Models of Growth and Development

Week 3 – Seeds of Growth

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Growth Models in Time

• Motivation: Some countries exhibit persistent growth while some don’t.
• GDP per capita PPP (IMF, 2008): 1st Qatar (86,000 USD); 181st Zimbabwe (268 USD). World Average: 10,433 (77 countries above)
• Significant clusters: Some regions grow while some others don’t
GDP Per Capita PPP

- Blue: Above average
- Orange: Below average
Source: Barro and Sala I Martin
Lucas (1988) on Development

- Huge differences in living standards (40 times)
- Difference in growth rates. E.g. between 1960-1980, yearly growth rates:
  - India: 1.4%
  - South Korea: 7%
  - Japan: 7.1%
  - U.S.: 2.3%
- But what do these rates mean in terms of living standards?
Time to double income

• An useful idea introduced by Lucas(1988) is the time to double your personal income. i.e. the “n” that solves $(1+g)^n=2$
  – India: $\ln(2)/\ln(1.014) \approx \ln(2)/0.014 = 49.5$ years
  – U.S.: 30.5 years
  – Japan: 10.1 years

I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action a government of India could take that would lead the Indian economy to grow like Indonesia’s or Egypt’s? If so, what, exactly? If not, what is it about the ‘nature of India’ that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.

Lucas(1988)
Lucas (1988)

- He argues:
  - We need a theory of economic development
    - But by theory he means “mathematical” theory
    - That allows to organize and explain these facts

This is what we need a theory of economic development for: to provide some kind of framework for organizing facts like these, for judging which represent opportunities and which necessities. But the term ‘theory’ is used in

I strongly recommend that you read the intro to this paper. Lucas, Robert E (1988), On the Mechanics of Economic Development, JME
This is the world “growth theorist” want to explain
The problems of making a “theory”

Source: Greg Clark, “A Farewell to Alms”.

Figure 1.1  World economic history in one picture. Incomes rose sharply in many countries after 1800 but declined in others.
There are 2 regimes

• The long-run world history seems to talk about 2 regimes
• First, the “Malthusian” period in which income per capita fluctuated around the survival income.
• But after the industrial revolution, modern growth first appears
Features of Modern Growth Models

• Strong emphasis in saving. Initial models took “savings” and the “Capital Output-Ratio” as given

• Harrod Domar: $s/\theta=(g+\delta)$
  – Policy recipe: either increase “s” or reduce “$\theta$”, capital-output ratio.

• Solow: Allows the capital output ratio to determine itself endogenously. In particular if you assume Y cobb douglas, $y/k=k^{(\alpha-1)}$, then $y/k$ decreases with k.
Solow Model

\[ \dot{k} = s \cdot f(k) - (n + \delta) \cdot k \]

Basic equation

\[ s \cdot f(k^*) = (n + \delta) \cdot k^* \]

Basic Solow: \( k^* \) is a steady state value

\[ c^* = (1 - s) \cdot f[k^*(s)]. \]

Steady state consumption is written as

\[ c^*(s) = f[k^*(s)] - (n + \delta) \cdot k^*(s) \]

Or replacing with the previous ...

The savings rate is fixed but we can actually choose one that will maximize steady state consumption

\[ f'(k_{gold}) = n + \delta \]

You can actually plot this result
Solow Model: Picture
Endogenous growth models

- So far,
  - The growth models considered start from some macroeconomic identities. Where are households? Lack of microfoundations
  - We have assumed that households will save a constant fraction of their wealth? How reasonable do you think that is? If your saving rate increases/decreases with your wealth then the implications of the model could be radically different
  - Solow model describes growth only during the transition. (This is however “solvable” introducing exogenous growth at a fixed rate).
Endogenous Growth Models

There is no level of $k$ that will equate growth to 0. Growth remain endogenous

\[ Y = AK \]
\[ \frac{\dot{k}}{k} = sA - (n + \delta) \]

The “Big Problem” of Solow are the decreasing returns!!!
• You can set up this as an individual who maximizes utility in a world populated by competitive firms (see Sala I Martin for details)
  – You need some extra transversality conditions “No ponzi game”

\[
\dot{k} = f(\hat{k}) - \hat{c} - (x + n + \delta) \cdot \hat{k} \\
\hat{c}/\hat{c} = \frac{\dot{c}}{c} - x = \frac{1}{\theta} \cdot [f'(\hat{k}) - \delta - \rho - \theta x]
\]

Two basic movement equations
How to get endogenous growth

• The trick is to get $f'(k)$ to be constant which was achieved in a number of ways:
  – Making state investment that increases the productivity of physical capital (adding $G$ in the Cobb-Douglas)
  – Making education increase the productivity of workers (Lucas, 1988). Technically more complicated
  – Adding capital spillovers (after Romer). Adding the aggregate capital into the equation.
Meet the Gloomy Guy

Thomas Robert Malthus
The Malthusian Economy
A simple Model

Figure 2.1 Long-run equilibrium in the Malthusian economy.
The Malthusian Economy

• Basic Facts
  – Stagnant standards of living: Calorie intake, heights etc. Downward jump when agriculture first appear (Cf. Jared Diamond)
  – Fertility:
    • Positive correlation between fertility and income.
    • Evidence of changes in fertility and some attempts to regulate it (e.g. European Fertility Pattern)
  – Life Expectancy:
    • Agriculture reduced life expectancy
    • Stagnated life expectancy until the modern times
The Malthusian Stagnation: Calorie Intake

- Standard of living close to subsistence levels. Transition from hunter gatherers to permanent settlements.

<table>
<thead>
<tr>
<th>Group</th>
<th>Period</th>
<th>Kilocalories</th>
<th>Grams protein</th>
</tr>
</thead>
<tbody>
<tr>
<td>England, farm laborers</td>
<td>1787–96</td>
<td>1,508</td>
<td>27.9</td>
</tr>
<tr>
<td>England, all</td>
<td>1787–96</td>
<td>2,322</td>
<td>48.2</td>
</tr>
<tr>
<td>Belgium, all</td>
<td>1812</td>
<td>2,248</td>
<td>—</td>
</tr>
<tr>
<td>Ache, Paraguay</td>
<td>1980s</td>
<td>3,827</td>
<td>—</td>
</tr>
<tr>
<td>Hadza, Tanzania</td>
<td>—</td>
<td>3,300</td>
<td>—</td>
</tr>
<tr>
<td>Alyware, Australia</td>
<td>1970s</td>
<td>3,000</td>
<td>—</td>
</tr>
<tr>
<td>Onge, Andaman Islands</td>
<td>1970s</td>
<td>2,620</td>
<td>—</td>
</tr>
<tr>
<td>Aruni, New Guinea</td>
<td>1966</td>
<td>2,390</td>
<td>—</td>
</tr>
<tr>
<td>!Kung, Botswana</td>
<td>1960s</td>
<td>2,355</td>
<td>—</td>
</tr>
<tr>
<td>Bayano Cuna, Panama</td>
<td>1960–61</td>
<td>2,325</td>
<td>49.7</td>
</tr>
<tr>
<td>Mbuti, Congo</td>
<td>1970s</td>
<td>2,280</td>
<td>—</td>
</tr>
<tr>
<td>Anbarra, Australia</td>
<td>1970s</td>
<td>2,050</td>
<td>—</td>
</tr>
<tr>
<td>Hiwi, Venezuela</td>
<td>1980s</td>
<td>1,705</td>
<td>64.4</td>
</tr>
<tr>
<td>Shipibo, Peru</td>
<td>1971</td>
<td>1,665</td>
<td>65.5</td>
</tr>
<tr>
<td>Yanomamo, Brazil</td>
<td>1974</td>
<td>1,452</td>
<td>58.1</td>
</tr>
</tbody>
</table>

The empirical evidence: demographics

Figure 2.6. Fertility and Mortality: England 1540-1870
Population and GDP

• Population mirrors GDP in pre-industrial times

Source: Galor (2005)
The Malthusian Stagnation: Heights

Table 3.10  Heights from Skeletal Remains by Period

<table>
<thead>
<tr>
<th>Period</th>
<th>Location</th>
<th>Observations</th>
<th>Height (centimeters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesolithic(^a)</td>
<td>Europe</td>
<td>82</td>
<td>168</td>
</tr>
<tr>
<td>Neolithic(^a,b)</td>
<td>Europe</td>
<td>190</td>
<td>167</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
<td>103</td>
<td>173</td>
</tr>
<tr>
<td>1600–1800(^c)</td>
<td>Holland</td>
<td>143</td>
<td>167</td>
</tr>
<tr>
<td>1700–1800(^c)</td>
<td>Norway</td>
<td>1,956</td>
<td>165</td>
</tr>
<tr>
<td>1700–1850(^c)</td>
<td>London</td>
<td>211</td>
<td>170</td>
</tr>
<tr>
<td>Pre-Dynastic(^d)</td>
<td>Egypt</td>
<td>60</td>
<td>165</td>
</tr>
<tr>
<td>Dynastic(^d)</td>
<td>Egypt</td>
<td>126</td>
<td>166</td>
</tr>
<tr>
<td>2500 BC(^e)</td>
<td>Turkey</td>
<td>72</td>
<td>166</td>
</tr>
<tr>
<td>1700 BC(^f)</td>
<td>Lerna, Greece</td>
<td>42</td>
<td>166</td>
</tr>
<tr>
<td>2000–1000 BC(^g)</td>
<td>Harappa, India</td>
<td>—</td>
<td>169</td>
</tr>
<tr>
<td>300 BC–AD 250(^h)</td>
<td>Japan (Yayoi)</td>
<td>151</td>
<td>161</td>
</tr>
<tr>
<td>1200–1600(^i)</td>
<td>Japan (medieval)</td>
<td>20</td>
<td>159</td>
</tr>
<tr>
<td>1603–1867(^h)</td>
<td>Japan (Edo)</td>
<td>36</td>
<td>158</td>
</tr>
<tr>
<td>1450(^i)</td>
<td>Marianas, Taumako</td>
<td>70</td>
<td>174</td>
</tr>
<tr>
<td>1650(^j)</td>
<td>Easter Island</td>
<td>14</td>
<td>173</td>
</tr>
<tr>
<td>1500–1750(^i)</td>
<td>New Zealand</td>
<td>124</td>
<td>174</td>
</tr>
<tr>
<td>1400–1800(^i)</td>
<td>Hawaii</td>
<td>—</td>
<td>173</td>
</tr>
</tbody>
</table>

Sources: \(^a\)Meiklejohn and Zvelebil, 1991, 133. \(^b\)Bennike, 1985, 51–52. \(^c\)Steckel, 2001. \(^d\)Masali,
The Malthusian Stagnation: European Fertility

• How did the EFP worked?
  – Fertility Control through late marriage or celibacy

<table>
<thead>
<tr>
<th>Country or group</th>
<th>Birth rate at age:</th>
<th>All births (20–44)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutterites</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>France</td>
<td>0.48</td>
<td>0.45</td>
</tr>
<tr>
<td>Germany</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.45</td>
<td>0.38</td>
</tr>
<tr>
<td>Scandinavia</td>
<td>0.43</td>
<td>0.39</td>
</tr>
<tr>
<td>England</td>
<td>0.43</td>
<td>0.39</td>
</tr>
</tbody>
</table>

*Source: Flinn, 1981, 86.*
## The EFP

### Table 4.2  Age of Marriage of Women and Marital Fertility in Europe before 1790

<table>
<thead>
<tr>
<th>Country or group</th>
<th>Mean age at first marriage</th>
<th>Births per married women</th>
<th>Percentage never married</th>
<th>Total fertility rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium(^a)</td>
<td>24.9</td>
<td>6.8</td>
<td>—</td>
<td>6.2(^*)</td>
</tr>
<tr>
<td>France(^a,b)</td>
<td>25.3</td>
<td>6.5</td>
<td>10</td>
<td>5.8</td>
</tr>
<tr>
<td>Germany(^a)</td>
<td>26.6</td>
<td>5.6</td>
<td>—</td>
<td>5.1*</td>
</tr>
<tr>
<td>England(^a)</td>
<td>25.2</td>
<td>5.4</td>
<td>12</td>
<td>4.9</td>
</tr>
<tr>
<td>Netherlands(^c)</td>
<td>26.5</td>
<td>5.4*</td>
<td>—</td>
<td>4.9*</td>
</tr>
<tr>
<td>Scandinavia(^a)</td>
<td>26.1</td>
<td>5.1</td>
<td>14</td>
<td>4.5</td>
</tr>
</tbody>
</table>


*Note:* * denotes values inferred assuming missing values at European average.
What about fertility elsewhere?

• Asia: Very low marital fertility
  • Misreporting?
  • Female infantice?

Table 4.3 Age-Specific Marital Fertility Rates outside Europe

<table>
<thead>
<tr>
<th>Country or group</th>
<th>Fertility rate at age:</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutterite&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>Northwestern Europe&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>China&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Japan&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>Roman Egypt&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.38</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Sources: <sup>a</sup>Table 4.1. <sup>b</sup>Lee and Feng, 1999, 87. <sup>c</sup>Bagnall and Frier, 1994, 143–46.
How high was fertility in the distant past?

- Forager societies:
  - Have very low fertility, comparable to those of Britain on the eve of the industrial revolution
- Maybe that’s why living standards moved sluggishly?

<table>
<thead>
<tr>
<th>Country or group</th>
<th>Fertility rate at age:</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hutterite(^a)</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>Northwestern Europe(^a)</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>China(^b)</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>Japan(^b)</td>
<td>0.29</td>
<td>0.25</td>
</tr>
<tr>
<td>Roman Egypt(^c)</td>
<td>0.38</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Sources: \(^a\)Table 4.1. \(^b\)Lee and Feng, 1999, 87. \(^c\)Bagnall and Frier, 1994, 143–46.
Correlation Income-Fertility in the Past
# Life Expectancy in the Malthusian World

<table>
<thead>
<tr>
<th>Group</th>
<th>Age</th>
<th>Life expectancy at 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magistrates, Canusium, Italy, AD 223(^a)</td>
<td>25</td>
<td>33</td>
</tr>
<tr>
<td>Ex-slaves, Italy, ca. AD 200(^a)</td>
<td>22.5</td>
<td>28</td>
</tr>
<tr>
<td><strong>England</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1300–48 (tenants)(^b)</td>
<td>20+</td>
<td>28</td>
</tr>
<tr>
<td>1350–1400 (tenants)(^b)</td>
<td>20+</td>
<td>32</td>
</tr>
<tr>
<td>1440–1540 (monks)(^c)</td>
<td>20</td>
<td>27</td>
</tr>
<tr>
<td>1600–1638 (testators)</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td><strong>England, 1750–99(^d)</strong></td>
<td>20</td>
<td>34</td>
</tr>
<tr>
<td>Rural Japan, 1776–1815(^d)</td>
<td>20</td>
<td>37</td>
</tr>
<tr>
<td>Rural China (Liaoning), 1792–1867(^d)</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td><strong>Modern foragers(^d)</strong></td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

*Sources: \(^a\)Duncan-Jones, 1990, 94–97. \(^b\)Razi, 1980. \(^c\)Harvey, 1993, 128. \(^d\)Tables 5.1 and 5.2.*
Correlation Life Exp-Income

The bar chart shows the child survival rate for different assets at death (£):
- 0-24
- 25-99
- 100-499
- 500+

The child survival rate increases as the assets at death increase.
Have you wondered why economics is called the “Dismal Science”

- Netherlands seems to escape the Malthusian trap in the 17\textsuperscript{th} century. Why?
  - Health environment: Mortality did not react a lot to income and neither did fertility.

![Figure 5.5 Real wages versus population in the Netherlands, 1500s–1810s.](image)
A Healthy Hell: Infantice in Polinesia

• There were many interactions between fertility and health
  – The tragedy of the “Bounty” or “When nature is too kind?”
  • Bottomline: If there are no “preventive” checks (abortion, birth control, celibacy, delayed marriage etc.) leads to “positive” checks (famine, violence, infanticide)
• In Polinesia, “Infanticide” seems to have been the norm (Cf. Jared Diamond, “Collapse”)
How healthy was Polinesia?

• It seems: A lot!!!!
  • No Mosquitos
  • Mild Weather

• But infanticide was pervasive. Until the arrival of christians, between 2/3 and 3/4 of the children were killed immediately after birth.

• One of the reasons why Europe did not resort to infanticide was maybe that it was not very healthy in the first place:
  • Weather
  • As Greg Clark puts it, “Europeans were dirty”

<table>
<thead>
<tr>
<th>Location</th>
<th>Troop nationality</th>
<th>Period</th>
<th>Death rate per thousand</th>
</tr>
</thead>
<tbody>
<tr>
<td>New Zealand</td>
<td>British</td>
<td>1846–55</td>
<td>9</td>
</tr>
<tr>
<td>Tahiti</td>
<td>French</td>
<td>1845–49</td>
<td>10</td>
</tr>
<tr>
<td>Cape Colony</td>
<td>British</td>
<td>1818–36</td>
<td>16</td>
</tr>
<tr>
<td>Canada</td>
<td>British</td>
<td>1817–36</td>
<td>16</td>
</tr>
<tr>
<td>Gibraltar</td>
<td>British</td>
<td>1817–36</td>
<td>21</td>
</tr>
<tr>
<td>Bombay</td>
<td>British</td>
<td>1830–38</td>
<td>37</td>
</tr>
<tr>
<td>Bengal</td>
<td>British</td>
<td>1830–38</td>
<td>71</td>
</tr>
<tr>
<td>Martinique</td>
<td>French</td>
<td>1819–36</td>
<td>112</td>
</tr>
<tr>
<td>Jamaica</td>
<td>British</td>
<td>1817–36</td>
<td>130</td>
</tr>
<tr>
<td>Senegal</td>
<td>French</td>
<td>1819–38</td>
<td>165</td>
</tr>
<tr>
<td>East Indies</td>
<td>Dutch</td>
<td>1819–28</td>
<td>170</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>British</td>
<td>1819–36</td>
<td>483</td>
</tr>
</tbody>
</table>

Unified Growth
Galor and Weil (1999)

• Basic Unified Growth model
• They define 3 regimes: “Malthusian Regime” (MR), “Post-Malthusian Regime” (PMR) and “Modern Growth Regime” (MG)
• Focus on 2 major sources of difference
  – Income per capita
  – Correlation between income per capita and population growth
Galor and Weil (1999)

- Malthusian Regime
  - No income growth
  - Corr(Income per capita, Population growth)>0

- Post-Malthusian Regime
  - Income per capita grows
  - Corr(Income per capita, Population growth)>0

- Modern Growth
  - Income per capita grows
  - Corr(Income per capita, Population growth)<0
Galor and Weil (1999)

- MR: Income stagnates. Better technology or more resources translate into larger (but not) wealthier population
- MGR: Income grows. Families choose fewer children (higher opportunity costs, higher value of mom’s time, children as a ec. Burden)
- But it is not so clear how things worked in the transition period.
Post-Malthusian Regime

- Income starts growing.
- Birth rates react faster than mortality rates. Fertility vs. Mortality transition. As a result, there is a period of high population growth.

Source: Galor (2005)

Figure 2.6. Fertility and Mortality: England 1540-1870
Key Questions

• What causes the reversal in the correlation between fertility and the demographic transition
• Why did income start to grow suddenly?
Building Blocks of the Model

• Catalyzer: faster technological progress
• 1. QQ tradeoff: When the return of human capital increases, parents substitute quantity for quality (Becker, 1981)
• 2. Feedback between education and technological progress: Educated children trigger faster technological progress; technological progress also increase with population
• 3. Decreasing returns: When population increase, wages fall. In absence of tech growth, malthusian conditions prevail.
Early Stages

- Stationary output.
- Slow technological growth.
- Slow income and population growth.
- Low returns to human capital.
- Hence, no incentive to substitute quality for quantity.
- “Pseudo Steady State”. Why so? Population grows (slowly) and eventually will accelerate growth.
  - In this model, the transition is “Inevitable”.
Transition Period

• Income growth affects the budget of the families in two ways
  – Income effect: Now families want to “demand” more children (Assumption, children are normal goods)
  – Substitution Effect: Now “quantity” is more expensive as the returns to human capital are higher. Parents fewer children

• During the transition period, the first effect is bigger than the second
Sustained Growth Period

• Eventually, the substitution effect comes to dominate.
• When that is happening we observe the demographic transition in its final stages
• When the demographic transition is completed, further increases in income translate in reduction in fertility
• Income keeps on growing.
The Model (Galor and Weil 2000)

• Setup
  – Overlapping generations. Infinite horizon.
  – 2 inputs: Land (fixed) (X) and “efficiency units” of labor (adjusted by productivity) (Ht)
  – Households decide over fertility and over the amount of efficiency units of labor

• Output:
  – $A_t$ is the “endogenous” level of technology

\[
Y_t = H_t^\alpha (A_t X)^{1-\alpha},
\]
The Model

• “Effective resources” are what matter, i.e. $X_A t$
• Output per worker is given by:

\[ y_t = h_t^\alpha x_t^{(1-\alpha)} = y(h_t, x_t) , \]

• Where lowercase denote “per capita” i.e. $x_t = X_t / L_t$
• Assumption. No property right for land (return=0). The wage per unit of efficiency of labor (ht) is simply the average product

\[ w_t = (x_t / h_t)^{1-\alpha} = w(h_t, x_t) , \]
The Model

- Household
  - Households care for their own consumption to be above a threshold ($\bar{c} > 0$) and for the aggregate income of their children, that is,

  \[ u^t = (c_t)^{(1-\gamma)}(w_{t+1}n_t h_{t+1})^\gamma, \]

- In order to generate child “quality” and child “quantity” parents have to invest time. To generate quality $e_{t+1}$, parents have to invest a fraction

  \[ \tau_q \cdot e_{t+1} + \tau^e e_{t+1} \]

  Of their time.
The Model

• Potential earnings are distributed between raising children and consuming

\[ w_t h_t n_t (\tau^q + \tau^e e_{t+1}) + c_t \leq w_t h_t. \]  

• Education reduces the adverse effects of technological progress

where

\[ h(e_{t+1}, g_{t+1}) > 0, \quad h_e(e_{t+1}, g_{t+1}) > 0, \quad h_{ee}(e_{t+1}, g_{t+1}) < 0, \quad h_g(e_{t+1}, g_{t+1}) < 0, \]
\[ h_{gg}(e_{t+1}, g_{t+1}) > 0, \quad \text{and} \quad h_{eg}(e_{t+1}, g_{t+1}) > 0 \quad \forall (e_{t+1}, g_{t+1}) \geq 0. \]
Solution

- Optimization problem

\[(7) \quad \{n_t, e_{t+1}\}
\]

\[= \arg\max\{w_t h_t [1 - n_t (\tau^q + \tau^e e_{t+1})]^{1-\gamma} \times \{(w_{t+1} n_t h(e_{t+1}, g_{t+1})\}^{\gamma}
\]

subject to

\[w_t h_t [1 - n_t (\tau^q + \tau^e e_{t+1})] \geq \bar{c};\]

\[(n_t, e_{t+1}) \geq 0.\]

- Under broad circumstances, it can be shown that the level of \(e_{t+1}\) chosen is a non-decreasing function of \(g_t\). (Ass. A1 and Lemma 1)
Solution

- Extra assumptions

\[(A1) \quad G(0, 0) = \gamma h(0, 0)\]
\[- \gamma h(0, 0) < 0.\]

\[(A2) \quad \int_{g_{t+1}} < 0 \quad \forall \ g_{t+1} > \dot{g}.\]
PROPOSITION 1: Under (A1)–(A2)

(a) Technological progress that is expected to occur between the first and second periods of children’s lives results in a decline in the parents’ chosen number of children and an increase in their quality, i.e.,

$$\frac{\partial n_t}{\partial g_{t+1}} \leq 0 \quad \text{and} \quad \frac{\partial e_{t+1}}{\partial g_{t+1}} \geq 0.$$  

(b) If parental potential income is below \( \bar{z} \) (i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality, i.e.,

$$\frac{\partial n_t}{\partial z_t} > 0 \quad \text{and} \quad \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if} \quad z_t < \bar{z}.$$  

(c) If parental potential income is above \( \bar{z} \), an increase in parental potential income does not change the number of children or their quality, i.e.,

$$\frac{\partial n_t}{\partial z_t} = \frac{\partial e_{t+1}}{\partial z_t} = 0 \quad \text{if} \quad z_t > \bar{z}.$$
Dynamics

• First regime: Subsistence consumption is binding

\[
\begin{align*}
  x_{t+1} &= \phi^a(e_t, g_t, x_t; L)x_t \\
  e_{t+1} &= e(g(e_t; L)) \quad \text{if } z_t \leq \bar{z} \\
  g_{t+1} &= g(e_t; L)
\end{align*}
\]  

(17)

• Second regime: Above subsistence consumption.

\[
\begin{align*}
  x_{t+1} &= \phi^b(e_t, x_t; L)x_t \\
  e_{t+1} &= e(g(e_t; L)). \quad \text{if } z_t \geq \bar{z}
\end{align*}
\]  

(18)
Education and Technology: Small

\[ e_{t+1} = e(g_{t+1}) \]

\[ g_{t+1} = g(e_t; L^t) \]

Technology Growth, \( g_t \)

Education, \( e_t \)
Education and Tecnology: Medium

\[ e_{t+1} = e(g_{t+1}) \]

\[ g_{t+1} = g(e_t; L^m) \]

Technology Growth, \( g_t \)

Education, \( e_t \)
Education and Technology: Large

\[ e_{t+1} = e(g_{t+1}) \]
\[ g_{t+1} = g(e_t; L^h) \]

**Figure 5. The Evolution of Technology and Education for a Large Population**
Towards a Phase Graph

1

Let the *Conditional Malthusian Frontier* be the set of all pairs \((e_t, x_t)\) for which, conditional on a given technological level \(g_t\), individuals' incomes equal \(\bar{z}\). Following the definitions of \(z_t\) and \(\bar{z}\), equations (6) and (11) imply that the Conditional Malthusian Frontier \(MM|g_t\), as depicted in Figures 6–8, is

\[
(20) \quad MM|g_t = \{(e_t, x_t) : x_t^{(1-a)}h(e_t, g_t)^a = \bar{c}l(1-\gamma)|g_t\}.
\]

2

The XX Locus.—Let XX be the locus of all triplets \((e_t, g_t, x_t)\), such that for a given population size the effective resources per worker, \(x_t\), are in a steady state:

\[
XX = \{(e_t, x_t, g_t) : x_{t+1} = x_t\}.
\]

3

Let \(XX|g_t\) be the locus of all pairs \((e_t, x_t)\), such that \(x_{t+1} = x_t\) for a given level of \(g_t\); that is,

\[
XX|g_t = \{(e_t, x_t) : x_{t+1} = x_t|g_t\}.
\]
Useful Lemmas

LEMMA 3: If (A1)–(A4) are satisfied, then for $z_t \geq \bar{z}$, there exists a unique value $0 < \hat{e} < e^h$, such that $x_t \in XX$. Furthermore, for $z_t \geq \bar{z}$

$$x_{t+1} - x_t \begin{cases} > 0 & \text{if } e_t > \hat{e} \\ = 0 & \text{if } e_t = \hat{e} \\ < 0 & \text{if } e_t < \hat{e}. \end{cases}$$

LEMMA 5: Let $(\hat{e}, \hat{x}) \in MM_{|g_t}$. If (A4) is satisfied, then

$$(\hat{e}, \hat{x}) = XX_{|g_t} \cap MM_{|g_t} \cap XX.$$
Phase Diagram for Small Economy

Figure 6. The Conditional Dynamical System for a Small Population
Phase Diagram for Medium Econ

---

**Effective Resources per Worker, \( x_t \)**

\( e_{t+1} = e_t \)

\( x_{t+1} = x_t \)

\( e_{t+1} = e_t \)

**Conditional Malthusian Steady-State Equilibrium**

\( e^u \)

\( \hat{e} \)

\( e^h \)

**Conditional Malthusian Frontier**

*Education, \( e_t \)*
Phase Diagram for Big Economy

Figure 8. The Conditional Dynamical System for a Large Population
Conclusions

• The model captures the transition from stagnation to growth

• The key feature is the interaction between education and technological growth
  – Alternative assumption: return of education as a function of the level of technology. This would imply that the takeoff is not related to population size.

• The transition involves an increase in the returns to education and the quantity-quality tradeoff
Conclusions

- Institutions/Culture is omitted from the picture. They might affect population growth and levels.
- Policy changes: The model suggests that public provision of school may help to accelerate the transition. Also, maybe migration (by relaxing the land constraint) at a moment when income was growing, helped to speed up the transition to modern growth in XiXth century Europe
Conclusion

• A good historical model but maybe not relevant for developing economies today:
  – Now technologies pour largely across borders, so the relationship between population and technological growth is broken

• But the main advantage is that the three regimes are captured within a unified framework!!!

• Citation:
Alternative theories of Human Capital Formation

• The central feature of the model is that there is Human Capital formation at a stage and that this interacts with the demographic transition.

• Basic model stresses that “educated people adapt themselves more easily to advancing technological environment”
  – New technologies demand skilled people
  – Debate: What about the industrial revolution?

• Are there any other mechanisms?
Other Mechanisms

• Doepke (2004), rising level of skill intensive industrial technology.
• Fernandez-Vilaverde (2005): Capital-Skill complementarity
• Galor et al (2003): International trade favors specialization in skill-intensive sectors
• Public policy: Doepke (2004) (and me!!) suggest that educational and child labor laws affected capital formation.
Reinforcing Mechanisms

• Drop in mortality and increase in life expectancy can reinforce human capital accumulation
  – Increased return in child investments
  – However, there seems to be little evidence for this...
    Investment in human capital is negligible until late in the 19th century.

• The usual channels to link mortality drop, human capital accumulation and fertility decline are:
Alternatives to generate the fertility transition: Through education

- Prolongation of life
- Increase in population density, making the transmission of human capital more efficient
- Interaction between health and education, increasing the returns to education
- Biased technological changed that reduced the demand for child labor
- Increased need of skilled workers, moved industrialists to propose a ban in child labor
- Cultural and genetic evolution. Individuals with higher valuation for offspring quality tended to survive, so then an increase in returns to human capital eventually triggers the demographic transition.
Alternatives to generate the fertility transition: Not Education

• Gender Gap (Galor and Weil, 1996)
  – Men have a comparative advantage in physical tasks
  – The industrial revolution just changed that
  – Relative increase in female labor demand
  – Increase participation / high opportunity costs triggered lower fertility.

• Increase in the relative level of education of women
  – Triggered lower fertility rates
Other Channels: Das Human Kapital

- This paper conjectures that industrialists pushed for universal education in response to skilled bias technological change.
- They consider that the “demise of the capitalist-worker class structure” far from being the result of class struggle (Marx, 1867) is the “by-product of a productive cooperation between capitalists and workers.”
Capitalists and Workers Cooperating?

• In initial stages of industrialization, physical capital matters.

• Society is then divided according to the ownership of the factor of production.

• Capital-Skill complementarities make “Human Capital” more valuable for capitalists.

• This triggered a “non-altruistic” shift in their preference towards the provision of public education.
Capitalists and Workers Cooperating?

- The diffusion of public education gives rise to a new “Middle Class”
- The widening of the middle classes causes the demise of the class structure.
- And all of these happened without “struggle” and as a result of the change in the socio-economic environment
3-Key Elements of this Model

1. **Capital-Skill Complementarity**: This is the reason why capitalists might be interested in education in the first place.

2. **Human Capital is embodied in individuals and its accumulation presents decreasing returns**: i.e. it is desirable to provide “universal” education.

3. **Sub-optimal level of investment in education**: In absence of public provision, people would invest less than optimal (e.g. borrowing constraints).
Welfare Implications

• Capitalists are willing to pay for it and they are better off as they reap-off part of the productivity benefits
• Workers are also better off as they not pay for the education they receive
• Capital Market Imperfections + Capital-Skill complementarities => Redistribution is Pareto-Improving
Universal vs. Industrial Education

• Why universal education?
  – Investment in primary education generated a wider pool for the managerial occupations
  – There seems to be some basic skills “Literacy” that matter for productivity in later stages of the Ind. Rev.

• A frictionless society?
  – Not really, but there is not a conflict between workers and capitalist
  – The conflict is rather among capitalist and land-owners
Capitalists vs. Landowners

• The struggle comes between the owners of the different factors of production
• Different factors of production means different skill complementarity
• In a word, land owners don’t need literate peasants
• Industrialist would push for an early introduction of universal education
Empirical Evidence

• The Balfour Act – UK 1902
• Education reform in the U.K. that created a national education system
• They gather data about how MPs voted in the third and final vote on the Balfour Act.
• Empirical Strategy: Ordered-Probit regression with respect to the skill-intensiveness of the MPs counties
Regression Analysis

• Ordered Probit: three outcomes “Yes”, “Abstain” and “No” and order matters

• Regressions include some controls
  – Income per capita
  – % Skill Intensive population in the county
  – % Nonconformist (The Balfour Act gave the Anglican church the power to regulate education)
  – National Dummies (Education could be interpreted as “English” neocolonization)
Regression Analysis

• Percentage urban. Urban locations have higher demand of education irrespective of industrial skilled labor demand

• Party affiliation: whether the MP is liberal

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>Abstain</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberal</td>
<td>19</td>
<td>98</td>
<td>92</td>
<td>209</td>
</tr>
<tr>
<td>Conservative</td>
<td>155</td>
<td>3</td>
<td>94</td>
<td>252</td>
</tr>
<tr>
<td>Unionist</td>
<td>43</td>
<td>0</td>
<td>26</td>
<td>69</td>
</tr>
<tr>
<td>Other</td>
<td>4</td>
<td>14</td>
<td>14</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>221</td>
<td>115</td>
<td>226</td>
<td>562</td>
</tr>
</tbody>
</table>
## Results

### TABLE 2

The effect of the weight of the skill-intensive sector on the support for the Balfour Act

<table>
<thead>
<tr>
<th>Exp. variable</th>
<th>(1) Vote on Balfour Act</th>
<th>(2) Vote on Balfour Act</th>
<th>(3) Vote on Balfour Act</th>
<th>(4) Vote on Balfour Act</th>
<th>(5) Vote on Balfour Act</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income per capita</td>
<td>-0.0011 (0.12)</td>
<td>-0.0030 (0.33)</td>
<td>-0.0033 (0.36)</td>
<td>-0.0118 (1.33)</td>
<td>-0.0120 (1.33)</td>
</tr>
<tr>
<td>% in skill-intensive</td>
<td>5.7298** (4.49)</td>
<td>4.1818** (3.09)</td>
<td>4.2554** (3.20)</td>
<td>2.6177* (1.93)</td>
<td>2.6171* (1.94)</td>
</tr>
<tr>
<td>occupations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% non-conformists</td>
<td>-1.9109** (3.95)</td>
<td>-1.4129 (1.59)</td>
<td>0.7349 (0.67)</td>
<td>0.7395 (0.67)</td>
<td></td>
</tr>
<tr>
<td>Scotland dummy</td>
<td>0.0215 (0.09)</td>
<td>-0.1246 (0.50)</td>
<td>-0.1216 (0.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wales dummy</td>
<td>-0.7289** (2.01)</td>
<td>-0.9086** (2.01)</td>
<td>-0.9057** (1.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Party affiliation</td>
<td></td>
<td></td>
<td>-1.7404** (11.44)</td>
<td></td>
<td>-1.7399** (11.46)</td>
</tr>
<tr>
<td>% urban</td>
<td></td>
<td></td>
<td></td>
<td>0.0246 (0.10)</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Absolute values of t-ratios are given in parentheses. Vote on Balfour: 2, Yes; 1, Abstain; 0, No. Party affiliation: 1, Liberal; 0, Conservative. Standard errors are adjusted by clustering by county.

* Indicates significance at 10%.
** Indicates significance at 5%.
Table 3: The effect of the weight of the skill-intensive sector on MP’s party affiliation

<table>
<thead>
<tr>
<th>Exp. variable</th>
<th>(1) Party affiliation</th>
<th>(2) Party affiliation</th>
<th>(3) Party affiliation</th>
<th>(4) Party affiliation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income per capita</td>
<td>-0.0139</td>
<td>-0.0119</td>
<td>-0.0112</td>
<td>-0.0009</td>
</tr>
<tr>
<td></td>
<td>[-0.00548]</td>
<td>[-0.00469]</td>
<td>[-0.0044]</td>
<td>[-0.0035]</td>
</tr>
<tr>
<td></td>
<td>(1.03)</td>
<td>(1.02)</td>
<td>(0.97)</td>
<td>(0.81)</td>
</tr>
<tr>
<td></td>
<td>(2.31)</td>
<td>(2.23)</td>
<td>(2.23)</td>
<td>(2.23)</td>
</tr>
<tr>
<td>% non-conformists</td>
<td>3.3908**</td>
<td>4.1208**</td>
<td>4.2180**</td>
<td>1.6664**</td>
</tr>
<tr>
<td></td>
<td>[1.3375]</td>
<td>[1.6272]</td>
<td>[1.6664]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.66)</td>
<td>(3.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scotland dummy</td>
<td>-0.3227</td>
<td>-0.3936</td>
<td>-0.1239</td>
<td>-0.1499</td>
</tr>
<tr>
<td></td>
<td>[-0.72]</td>
<td>(0.98)</td>
<td>(0.98)</td>
<td></td>
</tr>
<tr>
<td>Wales dummy</td>
<td>0.1406</td>
<td>0.0815</td>
<td>[0.03234]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.28)</td>
<td>(0.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% urban</td>
<td>-0.2887</td>
<td>-0.11402</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-0.11402]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.72)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chi-square P-value</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Total observations</td>
<td>455</td>
<td>455</td>
<td>455</td>
<td>455</td>
</tr>
<tr>
<td>Method</td>
<td>Probit</td>
<td>Probit</td>
<td>Probit</td>
<td>Probit</td>
</tr>
</tbody>
</table>

Note: Absolute values of t-ratios are given in parentheses. Marginal effects are reported in square brackets. Vote on Balfour: 2, Yes; 1, Abstain; 0, No. Party affiliation: 1, Liberal; 0, Conservative. Standard errors are adjusted by clustering by county.

*Indicates significance at 10%.

**Indicates significance at 5%.
Summary

• Percentage Skilled seems to predict the passing of the laws
• We cannot say anything about the magnitude of the marginal effects (non-linear model)
• Conceptual problem with the second regression. If “party affiliation” is endogenous, you need to find and instrument and perform the regression in Table 2 using 2SLS.
The Child Labor Connection (Hazan and Berdugo)

• Development trap set up: high child labor, low wages, low human capital accumulation.

• Technological progress makes human capital valuable triggering the quantity quality tradeoff

• The economy moves to a developed stage with high returns to human capital, low fertility and no child labor

• Child labor laws may be beneficial
Hazan and Berdugo, 2002

• Key assumptions:

1. Parents decide whether children work or go to school
2. Parents are altruistic (care about children future earnings)
3. The income of child labor goes to parents
4. Child rearing is time intensive
Hazan and Berdugo

• As in Galor and Weil (2000), human capital is an increasing and strictly concave function of the amount of time devoted to schooling.

• In particular, Hazan and Berdugo use

\[ h_{t+1} = h(\tau_t) = a(b + \tau_t)\beta \]
Hazan and Berdugo

• Individuals face a budget constraint that includes the possibility of sending children to work, with a productivity $\theta<1$ relative to a grown up and a cost of $z$ to raise them

$$(1 - zn_t)I_t + \theta(1 - \tau_t)n_tw^c = s_t.$$ 

• Schooling is free. All the effect occurs through “forgone costs” of child labor
Hazan and Berdugo

- Utility is also quite standard

\[ u^t = \alpha \ln (c_{t+1}) + (1 - \alpha) \ln (n_t I_{t+1}) \]

- As a result of optimization, Hazan and Berdugo define a function of the amount of education (τ) children get as a function of parental human capital (ht)
Solution

• 2 Cases depending on the parameters
  – Case 1: \( q \theta < z \), where
  – Case 2: \( q \theta > z \)

• In case 1, there is always an optimal \( \tau^* > 0 \), regardless of the education of the parents

\[
q \equiv \left( \frac{w_c}{a \omega} \right)^{1/\beta} \cdot \frac{1 - \beta}{\beta} + 1 + b
\]

\[
\tau^*_i = \begin{cases} 
\frac{\beta z - \beta \theta - b \theta}{\theta(1 - \beta)} & \text{if } h_t \leq w^c / \bar{\omega} \\
\frac{\beta \bar{\omega} h_t - \beta w^c \theta - b w^c \theta}{w^c \theta(1 - \beta)} & \text{if } w^c / \bar{\omega} \leq h_t \leq w^c \theta(1 + b) / \beta \bar{\omega} \\
1 & \text{if } w^c \theta(1 + b) / \beta \bar{\omega} \leq h_t.
\end{cases}
\]

• In case 2, \( \tau^* \) may be 0 for sufficiently low \( h_t \)
If parents are below a certain threshold of education, they will not invest time in educating their kids. Child labor!

Beyond a certain threshold, they don’t make children to work at all.

Intermediate situation with some child labor.

\[
\tau^*_t = \begin{cases} 
0 & \text{if } h_t < \tilde{h} \\
\beta z w h_t - \beta \omega^c \theta - b w^c \theta \over w^c \theta (1 - \beta) & \text{if } \tilde{h} \leq h_t \leq w^c \theta (1 + b) / \beta z \omega \\
1 & \text{if } w^c \theta (1 + b) / \beta z \omega \leq h_t 
\end{cases}
\]

where \( \tilde{h} \equiv q \theta w^c / z \omega. \)
Dynamic System

• Put the human capital formation equation with the optimal time devoted to children together to solve a dynamic system

• You get the function $\psi(h_t)$
Dynamic System (case 1)

– Unique and stable equilibrium
– High income
– Few children in hh
– No child labor
Dynamic System (case 2)

• Depending on the parameters, we can get 3 different situations
  • Situation 1
  • Low Income
  • Many Children
  • Child Labor
  • “Development trap”
Dynamic System (case 2)

• Situation 2
• 3 steady-states
• "Low", "High" are stable
• "Middle" is unstable
• History matters!
Dynamic System (case 2)

- Situation 2
- 2 steady-states
- "Low"
- "High"
- History matters!
Conclusions

• Extension with technology ➔ Just to make all the steady states into “pseudo” steady states

• With technological progress, you can assume that progressively you will move from the situations with low human capital and high fertility to development

• Negative relation between children and education (see paper) holds more generally, even when there is child labor
Conclusions

• Policy implications:
  – Introduction of compulsory schooling + redistribution to the elders

• Welfare Analysis
  – Basu and Van say that it is not Pareto improving because firms profits are lower after CSLs are enacted
  – Baland and Robinson (2000) prove that it could be improving if it induces certain changes in current and future wages
Conclusions

- The source of the problem in this model is like in Baland and Robinson (2000), child labor emerges because families are not only poor but they face credit constraints.

- So, the propose redistribution to the elders tries to compensate families for the decrease in income due to the banning of child labor.
A Note on Sources

• This presentation took a number of charts from the following sources:
  – Oded Galor (2005) chapter on the “Unified Growth”
  – Greg Clark (2007), A farewell to Alms, Ch. 1-5
  – Hazan and Berdugo (2002), Economic Journal