boils down to solving the following differential equation:

\[
 -\frac{\theta}{N^2} D_i^*(p_i^*) (D_{-i}^*)' (p_i^*) + (\alpha_i - p_i^*) (D_{-i}^*)' (p_i^*) + D_i^*(p_i^*) = 0, \tag{22}
\]

As it is usual in the literature, \( i \) will focus on the family of linear equilibria of the form \( D_i^* (p_i^*) = x_i - y_i p_i^* \), with \( x_i, y_i \geq 0 \). If all the other polluters use the same linear strategies, that is \( D_j^* (p_i^*) = \]

\[^{11}\]The constant \( \beta_i \) was ignored.

\[^{12}\]See for example Green (1999), Rudkevich (2005), Baldick et al. (2000)
\( x_j - y_j p_i^* \) for all \( j \neq i \), then for each polluter \( i \), equation (22) becomes:

\[
\frac{2\theta}{N^2} x_i \sum_{j=1, j \neq i}^{N} y_j - \alpha_i \sum_{j=1, j \neq i}^{N} y_j + x_i - \left( \frac{2\theta}{N^2} y_i \sum_{j=1, j \neq i}^{N} y_j - \sum_{j=1, j \neq i}^{N} y_j + y_i \right) p_i^* = 0, \tag{23}
\]

which has to hold for any \( p_i^* \). Therefore, the following system of equations characterizes the parameters \( x_i \) and \( y_i \):

\[
\begin{align*}
\frac{2\theta}{N^2} x_i \sum_{j=1, j \neq i}^{N} y_j - \alpha_i \sum_{j=1, j \neq i}^{N} y_j + x_i &= 0 \\
\frac{2\theta}{N^2} y_i \sum_{j=1, j \neq i}^{N} y_j - \sum_{j=1, j \neq i}^{N} y_j - y_i &= 0.
\end{align*}
\tag{24}
\]

In Appendix D it is shown that \( x_i = \alpha_i y_i \), for any \( i \) and \( y_i = \frac{N^2}{2\theta} \frac{N-2}{N-1} \) for all \( i \).\(^{13}\) Hence, the bid function of any bidder \( i \) is the piece-wise linear function\(^{14}\)

\[
D_i^*(p_i^*) = \begin{cases} 
\frac{N^2}{2\theta} \frac{N-2}{N-1} (\alpha_i - p_i^*) & \text{if } \alpha_i > p_i^* \\
0 & \text{if } \alpha_i \leq p_i^*.
\end{cases}
\tag{25}
\]

As equation (25) shows, the parameter \( \alpha_i \) is the maximum willingness to pay for the first permit of bidder \( i \). Hence, a simple comparative statics reveals that the higher the number of permits available, the lower the maximum willingness to pay (\( \partial \alpha_i / \partial N < 0 \)) and the higher the abatement cost, the higher the maximum willingness to pay for the permits (\( \partial \alpha_i / \partial \theta > 0 \)), which are both intuitive results. Finally, accounting for the relationship between \( \alpha_i \) and \( e_i \) mentioned above, equation (25) shows that the higher the business as usual emissions \( e_i \) of polluter \( i \), the more aggressive its bidding behavior in the primary auction.

Assuming that there exists a positive equilibrium price \( p_i^* \), such that \( \sum_{i=1}^{N} D_i^*(p_i^*) = E \), then

\[
p_i^* = \frac{1}{n} \sum_{i=1}^{n} \alpha_i - \frac{2\theta}{nN^2} \frac{N-1}{N-2} E,
\tag{26}
\]

where \( n \leq N \) is the number of polluters for which the demand is positive in the point where the fixed supply \( E \) is crossing the total demand. In other words, \( n \) is the number of winning bidders, that is those polluters that have the strongest bids in the auction.

Recall that bidding aggressiveness only differs with respect to the business-as-usual emissions, \( e_i \). Thus, given the assumed ordering of the polluters relative to \( e_i \), the bidders indexed from \( 1 \) to \( n \) are the winning bidders in the auction and those indexed from \( n+1, \ldots, N \) have zero initial endowments of permits. Obviously, \( n \) may coincide with \( N \) and then all bidders are successful in the primary auction. Formally, the number of successful bidders can be defined as

\[
n = \sup \left\{ n \in \{1, 2, \ldots, N\} \mid e_n > \frac{\sum_{j=1}^{n} e_j}{n} - \frac{E}{n} \frac{N-1}{N-2} \right\}.
\tag{27}
\]

\(^{13}\)Note that \( y_i = 0 \) is also an equilibrium, which means that all firms bid an empty schedule. However, this equilibrium is discarded from the analysis since it implies that the auction would be canceled.

\(^{14}\)Note the factor \( \frac{N^2}{2\theta} < 1 \) which captures the usual bid-shading property of the sealed-bid uniform price auction.
Using the previous notation where $\bar{e}_n$ is the average of the business-as-usual emissions of the first $n$ polluters awarded with permits in the auction and substituting the auction clearing price given by (26) into (25), the initial allocation for each emitter can be calculated. Thus, the first $n$ bidders receive

$$D^s_i = \frac{E}{n} + \frac{N-2}{N-1} (e_i - \bar{e}_n)$$ \hspace{1cm} (28)

permits each and the last $N-n$ bidders receive 0 permits.

Equation (28) shows that in a strategic environment with common knowledge, for any number $N > 2$ of auction participants, in the class of winning bidders, those with over (under) average business-as-usual emissions are rewarded (punished) relative to the symmetric equilibrium $\frac{E}{n}$.

### 5.3 Equilibrium

In Appendix E it is shown that the only plausible equilibrium is that in which all polluters earn a positive number of permits in the auctioning stage, i.e $n = N$. Therefore, accounting for the expression of $\alpha_i$ in equation (26), the final expression for $p^s_i$ is given by

$$p^s_i = p^s_2 - \frac{2\theta}{N^3(N-2)} E,$$

which is positive for any $N > 2$.

Farther, the equilibrium values of the initial permit endowment, the net trade and the amount of abatement can be calculated. Substituting $N$ for $n$ in (28), the initial endowment of permits for polluter $i = 1, \ldots, N$ is

$$D^*_{t_i} = \frac{E}{N} + \frac{N-2}{N-1} (e_i - \bar{e}_N),$$ \hspace{1cm} (30)

Next, the equilibrium net trade in the secondary market is given by

$$t^s_i = \frac{1}{N} (e_i - \bar{e}_N).$$ \hspace{1cm} (31)

Figure 1 in Appendix H illustrates the trading positions according to the business-as-usual emissions, as suggested by equation (31). Thus, those polluters with the business-as-usual emissions higher than the average emissions, $\bar{e}_N$, are net buyers in the secondary market and those with business-as-usual emissions smaller than $\bar{e}_N$ are net sellers in the secondary market. In other words, the anticipation of the strategic behavior in the secondary market prevents the high emitters from acquiring the desired level of permits, and part of these permits is ripped off by the small emitters. Note that the average emitter is not trading.

Finally, the equilibrium abatement level is

$$r^s_i = r^c_i + \frac{1}{N(N-1)} (e_i - \bar{e}_N)$$ \hspace{1cm} (32)

### 5.4 Discussion

It is now relevant to compare the prices of the two markets in order to assess the potential of gains from the price difference in the two markets for emissions permits. Following (29), the spread
between the secondary market price and the auction clearing price is given by

\[ p^*_2 - p^*_1 = \frac{2\theta}{N^3(N - 2)E}, \] (33)

which is obviously positive. Thus, strategic behavior benefits the low emitters, who make profits from the price difference, and it hurts the high emitters, who have to pay a higher price for supplementing their permits holdings. In the limit, as the number of polluters grows, the spread converges to zero, and they become indifferent between purchasing permits in the primary auction or in the secondary market.

Interestingly, the price spread increases both in the abatement cost parameter \( \theta \) and in the total number of permits supplied \( E \). This result is due to the different rates at which the two prices increase in \( \theta \) and decrease in \( E \). Specifically, \( p^*_2 \) increases in \( \theta \) at a higher rate than \( p^*_1 \) does, and it decreases in \( E \) at a lower rate than \( p^*_1 \) does, as a result of the direct versus indirect impact of these parameters on the two prices. Precisely, \( E \) directly affects the auctioning price because the fixed supply of permits is taken into account in a direct manner in the bidding strategies of the polluters. Moreover, the purpose of the auction is that of distributing \( E \) and thus it is natural that this parameter affects \( p^*_1 \) more. Conversely, \( \theta \) affects more the post-auction trading decisions and thus the secondary market price. This is because at this stage polluters have to close their positions by either buying/selling or abating. Therefore, the abatement cost directly affects their trading decisions. Note, however, that the price spread is independent of the business-as-usual emissions in the scheme.

With this in place the following proposition can be established:

**Proposition 5.2** In an emissions trading scheme with a sealed-bid uniform price auction as initial allocation method, polluters’ strategic behavior both in the auction and in the secondary market leads to a spot market price above the auction clearing price.

The result in Proposition 5.2 is intuitive. Since the regulator does not have any strategic role in this model, she does not counteract the polluters market power at the auction stage. Hence, this is a unilateral market on which all polluters act in the same direction, of depressing the clearing price. However, the secondary market is a bilateral market, as both the buyers and the sellers exercise market power. Therefore, since their interests diverge, there is less power for the players to drive the price in a given direction. In fact, as it was seen in Section 5.1, their market power cancels out and this price equals the competitive price.

Empirical evidence also support the result in Proposition 5.2. For example, Smith & Swierzbinski (2007) found that the price of the auction in the UK Emissions Trading Scheme, which was conducted using a descending clock format, was considerably above the secondary market price of the allowances. However, in the auction of the UK scheme, the regulatory body (the auctioneer) played the role of the buyer of abatement commitments from firms, as opposed to the one of the seller of emissions permits, as is the case in the model of the current paper.

A simple comparison between \( r^*_i \) and \( r^*_c \) reveals the direction of the inefficiency in the abatement activity. Precisely, combining equations (32) and (31), the difference between the strategic level
of abatement and its perfect competition counterpart is exactly equal to:

$$r_i^s - r_i^c = \frac{1}{N-1} t_i^s.$$  \hfill (34)

Hence, a net buyer ($t_i^s > 0$) will abate more than the efficient level $r_i^c$, when the polluters exercise market power. Conversely, a net seller ($t_i^s < 0$) will abate less than the efficient level. Thus, a non-competitive equilibrium will lead to the efficient level of abatement if and only if every polluter has no trading needs, that is her business-as-usual emissions coincide with the average level of the business-as-usual emissions in the scheme. Thus, equation (34) explicitly recovers the result identified by Lange (2012) in Proposition 3, using the relationship between the marginal abatement cost, the measure of market power and the permits’ price.

Let us now look at the initial allocations in the competitive versus strategic case and compare equations (10) and (30). From this comparison it is easy to see that the high emitters ($e_i > \bar{e}_N$) are under-allocated in the strategic case as compared to the price-taking case, while the low emitters ($e_i > \bar{e}_N$) are over-allocated in the strategic case in comparison with the efficient allocation.

It appears, thus, that the high emitters ($e_i \geq \bar{e}_N$) are punished twice due to the strategic behavior: in order to account for their emissions needs they have to, on the one hand, abate more than the efficient level and, on the other hand, purchase extra permits in the secondary market at a higher price than the auction clearing price. Intuitively, this result is due to the fact that the big polluters cannot credibly commit to bidding an empty schedule. Although relative to the true demands all polluters shade their bids in the same proportion, in absolute value the large polluters are forced to shade their bids more compared to the small polluters. Thus, they cannot earn sufficiently many permits at the auction and they must buy the extra permits from the small players, at a higher price. However, the secondary market trade is not able to fully restore the efficient allocation of permits due to strategic behavior, which is present also on this market. Thus, the large polluters have to abate more in order to meet their compliance needs.

6 Welfare analysis

With the equilibrium values from the strategic case one can calculate the social cost of strategic behavior, defined as the sum of the individual compliance costs. Thus, plugging the equilibrium values of $D_i^s$, $t_i^s$, $p_1^s$ and $p_2^s$ into the cost function from (4), the social cost reads:

$$\sum_{i=1}^{N} C_i^s = \frac{\theta}{N(N-1)^2} Var(e) + \theta N \left( \bar{e}_N^2 - \frac{\bar{e}^2}{N^2} \right) - \frac{2\theta \bar{e}^2}{(N-2)N^3}$$  \hfill (35)

Equations (35) shows that the higher the heterogeneity of the polluters ($Var(e)$), the higher the total cost of compliance when polluters act strategically. This result can be summarized in the following proposition:

**Proposition 6.1** The social cost of compliance with the environmental regulations when polluters act strategically is increasing in the variance of the business-as-usual emissions.

To see the intuition behind Proposition 6.1, let us first note that neither the auction clearing price, $p_1^s$, nor the secondary market price, $p_2^s$, depend on the variance of the distribution of the
business-as-usual emissions.\textsuperscript{15} Hence, for two business-as-usual emissions distributions with the same mean and different variances, these prices will be the same. Therefore, despite the fact that higher variability in the business-as-usual emissions leads to higher inefficiencies in the primary market and, thus, higher trading volume in the secondary market, in aggregation the costs related to these markets cancel out, since prices are the same. However, it is in the abatement cost where the variance of the distribution of the business-as-usual emissions matters. Recall that, for two different distributions of the business-as-usual emissions having the same mean, the total abatement in the scheme is constant and equal to \(\sum_{i=1}^{N} e_i - E\). To simplify the reasoning, let us suppose that polluter 1 is a low emitter and polluter 2 is a high emitter, i.e. \(e_1 < e_2\). The high emitter produces a higher level of abatement than the low emitter. Thus, the total abatement cost of a low emitter is lower than the total abatement cost of a high emitter. Therefore, if we reduce the business-as-usual emissions of the low emitter by an amount \(\epsilon\) and we increase the business-as-usual emissions of the high emitter by the same amount \(\epsilon\), the total cost of producing the given amount of abatement will increase.\textsuperscript{16}

This result is obviously driven by the inefficiencies in the abatement resulting from the strategic behavior. In turn, since in the price-taking setting the total abatement is shared evenly among polluters, the variance of the business-as-usual emissions does not play a role in the aggregate compliance cost of the scheme. This result is discussed below when I compare the welfare change from price-taking to strategic behavior.

Plugging the equilibrium values, both for the price-taking case and for the strategic case, into the cost function given by equation (4) and defining the individual loss from strategic behavior as \(\Delta C_i = C_i^s - C_i^c\), it follows that:

\[
\Delta C_i = \frac{\theta}{(N-1)^2}(t_s^i)^2 - \frac{2\theta E}{N^2(N-1)} t_s^i - \frac{2\theta E^2}{(N-2)N^4}
\]  
Equation (36), together with Assumption 3.2, reveals that all net sellers \((t_s^i < 0)\) and some of the net buyers (those who are relatively low-emitters, i.e. \(e_i \leq \bar{e}_N + \frac{N-1}{N} E \left[1 + \sqrt{1 + \frac{2}{N-2}}\right]\)) will benefit from strategic behavior, that is their total cost in the strategic behavior case is lower than the total cost in the price-taking case (see Appendix F for the details of this result).\textsuperscript{18} Moreover, if the business-as-usual emissions of the largest emitter \((e_1)\) are below \(\bar{e}_N + \frac{N-1}{N} E \left[1 + \sqrt{1 + \frac{2}{N-2}}\right]\), then all polluters are individually better off acting strategically. Hence, keeping the average emissions constant, this condition is equivalent to saying that the variance of the business-as-usual emissions would be reduced. In other words, there is an upper bound on the variance of the emissions below which all polluters are better off acting strategically.

Further, the social loss from strategic behavior, defined as the sum of the individual losses, is

\textsuperscript{15}This result is due to both the linearity of the net trade functions in the secondary market and of that of the bids in the primary auction.

\textsuperscript{16}Note that this change in the business-as-usual emissions increases the variance of the distribution of the business-as-usual emissions, keeping their mean constant.

\textsuperscript{17}This interpretation was suggested by the intuition given to Proposition 1 in Long & Soubeyran (1997)

\textsuperscript{18}To see this, it suffices to solve the inequality \(\Delta C_i < 0\), given by equation (36), as a second degree equation in \(t_s^i\).

\textsuperscript{19}Note that \(\bar{e}_N + \frac{N-1}{N} E \left[1 + \sqrt{1 + \frac{2}{N-2}}\right] \approx \bar{e}_N + 2E\). Therefore, this condition is likely to hold in a real ETS.
given by

\[ \sum_{i=1}^{N} \Delta C_i = \frac{\theta}{N(N-1)^2} Var(e) - \frac{2\theta E^2}{N^3(N-2)} \]  

(37)

With respect to equation (37), three things should be noted. First, it becomes apparent that the social loss from strategic behavior increases in the variability of the business-as-usual emissions. As in Proposition 6.1, this is the result of the inefficient distribution of the abatement burden over the set of polluters, driven by the strategic behavior. Particularly, when the players are identical with respect to their business-as-usual emissions, i.e. \( Var(e) = 0 \), strategic behavior is socially preferable. Intuitively, this is due to the fact that their concerted strategic effort resembles that of a monopoly, which they are able to exercise in the auctioning stage, decreasing the price relative to the one in the secondary market and relative to the price-taking case. Despite the fact that the outcome is the same as in the competitive case, i.e. no secondary market trade and even distribution of the permits in the auctioning stage, the strategic behavior leads to a lower auction clearing price than in the case of perfect competition, thus decreasing the total cost of compliance. Thus, the gain from strategic behavior is a trade-off between the price gain and polluters’ heterogeneity.

This result relates to Proposition 2 in Malueg & Yates (2009), who found that the difference in the aggregate costs between the strategic behavior and the price-taking behavior is proportional to the variation in the marginal abatement cost at the permits’ endowment. Therefore, they obtain that homogeneity leads to the same social cost regardless of the market behavior of the polluters. This is due to the fact that in their model the initial allocation is free of charge, i.e. the first market is missing. In my model, the difference in the two costs is driven by the spread created between the prices of the two markets as a result of the strategic behavior. Therefore, the model of my paper shows how the method of the initial allocation bears importance for the overall efficiency of the scheme, if emitters choose to act strategically.

Second, assuming that the variance of the business-as-usual stays constant while the number of polluters in the scheme grows, the social loss from strategic behavior converges to zero. This is consistent with the intuition, since a high number of market participants leads to the competitive outcome. Third, the social loss in (37) is a decreasing function of the amount of permits issued in the scheme. Moreover, there exists a lower bound of the total number of permits \( E \) above which the total cost from strategic behavior is lower than the total cost from the price taking behavior. Specifically, the following proposition can be established:

**Proposition 6.2** For a given distribution of the business-as-usual emissions, strategic behavior is socially preferable if and only if

\[ E > \frac{N}{N-1} \sqrt{\frac{N-2}{2} Var(e)}. \]  

(38)

The reason behind Proposition 6.2 is the spread created between the auction price and the secondary market price, which becomes larger as the fixed supply of permits increases (see equation (33)). Thus, Proposition 6.2 says that there is a threshold of the fixed supply above which the auction clearing price becomes so low that the strategic behavior results in a lower compliance cost of the scheme relative to the case of the price-taking behavior. Thus, there is a threshold of
$E$ above which the price gain is big enough to overcome the inefficiencies in the initial abatement resulted from the strategic behavior. Therefore, a large enough $E$ has the potential to create a low enough price in the auctioning stage, such that it provides incentive for strategic behavior. This result is missing in Malueg & Yates (2009a), since in their model the initial allocation is grandfathering rather than auctioning. Instead, in their model, the strategic behavior always leads to a higher aggregate abatement cost as compared to the non-strategic behavior. Hence, the method of initial allocation matters in a scheme where the polluters can exercise market power to the extent that the aggregate abatement cost is lower than in the case in which the polluters did not recognize their potential for strategic behavior.

Note that Assumptions 3.2 and 5.1 put a lower and an upper bound, respectively, on the fixed supply of permits in the scheme. Precisely,

$$N(\bar{e}_N - e_N) \leq E \leq \frac{N^2(N - 1)}{N(N - 1) + 1} \bar{e}_N,$$

(39)

where $e_N$ is the business-as-usual emissions level of the emitter with the smallest business-as-usual emissions. Therefore, whether the strategic behavior leads to a lower aggregate abatement cost than the price-taking behavior depends only on the number of emitters in the scheme, the average emissions and the business-as-usual emissions of the smallest emitter in the scheme.

For convenience, let us further introduce some notations:

$$E_{\text{min}} = N(\bar{e}_N - e_N)$$

(40)

$$E_{\text{max}} = \frac{N^2(N - 1)}{N(N - 1) + 1} \bar{e}_N$$

(41)

and

$$\hat{E}(\text{Var}(e)) = \frac{N}{N - 1} \sqrt{\frac{N - 2}{2} \text{Var}(e)}$$

(42)

It appears that, in this model, if the regulator wants to influence the strategic behavior of the polluters, it suffices to have information about the variance of the distribution of the business-as-usual emissions in the scheme. Hence, if the regulator cares about raising revenue from the auction, she should choose the level of $E$ such that to induce price-taking behavior which results in a higher auction clearing price than the strategic behavior does. Therefore, provided that she has perfect information about the variance of the distribution of the business-as-usual emissions, she should always choose a fixed supply of permits below the curve $\hat{E}(\text{Var}(e))$.

For analyzing the regions in which strategic behavior is socially preferable, it is enough to fix the number of emitters, $N$, their average emissions, $\bar{e}_N$ and the business-as-usual emissions level of the smallest emitter, $e_N$, which by (39) has to be larger than $\frac{1}{N(N - 1) + 1} \bar{e}_N$. Fixing these, equations (40) and (41) imply a band in which the supply or permits can move, i.e. $E_{\text{min}}$ and $E_{\text{max}}$ are also fixed. Let also $\text{Var}^{\text{min}}$ be the lower bound of the variance of $e_i$’s resulted by fixing $N$, $\bar{e}_N$ and $e_N$ and $\text{Var}^{\text{max}}$ the maximum of this variance. The lower and the upper bounds of the variance will obviously depend on the values of the remaining $(N - 1)$ business-as-usual emissions, such that their average is $\bar{e}_N$. Formally, for any triplet $(N, \bar{e}_N, e_N)$ such that $e_N \geq \frac{1}{N(N - 1) + 1} \bar{e}_N$, let $\text{Var}^{\text{min}} = \min_{\{e_1, \ldots, e_{N - 1}\}} \{\text{Var}(e)\}$ and $\text{Var}^{\text{max}} = \max_{\{e_1, \ldots, e_{N - 1}\}} \{\text{Var}(e)\}$. For the existence
of a region of $\mathcal{E}$ in which the scheme as a whole is better-off acting strategically, it must be that:

$$\hat{E}(\text{Var}(e)) \leq \mathcal{E}_{\text{max}}.$$  \hspace{1cm} (43) 

In Appendix G it is shown that inequality (43) holds for any $N > 2$ and for any choice of $e_i$’s, that is for any $\text{Var}(e)$. This implies that under the assumptions of this paper, the variance of the business as usual emissions is upper-bounded by a quantity which depends on the average emissions and on the number of emitters in the scheme. Let us denote the upper bound of the variance by $\text{Var}^{\text{max}}$. If we regard $\hat{E}$ as a function of $\text{Var}(e)$, this implies that $\hat{E}$ is defined on $\text{Var}^{\text{min}}$ to $\text{Var}^{\text{max}}$. In addition, as it can easily be seen in (42), $\hat{E}$ is an increasing function of $\text{Var}(e)$. Therefore, let us denote by $\hat{E}_{\text{min}}$ and $\hat{E}_{\text{max}}$ the values of $\hat{E}$ evaluated at $\text{Var}^{\text{min}}$ and $\text{Var}^{\text{max}}$, respectively.

With these notations at hand and with the result from Proposition 6.2 and inequality (43), let us further identify the regions on which strategic behavior is socially preferable, i.e. the aggregate abatement cost is lower in the case of strategic behavior. First, let us note that depending on the values of $\hat{E}_{\text{min}}$ and $\hat{E}_{\text{max}}$ relative to $\mathcal{E}_{\text{min}}$, which result from different choices of the triplet $(N, \bar{e}_N, e_N)$, three cases should be discussed:

(a) $\mathcal{E}_{\text{min}} \leq \hat{E}_{\text{min}}$

(b) $\hat{E}_{\text{min}} \leq \mathcal{E}_{\text{min}} \leq \hat{E}_{\text{max}}$

(c) $\hat{E}_{\text{max}} \leq \mathcal{E}_{\text{min}}$

Let us now consider each of these cases in turn and characterize the situations in which strategic behavior pays off, as a function of the fixed supply of permits and the variance of the business-as-usual emissions, which depends on the values of $e_i$’s, $i = 1, \ldots, N - 1$. Thus, Figures 2 to 4 in Appendix H identify the regions in which the social cost is lower in the case of strategic behavior (SB) and of price-taking behavior (PTB), respectively. Hence, for regions marked by SB the strategic behavior provides a lower social cost than the price-taking behavior. Similarly, for regions marked by PTB favor price-taking behavior. The thick lines in these graphs delineate the regions in which the scheme operates, in the space defined by $\mathcal{E}$ and $\text{Var}(e)$, for each of the cases (a) to (c).

First, Figure 2 illustrates case (a) above. In this case, if $\mathcal{E}$ is larger than $\hat{E}$ evaluated at the smallest variance of $e_i$’s, $i = 1, \ldots, N - 1$ after fixing the triplet $(N, \bar{e}_N, e_N)$, strategic behavior is more likely to be socially preferable if the variance is small. If, instead, the variance of the business-as-usual emissions is large, for the strategic behavior to pay off at the aggregate level, the fixed supply of permits must be very large.

In fact, a closer look at the condition (a) shows that this is a case of a scheme in which the smallest possible variance of the business-as-usual emissions is large enough. Therefore, as discussed before, for the strategic behavior to pay off, the price spread has to be large enough (large $\mathcal{E}$) to cancel the effect of the high heterogeneity which affects directly the total abatement cost of the scheme.

Case (b) is illustrated in Figure 3. For this case it is interesting to note that there is a region of the variance of $e_i$’s for which strategic behavior is socially preferable regardless of the value of $\mathcal{E}$ between $\mathcal{E}_{\text{min}}$ and $\mathcal{E}_{\text{max}}$. Condition (b) is equivalent with the maximum possible variance of
the business-as-usual emissions in the scheme, resulting from fixing the above-mentioned triplet, being above a certain threshold. However, this condition does not impose any restriction on the minimum possible variance resulting from fixing the same triplet. Therefore, there exists room for strategic behavior to be socially preferable even for small values of the fixed supply of permits, i.e. small price spread. Precisely, if the variance of the distribution of the business-as-usual emissions is low enough, strategic behavior always pays off and the regulator cannot influence this behavior through the choice of the number of permits in the scheme.

This result becomes stronger for case (c) illustrated in Figure 4. In this case, regardless of the value of $\bar{E}$ and of the variance resulted by fixing the triplet $(N, \bar{e}_N, e_N)$, strategic behavior is always socially preferable and the regulator has no room to influence this by adjusting the supply of permits in the scheme. This is because condition (c) puts an upper bound on the maximum possible variance that can result after fixing $N$, $\bar{e}_N$ and $e_N$. Thus, it is equivalent to saying that the firms are not too heterogeneous, so the aggregate abatement cost can be easily neutralized by the price spread created when polluters act strategically.

In sum, this analysis shows that only high heterogeneity among the polluters allows the regulator to manipulate their market behavior by reducing the emissions cap to induce competitive behavior. However, this analysis did not take into account the regulator as a profit maximizer, who could choose the emissions cap to maximize its revenue from auctioning permits. Therefore, at this point, it cannot be said which type of behavior would be preferred by the regulator. The comparison of the total compliance costs shows that there are emissions trading schemes, i.e. the one depicted in 4, for which strategic behavior is socially preferable.

### 7 Conclusion

This paper developed a full-information model of an emissions trading scheme in which all the participants in the scheme are allowed to exercise market power in the markets for emissions permits. The market power arises endogenously from polluter's business-as-usual emissions level, which is the sole source of heterogeneity in this model. The permits are allocated via a uniform price sealed-bid auction and this is followed by a secondary market in which the polluters can trade permits. The anticipation of the exercise of market power in the after-auction market influences the bidding strategies of the polluters when permits are initially allocated to the participants in the scheme. Consequently, the initial allocation is inefficient: the low emitters are awarded too many permits in the auction and the high emitters are awarded too few as compared to the non-strategic first-best. In addition, due to the strategic behavior, the high emitters abate more than in the non-strategic case, while the low-emitters abate less, and this depends on the individual level of the business-as-usual emissions level. While in the perfect competition case the secondary market is redundant as the permits are efficiently allocated right in the auctioning stage, in the strategic case there exists trade at a price which is unambiguously above the auction clearing price.

The welfare analysis reveals that the strategic behavior favors all net sellers (emitters with the business-as-usual emissions below the average) and some net buyers, namely those with the lowest business-as-usual emissions. At the individual level, the high emitters are punished twice by the collective exercise of market power: first because they have to buy the deficit of permits in the

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20 To check this one should solve for $\text{Var}(e)^{\text{max}}$ in the inequality from (c).
second, because they abate more than in the perfect competition case. At the aggregate level, the social loss from strategic behavior increases in the variability of the business-as-usual emissions and decreases in the supply of permits in the scheme. Therefore, the higher the supply of permits, the more likely for the strategic behavior to be cost-effective at aggregate level.

Moreover, there are emissions trading schemes for which strategic behavior is socially preferable regardless of the choice of the regulator with respect to the fixed supply of permits. In particular, such an emissions trading scheme is characterized by certain values of the parameters $N, \bar{e}_N, e_N$ such that the possible resulting distributions of the business-as-usual emissions have low variances. In fact, having fixed the domain of the supply of permits in the scheme, whether the regulator can influence or not the type of competition in the permits markets of this model, depends only on the variance of the distribution of the business-as-usual emissions.

This analysis, therefore, raises a few questions with regard to the emissions trading schemes in which initial allocation is auction (e.g. the third phase of the EU ETS, the California ETS). Given that the electricity producers have the highest needs for emissions, this model suggests that they may be the "biggest losers" of the future scheme, provided that the active part of the market is thin enough and all polluters act strategically. The second implication of this model is that the permission of non-polluters to bid for permits in the auction, which, for example, is already stipulated in the auction regulations of the EU ETS, is justified. Apart from the practical considerations of ensuring market liquidity, their presence may also have the role of enlarging the number of the market participants, avoiding thus the exercise of market power. However, whether this is socially preferable depends on the variance of the distribution of the business-as-usual emissions.

Further steps and extensions of this model are worth considering. First, modeling heterogeneity in the abatement costs appears to be a more realistic approach than homogeneity, although it poses analytical difficulties. This approach may turn out to be important in particular when resorting to quantitative evaluations of the effect of market power. Second, as in some ETSs the auction revenue is re-distributed to its members, it would be useful to investigate how this rule affects the outcome of the strategic behavior. Third, it is worth considering the regulator as a strategic player and model its decision on choosing the fixed supply of permits such that to maximize its revenue from the auction.

Appendix A: The Kuhn-Tucker conditions for the constrained problem

In order to apply the standard Kuhn-Tucker conditions, I transform problem (3) into a maximization problem. I solve it for the second stage of the game, when polluters make secondary market trading decisions, under the assumption that they behave as price-takers. This reads:

$$\max_{t_i} C_i = -\theta(e_i - D_i - t_i)^2 - p_2 t_i - p_1 D_i,$$

$$- e_i + D_i + t_i \leq 0$$

$$- t_i - D_i \leq 0,$$

where $D_i$ is taken as given.
Let $\lambda_i$ and $\mu_i$ be the Lagrange multipliers associated with the first and the second constraint, respectively. Then, the Lagrange function is:

$$\mathcal{L}(t_i, \lambda_i, \mu_i) = -\theta(e_i - D_i - t_i)^2 - p_2 t_i + \mu_i(t_i + D_i)$$

and the necessary conditions for a maximum are:

$$\begin{cases} 
2\theta(e_i - D_i - t_i) - p_2 - \lambda_i = 0 \\
e_i - D_i - t_i \geq 0; \quad \lambda_i \geq 0 \\
t_i + D_i \geq 0; \quad \mu_i \geq 0 \\
\lambda_i(e_i - D_i - t_i) = 0 \\
\mu_i(t_i + D_i) = 0
\end{cases} \quad (A.2)$$

**Case 1:** $\lambda_i > 0$ and $\mu_i = 0$.

This implies that $e_i - D_i - t_i = 0$ and $t_i = -D_i$. These provide $e_i = 0$. Thus, this is the case of a non-polluter. However, a non-polluter would not be part of the ETS, or she would be a speculator. Although introducing speculators into the model would be an interesting case to consider, I leave it for further research and I only focus here on polluters.

**Case 2:** $\lambda_i = 0$ and $\mu_i > 0$.

This amounts to $2\theta(e_i - D_i - t_i) - p_2 + \mu_i = 0$ and $t_i = -D_i$, from which it follows that $\mu_i = -2\theta e_i + p_2$. Since $\mu_i > 0$, then it must be that $2\theta e_i < p_2$. Hence, this is the case of a polluter whose marginal abatement cost at the level of her business-as-usual emissions is below the secondary market price. This type of ETS participant would sell in the secondary market all the permits she buys in the primary market ($t_i = -D_i$), and she would abate everything. While this may be a realistic case to consider, I leave these type of ETS participants out of the current analysis because they resemble again the case of a speculator. Since their supply of permits on the secondary market is inelastic, they have no effect on the price, i.e. their business-as-usual emissions would not enter the formula for the secondary market price.

**Case 3:** $\lambda_i > 0$ and $\mu_i = 0$.

This case obtains that $2\theta(e_i - D_i - t_i) - p_2 - \lambda_i = 0$ and $e_i - D_i - t_i = 0$. The latter equality is equivalent with the level of abatement being equal to 0. This further provides $\lambda_i = -p_2$. Since $p_2$ is positive, this would imply the $\lambda_i$ is negative, which is a contradiction.

**Case 4:** $\lambda_i = 0$ and $\mu_i = 0$.

This is the interior solution case in which both constraints are slack, resulting in

$$t_i = e_i - D_i - \frac{1}{2\theta}p_2.$$ 

This is the case considered throughout the paper.

**Appendix B: Strategic behavior with intercept and slope**

Let us focus on linear equilibria. Thus the net trade function for each trader $i$ is given by

$$t_i^*(p_2) = a_i - b_i p_2.$$ 

Each polluter chooses the price to minimize her compliance cost function $C_i^*(p_2) = \theta(e_i - D_i - t_i)^2 + p_2 t_i + p_1 D_i$ under the market clearing condition $t_i^*(p_2) + t_{-i}^*(p_2) = 0$, where $t_{-i}^*(p_2) = \sum_{j \neq i} t_j^*(p_2) = \sum_{j \neq i} a_j - (\sum_{i \neq j} b_j)p_2 = a_{-i} - b_{-i}p_2$. The first order condition with respect to $p_2$ reads:

$$2\theta(e_i - D_i - t_i^*) (t_{-i}^*(p_2))' + t_i^*(p_2) + p_2 (t_{-i}^*(p_2))' = 0$$

After grouping around $p_2$ it yields:

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\[-2\theta (e_i - D_i - a_i) b_{-i} + a_i + (-2\theta b_i b_{-i} - b_i + b_{-i}) p_2 = 0,\]

which has to hold for any \( p_2 \). Therefore, the following system of equations results, \( \forall i \):

\[
\begin{align*}
-2\theta (e_i - D_i - a_i) b_{-i} + a_i &= 0 \\
-2\theta b_i b_{-i} - b_i + b_{-i} &= 0
\end{align*}
\]

(A.3)

From the second equation of the system we have that \( b_i = \frac{b_{-i}}{2\theta b_{-i} + 1}, \forall i \). As Rudkevich (1999) showed in Lemma1, this system has a unique positive solution. Due to the polluters symmetry in the abatement cost, the solution of the system must be the symmetric one as well. Therefore,

\[
b_i = \frac{N - 2}{2\theta(N - 1)}, \forall i.
\]

Substituting it in the first equation of (A.3), the intercept of the net trade function is:

\[
a_i = \frac{N - 2}{N - 1} (e_i - D_i), \forall i.
\]

Finally, the net trade function for any polluter \( i \) is given by:

\[
t_i = \frac{N - 2}{N - 1} \left( e_i - D_i - \frac{1}{2\theta} p_2 \right)
\]

The market clearing condition \( \sum_{i=1}^{N} t_i = 0 \), provides the equilibrium price \( p_2 = 2\theta \left( \bar{e}_N - \frac{E}{N} \right) \).

**Appendix C: The derivation of the intercept of the net trade function in the strategic case**

Adding up equations (14) for all \( i \) it yields:

\[
\sum_{i=1}^{N} a_i = \frac{N}{N + 1} \left( \sum_{i=1}^{N} e_i - E \right) + \frac{1}{N + 1} \sum_{i=1}^{N} a_i,
\]

from which it immediately follows that \( \sum_{i=1}^{N} a_i = \sum_{i=1}^{N} e_i - E \). Therefore, in equation (14), \( \sum_{i=1}^{N} e_i - E - a_i \) can be substituted for \( a_{-i} \) and it yields:

\[
a_i = \frac{1}{N^2 - 1} \left( \sum_{i=1}^{N} e_i - E - a_i \right) + \frac{N}{N + 1} (e_i - D_i),
\]

from which solving for \( a_i \) it produces equation (15).
Appendix D: The coefficients of the bidding schedules for the strategic case

Solving for $\sum_{j=1, j\neq i}^N y_j$ from the first equation of system (24) it yields:

$$\sum_{j=1, j\neq i}^N y_j = \frac{x_i}{\alpha_i - \frac{2\theta}{N^2}} x_i$$

Substituting this into the second equation of the system it provides $y_i\alpha_i = x_i$. Therefore,

$$\sum_{j=1, j\neq i}^N y_j = \frac{y_i}{1 - \frac{2\theta}{N^2} y_i}.$$

Because polluters are symmetric in the marginal abatement costs, it follows that $y_i = y_j = y, \forall i \neq j$. Thus, substituting and solving for $y$ in the above equation, it obtains

$$y_i = \frac{N^2 - 2}{2\theta N - 1}, \forall i.$$

Appendix E: The equilibrium of the strategic case

In relation to the equilibrium, three important thresholds can now be defined. First, $A = (N-1)\bar{e}_N - (N-2)\bar{e}_n + (N-1)\bar{E} \left(\frac{1}{n} - \frac{1}{N}\right)$ defines polluter’s net trading position on the secondary market. In particular, if $e_i > A$ then the polluter is a net buyer, while polluters with $e_i < A$ are net sellers on the secondary market. Second, $B = \bar{e}_n - \bar{E} \left(\frac{N-1}{n} - \frac{N-2}{N}\right)$ is the threshold which determines whether a polluter is awarded permits in the auctioning stage. All polluters with business-as-usual emissions above this threshold receive permits $D_i > 0$ in the auction. Polluters with $e_i$ below $B$ are not awarded any permits in the initial allocation stage (see equation (27) which defines the successful bidders). Finally, let $C = \bar{e}_N - \frac{\bar{E}}{N}$ and according to Assumption 3.2, $e_i > C$ for all $i$. Thus, according to the business-as-usual emissions level, all polluters lie to the right of $C$. Moreover, under the Assumption 3.1, $C \geq 0$. Further, it is important to note that $A = (N-1)C - (N-2)B$.

Because $e_i \geq e_{i+1}$ for any $i$, it follows that $e_n \geq e_N$ for any $n \leq N$. Therefore, the relationship between $B$ and $C$ is ambiguous, which imposes the discussion of two possible equilibria: (i) one for the case in which $B > C$ and (ii) one for the case in which $B \leq C$. Let us consider each of them in turn.

i) $B > C$. From this it follows that $0 > (N-2)C - (N-2)B$. Further, $C > (N-1)C - (N-2)B$. Note that the right-hand side of this inequality is exactly $A$. Therefore, $C > A$. Hence, $B > C$ implies that $A < C < B$. Since by Assumption 3.2 all polluters’ business-as-usual emissions lie to the right of $C$, this equilibrium would imply that all emitters are net buyers in the secondary market. Obviously, this is not a plausible equilibrium.

(ii) If $B \leq C$, it also follows that $C \leq A$. Hence, $B \leq C \leq A$. Again, because by Assumption 3.2 all polluters lie to the right of $C$, this equilibrium amounts to all polluters being successful in

\footnote{From $B \leq C$ it follows that $0 \leq (N-2)C - (N-2)B$. Further, $C \leq (N-1)C - (N-2)B$. Note that the right-hand side of this inequality is exactly $A$. Hence, $C \leq A$.}
the auction. Recall from (27) that \( n \) designates the number of the winning bidders, that is the \( n \) heaviest polluters. Since we have shown that all polluters participating in the scheme are awarded permits in the auction, this is equivalent to \( n = N \). Finally, simple calculations show that \( A = \bar{e}_N \).

**Appendix F: The individual change in welfare**

The change in welfare favors the strategic behavior, i.e. \( \Delta C_i < 0 \), if and only if

\[
\frac{N-1}{N^2} E \left( 1 - \sqrt{1 + \frac{2}{N-2}} \right) \leq t_i^* \leq \frac{N-1}{N^2} E \left( 1 + \sqrt{1 + \frac{2}{N-2}} \right). \tag{A.4}
\]

Further, accounting for the value of \( t_i^* \), the double inequality in (A.4) reduces to

\[
\bar{e}_N - \frac{E}{N} + \frac{E}{N} \left( 1 - \frac{N-1}{N} \sqrt{1 + \frac{2}{N-2}} \right) \leq e_i \leq \bar{e}_N - \frac{E}{N} + \frac{E}{N} \left( 1 + \frac{N-1}{N} \sqrt{1 + \frac{2}{N-2}} \right) \tag{A.5}
\]

Since \( 1 - \frac{N-1}{N} \sqrt{1 + \frac{2}{N-2}} < 0 \), \( \forall N > 2 \) and by Assumption 3.2 \( \bar{e}_N - \frac{E}{N} \leq e_i, \forall i \) it follows that the first inequality in (A.5) holds for all \( i \). This means that the double inequality in equation (A.4) captures all net sellers and some net buyers, namely those with low business-as-usual emissions (see the second inequality in equation (A.5)).

**Appendix G: Proof of inequality (43)**

Using the well-known identity \( \text{Var}(e) = \frac{1}{N} \sum_{i=1}^{N} e_i^2 - \frac{1}{N^2} \left( \sum_{i=1}^{N} e_i \right)^2 \), inequality (43) becomes

\[
\frac{N(N-1)}{1 + N(N-1)} \left( \sum_{i=1}^{N} e_i \right) \geq \frac{N}{N-1} \sqrt{\frac{N-2}{2} \left( \frac{1}{N} \sum_{i=1}^{N} e_i^2 - \frac{1}{N^2} \left( \sum_{i=1}^{N} e_i \right)^2 \right)}
\]

Since both sides are positive, the square root can be eliminated and after some algebraic manipulations it arrives at:

\[
\frac{N(N-2)}{2(N-1)^2} \sum_{i=1}^{N} e_i^2 < \left( \frac{N-2}{2(N-1)^2} + \frac{N^2(N-1)^2}{(1+N(N-1))^2} \right) \left( \sum_{i=1}^{N} e_i \right)^2
\]

Since \( e_i > 0, \forall i \), we have that \( \sum_{i=1}^{N} e_i^2 < \left( \sum_{i=1}^{N} e_i \right)^2 \). Therefore, for the above inequality to hold it suffices to show that

\[
\frac{N(N-2)}{2(N-1)^2} < \left( \frac{N-2}{2(N-1)^2} + \frac{N^2(N-1)^2}{(1+N(N-1))^2} \right).
\]

After some algebraic manipulations, this reduces to

\[
0 < N^3(N^2 - 2N - 1) + N(6N - 5) + 2
\]
It is now easy to see that each term of the right-hand side of the above inequality is increasing in \( N \) and that the whole right-hand side is positive for any \( N \geq 1 \). Thus, inequality (43) is proven.

**Appendix H: Figures**

Figure 1: Trading positions

\[
\begin{align*}
\text{SELLERS} & \quad \text{BUYERS} \\
\epsilon_N & \quad \Leftarrow \text{small emitters} \quad \epsilon_N & \quad \Rightarrow \text{big emitters} \\
\end{align*}
\]

Figure 2: (a) \( E_{\min} \leq \hat{E}_{\min} \).

Figure 3: (b) \( \hat{E}_{\min} \leq \overline{E}_{\min} \leq \hat{E}_{\max} \).
References


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