Market Structure: Competition vs. Monopoly

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Introduction

In this section, we will start to study how the actions of consumers and firms determines market prices. We will make different assumptions on how the agents treat prices, in particular, whether they consider that these are beyond or within their control.

1. **Competitive Markets**: How does the interaction of agents who think that prices are beyond their control determines the equilibrium quantities and prices in a given market.

2. **Monopoly**: What happens if some agents think that they have a dominant position and that can control prices. What are the quantities and prices in this case?
Outline

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Competition in Markets

- A **Competitive Firm** is one that takes the market price of output as being given and outside its control.

- Let \( \bar{p} \) be the market price. Then, the demand curve facing a competitive firm takes the form,

\[
D(p) = \begin{cases} 
0 & \text{if } p > \bar{p} \\
\text{any amount} & \text{if } p = \bar{p} \\
\infty & \text{if } p < \bar{p}
\end{cases}
\]

- From the equation above it is clear that a competitive firm would want to sell at market prices. If they try to sell above market prices, they will sell 0 units. If they sell below market price, then they will unnecessarily forgo profits as they will sell their output at a lower price.

- **Discussion:** How reasonable is this assumption in the real world? Think of competition as a “polar” case.
The Profit Maximization Problem

- The competitive firm chooses output as to maximize profits
  \[ \pi = \max_y (py - c(y)) \]
- FOC and SOC are: FOC:
  \[ p = c'(y^*) \]  
  SOC:
  \[ c''(y^*) \geq 0 \]
- We will often assume that SOC are fulfilled with strict inequality. We will call this the regular case.
- Define the inverse supply function \( p(y) \) as the price at which the optimal quantity produced by the firm is exactly \( y \). Under the regular assumption, this function is given by the FOC
  \[ p(y) = c'(y) \]
The Supply Function and Comparative Statics

- Define the **supply function** as the profit-maximizing output at each price.

\[ p \equiv c'(y(p)) \quad (3) \]

- How does the output of the firm changes with prices? We differentiate (3) with respect to \( p \)

\[ 1 \equiv c''(y(p))y'(p) \implies y'(p) = \frac{1}{c''(y(p))} > 0 \quad (4) \]

- We focused on the interior solution. Note that if we assume fixed costs (\( F \)), then it becomes apparent that firms would choose to produce so long as the lose less than \( F \), the amount they will lose if they chose not to produce at all.

\[ \pi(y(p)) \geq -F = p y(p) - c_v(y(p)) - F \geq -F \implies p \geq \frac{c_v(y(p))}{y(p)} \]
The Industry Supply Function

- The **industry supply function** is the sum of the individual firm supply functions. If there are \( m \) firm in the industry, the supply function is,

\[
Y(p) = \sum_{i=1}^{m} y_i(p).
\]

- In analogy to the individual firm case, the **industry inverse supply function** is just the inverse of this function.

**Example:** Different cost functions. Assume two firms, \( c_1(y) = y^2 \) and \( c_2(y) = 2y^2 \). FOC imply that \( p = c'(y) \) so that \( p = 2y_1(p) \) and \( p = 4y_2(p) \). The industry demand function is just

\[
Y(p) = \sum_i y_i(p) = p/2 + p/4 = 3/4p \Rightarrow \text{IISF} : p = \frac{4}{3}Y
\]

**Example:** Identical cost functions. \( m \) firms with cost function \( c(y) = y^2 + 1 \). FOC yield \( y_i(p) = p/2 \). The industry supply function is

\[
Y(p) = \sum_{i=1}^{m} y_i(p) = \sum_{i=1}^{m} p/2 = mp/2 \Rightarrow \text{IISF} : p = \frac{2}{m}Y
\]
Market Equilibrium

- An **equilibrium price** is a price where the amount demanded equals the amount supplied. Why do we call it “equilibrium”? If supply and demand are not equal, then some agents have incentives to change their behavior. e.g. excess demand would drive some consumers out of the market while driving producers into the market.

- Let $x_i$ with $i = 1, 2, ..., n$ denote the demand function of each of the $n$ consumers and $y_j$ denote the supply function of each of the $m$ firms. An **equilibrium price** fulfills the following equation,

$$\sum_{i=1}^{n} x_i(p) = \sum_{j=1}^{m} y_j(p)$$

(5)
Market Equilibrium: An example

- Assume the aggregate demand function is $X(p) = a - bp$ and the m firms have identical cost function $c(y_j) = y_j^2 + 1$. We have shown that the industry supply function is $Y(p) = mp/2$. (5) implies that in equilibrium $X(p) = Y(p)$. Thus

$$a - bp = \frac{mp}{2} \implies p^* = \frac{2a}{m + 2b}$$

- In this example, as the number of firms grows, the inverse supply function becomes flatter and the equilibrium price is reduced. Does this property generalize?

- Consider a generic demand function $X(p)$ and m identical $y(p)$. Then (5) implies, $X(p) = my(p)$. Treat m as an implicit function of p, $X(p(m)) \equiv my(p(m))$. We can differentiate this equation:

$$X'(p)p'(m) = y(p) + my'(p)p'(m) \implies p'(m) = \frac{y(p)}{X'(p) - my'(p)} < 0$$
So, in general, with identical cost functions. If the supply function is positively sloped, then the equilibrium price should go down as \( m \) increases.

In the previous calculations we assumed \( m \) is fixed. However, it would be reasonable to endogeneize the number of firms. This takes us to the **entry** and **exit** models. There could be different models depending on the assumption on entry and exit costs.

Assume 0 entry/exit cost and perfect foresight. Let define the **break-even price** \( p^* \) as the price for which the supply function yields 0 profits: \( p y(p) - c(y) = 0 \) and \( p = c'(y) \) implies \( c'(y) = c(y)/y \).
We have seen that if there are many firms in an industry, the supply function will be very flat. This, together with the break-even condition suggests that the supply curve of a competitive industry with free entry is simply a flat line that touches the minimum average cost.

The equilibrium price could be larger than the break-even price if entry is inhibited by potential entrants perceive that their entrance would yield negative profits.

Recall that “economic profits” are rents. In this case, this would be a rent for being first in a given market.
Welfare Analysis: Representative Consumer

- In this section we investigate the welfare implications of equilibrium. We will use the representative consumer approach.

- Let's assume that the market demand curve $x(p)$ is generated by maximizing utility of a single representative consumer who has utility $u(x, y) = u(x) + y$, where $x$ is the good we are interested in studying and $y$ is a kind of aggregate good, often assumed to be money.

- We have seen that these preferences (quasilinear) produces FOC $u'(x(p)) = p$, from which the demand curve can be obtained. Recall that this demand curve is independent from income. This is useful as simplifies greatly welfare analysis (EV=CS=CV)
Now, let’s focus on the other side of the market. We can assume a representative firm with cost function \( c(x) \), interpreted as the cost of producing \( x \) units of the first good are \( c(x) \) units of good y. We assume \( c(0) = 0 \), \( c'(x) > 0 \) and \( c''(x) > 0 \). This will ensure that we have a unique solution to the firm’s problem. In this point \( p = c'(x) \)

Market equilibrium occurs when \( p = c'(x) = u'(x) \). This means that the marginal utility (marginal willingness to pay) of the good must be equal to the cost of producing it at the equilibrium.
Imagine that we do not have a competitive market. We just have a consumer that has the quasilinear utility described earlier and access to the same technology. Let’s call his problem the “Welfare Maximization Problem”

WMP: $\max_{x,y} u(x) + y$ subject to $c(x) + y = \omega$. $\omega$ is to be interpreted as the original endowment of good y. Replacing, $\max_x u(x) + \omega - c(x)$

$FOC \Rightarrow u'(x) = c'(x)$ which is the same condition than when we set up a competitive market. In particular, under this arrangement the quantities produced and consumed of x are going to be the same!
Let define the **consumer’s surplus** in this context as the difference between the utility achieved by some output level minus the cost of the goods. In this case, \( CS(x) = u(x) - px \). Similarly, define **producer’s surplus** at production level \( x \) as the total receipts for producing \( x \) minus the cost of production. \( PS(x) = px - c(x) \)

It is evident that the welfare maximization problem could be rewritten as:

\[
\max_x u(x) + \omega - c(x) = \max_x CS(x) + PS(x) + \omega = \max_x CS(x) + PS(x)
\]

where the last equality follows from the fact that \( \omega \) is a constant.

So, a competitive market yields the same solution than directly maximizing welfare. In particular, these problems are equivalent to maximizing **total surplus**, \( TS = CS + PS \)
Several Consumers and Firms

- So far we considered the case of a single consumer and firm. This is easily extended to the case of many consumers indexed by $i, i = 1, 2, ..., n$ and firms indexed by $j$ with $j = 1, 2, ..., m$

- Each consumer has a quasilinear utility function $U_i(x_i, y_i) = u_i(x_i) + y_i$ and an initial endowment equal to $\omega_i$ (as before, this is measured in terms of the $y$-good). Each firm faces a cost function that tells them the cost of producing $z_j$ units of the $x$-good is $c(z_j)$

- Now the candidate for welfare maximum is the combination that maximizes the combined utility of the $n$ consumers that is feasible, i.e. the $j$-firms can jointly produce that output level. More formally,
Several Consumers and Firms

\[
\max_{\{x_i, y_i\}} \sum_{i=1}^{n} u_i(x_i) + \sum_{i=1}^{n} y_i
\]

subject to

\[
\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \omega_i - \sum_{j=1}^{m} c_j(z_j)
\]

and

\[
\sum_{i=1}^{n} x_i = \sum_{j=1}^{m} z_j
\]

- The third condition is the new one: it simply says that at the optimum firms have to choose to produce in aggregate exactly how much consumers choose to consume. Putting 1 restriction into the objective function, the problem becomes...
Several Consumers and Firms

\[
\max \left\{ x_i, z_j \right\} \sum_{i=1}^{n} u_i(x_i) + \sum_{i=1}^{n} \omega_i - \sum_{j=1}^{m} c_j(z_j)
\]

subject to

\[
\sum_{i=1}^{n} x_i = \sum_{j=1}^{m} z_j
\]

let \( \lambda \) denote the lagrange multiplier, then FOC imply

\[
x_i : \ u_i'(x_i^*) - \lambda = 0
\]

\[
z_j : \ - c_j'(z_j^*) + \lambda = 0
\]

\[
\lambda : \ \sum_{i=1}^{n} x_i^* = \sum_{j=1}^{m} z_j^*
\]
Again, in this case we have a similar condition that will apply if we had an optimum price \( p^* = \lambda \). So, the market equilibrium maximizes welfare, *at least* to the extent that can be measured by the sum of utilities of individuals!

Related to that point, total utility will typically depend on the distribution of the initial endowment \( \omega_i \). In the case of quasilinear utilities, total utility is independent of the distribution on endowments and any pattern of \( \omega_i \) is consistent with equilibrium.
Pareto Efficiency

- A competitive equilibrium maximizes the sum of utilities. It is not obvious why this should be the relevant objective function to maximize.

- A more general objective would be that it is not possible to improve one agent’s welfare without reducing the welfare of somebody else. This is the idea of **Pareto efficiency**. If I can make at least at least one agent better off, without reducing the welfare of the rest, then there is still room for improvement.

- When I reach a point in which I cannot improve anybody without harming the rest, then I am at a Pareto efficient point. Note that “pareto efficiency” is relatively free of moral judgements. For instance, if people have positive marginal utilities, then a combination in which one agent has everything and the rest has nothing will be a pareto efficient point!
Pareto Efficiency

Imagine that you have 2 agents in the economy. The total availability of good $x$ and $y$ in the economy is $(\bar{x}, \bar{y})$. We would like to find the possible bundles that would maximize the utility of 1 subject to the constraint that agent 2 gets at least as much utility as before $(\bar{u})$.

$$\max_{x_1,y_1} u_1(x_1) + y_1$$  \hspace{1cm} (6)

subject to

$$u_2(\bar{x} - x_1) + \bar{y} - y_1 = \bar{u}$$  \hspace{1cm} (7)

FOC $\Rightarrow u'_1(x^*_1) = u'_2(\bar{x} - x^*_1) = u'_2(x^*_2)$

So Pareto efficiency requires that the marginal utility of the consumption is equalized across consumers.
Pareto Efficiency

- Note that the competitive equilibrium satisfies the necessary condition for pareto efficiency (equalized marginal utilities):
  \[ p^* = u'_1(x_1^*) = u'_2(x_2^*) \]
- It turns out that this result holds in general (we will study the welfare theorems later) and not only in the case of quasilinear preferences.
Pareto Efficiency and Welfare

It may seem puzzle that the same $x_1^*, x_2^*$ solve both the welfare and pareto optimality problems. Let’s have a further look on that. Consider the 2 agent case as before with endowments $(\bar{x}, \bar{y})$

The “welfare maximization problem looks like:

$$\max_{x_1, y_1, x_2, y_2} u_1(x_1) + y_1 + u_2(x_2) + y_2 \text{ subject to } x_1 + x_2 = \bar{x}; y_1 + y_2 = \bar{y}$$

which can be simplified to

$$\max_{x_1, y_1} u_1(x_1) + y_1 + u_2(\bar{x} - x_1) + \bar{y} - y_1$$

The “pareto efficiency” problem instead is framed as

$$\max_{x_1, y_1} u_1(x_1) + y_1 \text{ subject to } u_2(\bar{x} - x_1) + \bar{y} - y_1 = \bar{u}$$

which is simplified to

$$\max_{x_1, y_1} u_1(x_1) + y_1 + u_2(\bar{x} - x_1) + \bar{y} - \bar{u}$$
The two problems look identical except for the fact that $y_1$ vanishes in the welfare problem but not in the pareto optimality problem.

This simply means that while any $y_1$ would be consistent with the welfare equilibrium, only the $y_1$ such that the utility of 2 remains fixed at $\bar{u}$ will be consistent in the pareto optimality problem.

Summarizing, while any $(y_1, y_2)$ would solve the first problem, only one specific $y_1$ would solve the second problem. A special feature of quasilinear utility generates this. Under QL utility, $x_1^*, x_2^*$ is determined independently of income $(y_1, y_2)$. So the set of all pareto optima allocations is simply $(x_1^*, x_2^*, y_1, y_2)$ where $y_1 + y_2 = \bar{y}$. (interior solution of QL preferences is assumed).
Taxes and Subsidies

- A natural change in economic environment is caused by taxes and subsidies. They motivate the idea of comparative statics in economics.
- A tax will generate a wedge between the price consumers pay for the good (demand price $p_d$) and the price producers receive for their output (supply price, $p_s$).
- Let us consider a **quantity tax**, that is, a tax levied on the amount of a good consumed. If there is a tax of $t$ per each unit of the good consumed, then the $p_d = p_s + t$.
- Alternatively, a **value tax** is a tax levied on the expenditure in a given good. If the rate at which expenditure is taxes is $\tau$, then $p_d = (1 + \tau)p_s$. This is the case of the VAT.
- Subsidies are simple “negative taxes”
Taxes and Subsidies

- How does equilibrium change in the presence of subsidies or taxes?

- In the case of the unit tax, the familiar demand equal supply equation $D(p) = S(p)$ still hold but we have to acknowledge that demand reacts to the demand price while supply depends on the supply price. Solving, $D(p_d) = S(p_d - t)$ or $D(p_s + t) = S(p_s)$.

- In the case of value taxes the same concept is equal, but the relation between the prices is slightly different. In that case $D(p_d) = S(p_d/(1 + \tau))$ or $D(p_s(1 + \tau)) = S(p_s)$ would yield the result.

- **Deadweight loss** Since taxing will typically increase prices for consumers and decrease it for producers, it is related to lower production and consumption. The loss in welfare due to the introduction of the tax is called deadweight loss.
Outline

1 Topics

2 Competitive Markets

3 Monopoly
   - Monopolist Problem
   - Comparative Statics and Welfare
   - Quality choices
   - Price discrimination
     - First-degree price discrimination
     - Second-degree price discrimination
     - Third-degree price discrimination
Monopoly: Situation in which one firm is the only seller in one market. Note that the definitions is sensitive to how we measure a “market”. Many firms participate in the demand for beverages, but only one in the demand for the mineral water branded “Evian”. If a firm has a demand defined over the good it sells (e.g. consumers are not indifferent to the brand of the mineral water), then “Evian” can behave as a monopolist in her market, setting up prices or production levels.

Under competitions, firms were price-takers. Under monopoly, firms would be price-makers.

The monopolist faces 2 constraints:

1. **Technological**: Just as before there is a technology to transform inputs into output. We will summarize this in the cost function $c(y)$

2. **Market**: Consumers’ demand is a function of how much I want to charge for my output (this problem did not exist before).
The monopolist profit maximization problem is:

$$\max_{p,y} py - c(y) \quad \text{s.t.} \quad D(p) \geq y$$

Typically, the consumer will want to produce exactly the amount of goods that he is selling. In that case $D(p) = y$ and the problem simplifies to:

$$\max_{p} pD(p) - c(D(p))$$

In general, it will be more useful for us to present the problem as one of choosing quantities. Using the inverse demand function $p(y) = D^{-1}(y)$. Then,

$$\max_{y} p(y)y - c(y)$$

The first and second order conditions for maximization require that:
FOC and SOC

\[ p(y) + p'(y)y - c'(y) = 0 \]  \hspace{1cm} (8)

\[ p'(y) + p''(y)y + p'(y) - c''(y) = 2p'(y) + p''(y)y - c''(y) \leq 0 \] \hspace{1cm} (9)

- The first condition is the familiar “Marginal Revenue=Marginal Cost” condition. Let define \( r(y) = p(y)y \) as the revenue of the firm. Clearly, equation (8) is just \( r'(y) = c'(y) \).
- Economic intuition for the change in revenue:
  1. **DIRECT EFFECT**: If he increases his sales by a small quantity \( dy \) he makes an additional \( pdy \).
  2. **INDIRECT EFFECT**: In order to increase his sales he has to reduce prices slightly over all units. That is, let \( dp/dy \) be the change in prices necessary to sell \( dy \) units more, now the indirect effect amounts to \( y(dp/dy)dy \). Since \( dp/dy < 0 \), this goes against the direct effect. Note that \( IE + DE = pdy + ydp = dr(y) \).
Elasticity condition

- Note that,
  \[ r'(y) = \frac{dp(y)}{dy} y + p(y) = p(y)[\frac{dp(y)}{dy} \frac{y}{p(y)} + 1] = p(y)[1 - \frac{1}{\epsilon(y)}] \]
  
  where \( \epsilon(y) \) is just the price-elasticity of demand, i.e. the percent variation in demand due to a one percent in prices.

\[ \epsilon(y) = -\frac{dy/y}{dp(y)/p(y)} = \frac{dy}{dp(y)} \frac{p(y)}{y} \]

- Note that the monopolist would never operate at a point in which \( \epsilon < 1 \).

1. **Economic argument:** If \( \epsilon < 1 \), and I increase prices by 1 percent my demand decreases \( \epsilon \) percent. Since \( \epsilon < 1 \), my overall revenue goes up and my total costs go down (I am producing less!). Hence, profits increase which is inconsistent with maximizing behavior in the first place. It is easy to see that revenue increases.

\[
q^1 = q^0(1 - (\epsilon/100)) \quad \text{and} \quad p^1 = p^0(1.01), \quad \text{then} \quad p^1 q^1 = (1 + 0.01)(1 - (\epsilon/100))p^0 q^0 \approx (1 + 0.01 - (\epsilon/100))p^0 q^0 > p^0 q^0.
\]

2. **Mathematical argument.** If \( \epsilon < 1 \) then \( r'(y) < 0 \) and since \( c'(y) > 0 \), FOC cannot be fulfilled.
Graphical Example

- Note that FOC imply \( r'(y) = c'(y) \). SOC, in turn, require that \( r''(y) + c''(y) \leq 0 \). That is, that the point where \( r'(y) = c'(y) \) the slope of \( r'(y) \) is smaller than the slope of \( c'(y) \). In other words, it requires that \( r'(y) \) crosses \( c'(y) \) from above.
- Note also that \( r'(y) = p(y) + p'(y)y \). Since demands are usually negatively sloped, then \( r'(y) < p(y) \), the marginal revenue function lies below the demand function.
- Graph †
- Linear Case: Assume \( p(y) = a - by \) and \( c(y) = cy \). Then \( y^* = \frac{a-c}{2b} \) and \( p^* = \frac{a+c}{2} \). †
- Constant Elasticity Demand: \( p^* = \frac{c}{1-\frac{1}{\epsilon}} \), i.e. the optimal price is just a constant markup over cost. †
Comparative Statics

- Assume for simplicity that marginal cost is fixed at $c$. So that $c(y) = cy$
- Define $y(c)$ as the optimal response to changes in marginal cost. Then $p'(y(c))y + p(y(c)) \equiv c$
- The implicit function theorem yields: $y'(c) = \frac{1}{p''(y(c))y(c)+2p'(y(c))}$
- The chain rule implies that: $\frac{dp}{dc} = \frac{dp}{dy} \frac{dy}{dc} = \frac{p'(y(c))}{p''(y(c))y(c)+2p'(y(c))}$
- Simplifying: $\frac{dp}{dc} = \frac{1}{\frac{p''(y(c))y(c)}{p'(y(c))}+2}$
- Linear function $\Rightarrow \frac{dp}{dc} = \frac{1}{2}$. Constant elasticity of demand $\Rightarrow \frac{dp}{dc} = \frac{\epsilon}{\epsilon-1} \cdot \frac{1}{2}$
- Note that in the case of constant elasticity, the price increases more than proportionally when cost increase!  

\[1\text{In the book some formulas differ. This is because } \epsilon = -\epsilon_{\text{Varian}}\]
We have seen that if markets are competitive $p = c'(y)$ yields a quantity that is Pareto efficient.

We have seen that $r(y)$ is always below the inverse demand $p(y)$, so that monopoly must produce lower quantities (look at the graph). However, how severe is this inefficiency is yet to be studied.

Assume a 1-consumer economy with quasilinear utility $u(x) + y$. This yields a nice inverse utility function $p(x) = u'(x)$. Let $c(x)$ denote the cost (in terms of $y$) of producing $x$ units of the $x$-good. A welfare function is the one that maximizes utility $y = m - c(x)$ and $m$ is fixed, this is like maximizing

\[ W(x) = u(x) - c(x) \]
Welfare Considerations

- At the social optimum, FOC require that \( u'(x_{so}) = c'(x_{so}) \). As we saw, monopoly satisfies the relation, \( p'(x_m)x_m + p(x_m) = c'(x_m) \). Use the fact that \( p(x) = u'(x) \), then this condition implies that

\[
 u''(x_m)x_m + u'(x_m) = c'(x_m) \tag{10}
\]

- The derivative of the social welfare function evaluated at the monopoly equilibrium \( x_m \) is

\[
 W'(x_m) = u'(x_m) - c'(x_m) = -u''(x_m)x_m > 0 \tag{11}
\]

where I have used (10) and the fact that \( u(.) \) is a concave function.

- This means that at the monopolist quantities, we can still improve welfare as there are people willing to pay more than what it costs to produce one more unit of \( x \). Graph \( \dagger \)
So far, we assumed that the only dimension of the monopolist choice were quantities. What if the monopoly could decide over the quality of the good produced? Let $q$ denote the units of “quality”. It is reasonable to assume that $\frac{\partial u(x, q)}{\partial q} > 0$ and $\frac{\partial c(x, q)}{\partial q} > 0$. That is, consumers value quality and quality is costly.

The monopolist maximization problem is now:

$$\max_{x,q} p(x, q)x - c(x, q).$$

FOC for this problem are:

$$p(x_m, q_m) + \frac{\partial p(x_m, q_m)}{\partial x} x_m = \frac{\partial c(x_m, q_m)}{\partial x}$$  \hspace{1cm} (12)

$$\frac{\partial p(x_m, q_m)}{\partial q} x_m = \frac{\partial c(x_m, q_m)}{\partial q}$$  \hspace{1cm} (13)
Welfare Considerations

- Adding quality to the welfare function we saw before $W(x, q) = u(x, q) = c(x, q)$. The first order conditions evaluated, not at the optimum, but at the monopolist equilibrium yield:

$$\frac{\partial W(x_m, q_m)}{\partial x} = \frac{\partial u(x_m, q_m)}{\partial x} - \frac{\partial c(x_m, q_m)}{\partial x} = -\frac{\partial p(x_m, q_m)}{\partial x} x_m > 0$$

where I have used the fact that preferences are quasilinear and equation (12) to get the second equality.

- Similarly, you can use FOC for the amount of quality and equation (13) to get:

$$\frac{\partial W(x_m, q_m)}{\partial q} = \frac{\partial u(x_m, q_m)}{\partial q} - \frac{\partial c(x_m, q_m)}{\partial q} = \frac{\partial u(x_m, q_m)}{\partial q} - \frac{\partial p(x_m, q_m)}{\partial q} x_m$$
As it was before, the first equation says that keeping quality fixed, an increase in quantity will increase social welfare.

The effect of an increase of quality on welfare are, however, ambiguous. Clearly, the second part (the marginal cost) is positive and so is the first part (marginal utility). At the monopolist quantities, it is not clear whether the whole expression will be positive or negative!

Rearranging the second equation:

\[
\frac{1}{x_m} \frac{\partial W(x_m,q_m)}{\partial q} = \frac{\partial}{\partial q} \left[ \frac{u(x_m,q_m)}{x_m} - p(x_m, q_m) \right]
\]

The derivative of the welfare with respect to quality is proportional to the derivative of the average willingness to pay minus the marginal willingness to pay. Unless that these two are equalized at the monopolist solution \((x_m)\), this will fail to produce the optimal quantity of “quality”.

Welfare Considerations
Price discrimination occurs when a monopolist can sell different units of the same good at different prices, either to the same or different consumers.

Recall that the main problem why the monopolist sold too few units is because in order to sell an extra unit, it had to reduce the price of all the previously sold units. If the monopolist finds a way to sell at different prices the same good, then he will sell more units.

A key issue is that the markets should be fragmented, in the sense, that if there are two prices but people can sell and buy between them, then everybody would have an incentive to arbitrage, i.e. buy in the market that sells cheap and sell it in the market that buys at the expensive price.
**Idea**

- The easy way to discriminate is to do it with respect to some category that is endogenous to consumers, i.e. age.

- If the discrimination is done over a choice variable for the consumer (quantities consumed, time of the purchase) then the monopolist has to structure his pricing in such a way that the consumers will sort themselves out in the right category.

- There are three types of price-discrimination
  
  1. First-degree price discrimination/perfect price discrimination: The monopolist can charge to each consumer exactly a price equal to her willingness to pay for that unit.

  2. Second-degree price discrimination: When prices differ according to the number of units bought but not on the consumer. e.g. quantity discounts.

  3. Third-degree price discrimination: Different consumers get charged different prices.
First-degree price discrimination

Imagine that the monopolist faces only one agent. He gets to offer a “take-it-or-leave-it” combination of output and (overall) price \((r^*, x^*)\). Assuming linear costs as before, his profit maximization problem would be:

\[
\max_{r,x} r - cx \quad \text{s.t.} \quad u(x) \geq r
\]

where the constraint just says that the consumer should be willing to accept the deal, so he should get at least enough utility from consuming the good than what he pays for it.

FOC for this problem, noting that the monopolist would never choose \(r^* < u(x)\), is:

\[u'(x^*) = c \Rightarrow r^* = u(x^*)\] (14)
Features of FDPD

There are a few interesting characteristics of this solution:

1. The amount produced coincides with the amount produced under competition (see (14)). The monopolist chooses a *Pareto optimum* quantity.
2. However, in sharp contrast with the competitive solution, the monopolist gets *all* the surplus (both the consumers and the producers)!
3. The reason for this is that the consumer can sell now additional units without lowering the price of the units he had “previously” sold.
Example

Assume that the monopolist can break up its production in n-pieces \( x = n\Delta x \) and, crucially, that he can charge a different price for each fraction \( p_i \), where \( i \) stands for the order in which the fraction is sold. As usual, assume quasilinear preferences on the consumer.

Then he would charge a price so that the consumer is just indifferent between buying it or not. For the first \( \Delta x \) units in the market we will equalize

\[
\underbrace{u(0) + m}_{\text{He doesn't consume the good}} = \underbrace{u(\Delta x) + m - p_1}_{\text{He consumes the first } \Delta \text{ units}} \implies u(0) = u(\Delta x) - p_1
\]

Similarly for the second fraction, the consumer would already be consuming \( \Delta x \), so the monopolist would equalize:

\[
u(\Delta x) + m - p_1 = u(2\Delta x) + m - p_1 - p_2
\]
You can continue up until the last unit of the good.

\[ u((n - 1)\Delta x) = u(n\Delta x) - p_n \]  \hspace{1cm} (16)

Summing up all these n equations we get:

\[ \sum_{i=1}^{n} u((i - 1)\Delta x) = \sum_{i=1}^{n} u(i\Delta x) - \sum_{i=1}^{n} p_i \]  \hspace{1cm} (17)

Canceling out all the intermediate terms in the sum of u(.)’s,

\[ u(0) = u(n\Delta x) - \sum_{i=1}^{n} p_i \]  \hspace{1cm} (18)

Normalizing \( u(0) = 1 \) and using the fact that \( x = n\Delta x \),

\[ u(x) = \sum_{i=1}^{n} p_i \]  \hspace{1cm} (19)
What equation (19) says is that the sum of the *marginal* willingness-to-pay should equal the *aggregate* willingness to pay for that amount of $x$.

So as long as the firm can charge a different price each different unit of the same good, she would be producing at a point in which everybody who is willing to pay more than what the good costs, would get the good. It is not crucial the assumption of a take-it-or-leave-it offer we did before!
Second-degree price discrimination

- This is non-linear prices. That is, charging different prices according to the quantities bought (e.g. quantity discounts).
- Assume there are 2 consumers with utility functions $u_1(x_1) + y_1$ and $u_2(x_2) + y_2$, where $u_2(x) > u_1(x)$ and $u'_2(x) > u'_1(x)$. We will call 2 the *High-demand consumer* and 1 the *low-demand consumer*.
- The property that the agent with the largest utility (and average utility) of $x$, also has the largest marginal utility at $x$ is called **single crossing property**. This condition implies that the indifference curves can cross at most once.
- Now the monopolist gets to choose a function $r_i = p(x_i)x_i$ that shows how prices change with $x_i$.
The monopolist has 2 set of restrictions. First, consumers should find it convenient to purchase the good

\[ u_1(x_1) - r_1 \geq 0 \]

\[ u_2(x_2) - r_2 \geq 0 \]

Second, each consumer has to **self-select** himself into the group that maximizes monopolist income

\[ u_1(x_1) - r_1 \geq u_1(x_2) - r_2 \]

\[ u_2(x_2) - r_2 \geq u_2(x_1) - r_1 \]
These equations imply the following restrictions. In general, only one of each restrictions will be binding as the monopolist wants to maximize $r_1$ and $r_2$.

$$r_1 \leq u_1(x_1) \quad (20)$$

$$r_1 \leq u_1(x_1) - u_1(x_2) + r_2 \quad (21)$$

$$r_2 \leq u_2(x_2) \quad (22)$$

$$r_2 \leq u_2(x_2) - u_2(x_1) + r_1 \quad (23)$$

It turns out that with the single crossing property it is enough to know which inequality will be binding.
Idea

- Assume (22) is binding. Then \( r_2 = u_2(x_2), ((23)) \Rightarrow u_2(x_1) \leq r_1. \)
- But this last inequality leads to a contradiction.

\[
u_1(x_1) < u_2(x_1) \leq r_1 \leq u_1(x_1). \quad \text{Where the first inequality comes from single crossing and the third comes from (20). Then (23) should be binding.}
\]

- Let’s turn to the other set of restrictions. Assume (21) is binding. Then \( r_1 = u_1(x_1) - u_1(x_2) + r_2. \) Substitute \( r_2 \) from (23) that we now know is binding. Then,

\[
\mathcal{r}_1 = u_1(x_1) - u_1(x_2) + u_2(x_2) - u_2(x_1) + \mathcal{r}_1
\]

- This last equality means that

\[
\int_{x_1}^{x_2} u_1'(x)dx = \int_{x_1}^{x_2} u_2'(x)dx \quad (24)
\]

- But this contradicts the single crossing property: \( u_2'(x) > u_1'(x). \) Then (20) is binding.
Putting these two results together we get that $r_1 = u_1(x_1)$ so that the low demand consumer is charged his willingness to pay. Also, $r_2 = u_2(x_2) - u_2(x_1) + r_1$, so the high demand agent is charged the maximum amount possible provided that he would still consume $x_2$.

The profits of the consumer are given by

$$\pi = [r_1-cx_1] + [r_2-cx_2] = u(x_1) - c(x_1) + u_2(x_2) - u_2(x_1) + u(x_1) - cx_2$$

where we have used the two conditions from above.

FOC yield:

$$u_1'(x_1) - c + u_1'(x_1) - u_2'(x_1) = 0 \quad (25)$$

$$u_2'(x_2) - c = 0 \quad (26)$$
(25) can be rearranged to show that:

$$u'_1(x_1) = [u'_2(x_1) - u'_1(x_1)] + c > c$$  \hspace{1cm} (27)

where the last inequality follows from single crossing (i.e. $u'_2(x) > u'_1(x)$ for all $x$).

From (26) and (27) there are 2 outstanding facts. Under non-linear prices, the consumer with “low demand” consumes too few units and too expensive (the monopolist fixes the quantities/price at a point in which the marginal willingness to pay exceeds the cost of producing it. Recall, at a socially optimal consumption bundle, the marginal willingness to pay has to be equal to marginal cost.

The high demand consumer would consume exactly as many units as socially efficient (that is, marginal willingness to pay equals the marginal cost).
Final thoughts

- In general, we will always expect the high-demand to consume the right quantity. This is so because if the high type were consuming less, the monopolist could reduce the price he’s facing and (since price is above marginal cost) still make a profit. Unlike the non-discrimination case, the change in price would only affect the high type and not the rest of the buyers!
Third-degree price discrimination

- In this case, prices are the same for consumers in the same group, but vary across different groups of consumers. Very common situation: discounts for the elderly, discount for the young etc.
- In this case, the separation between the two groups is very simple. Assume the cost of the units is the same regardless of the group the monopolist is selling to. Let $p_i(x_i)$ denote the inverse demand function. Then the monopolist is seeking to maximize $p_1(x_1)x_1 + p_2(x_2)x_2 - cx_1 - cx_2$.
- FOC in this case are,
  $$p'_i(x_i)x_i + p_i(x_i) = c$$
- Which leads to a elasticity condition (see above),
  $$p_i(x_i)[1 - \frac{1}{\epsilon_i}] = c$$
This conditions them mean,

\[ p'_1(x_1)[1 - \frac{1}{\epsilon_1}] = p'_2(x_2)[1 - \frac{1}{\epsilon_2}] \]

That is if \( \epsilon_1 < \epsilon_2 \) then \( p_1 > p_2 \). Is it intuitive?

Think about it, you are charging more to the group the the most inelastic demand. Since they are not so responsive to prices, you can charge them more. The group with elastic demands, would reduce significatively their demand when you increase the price.

Think about next time you get a student discount!
In the previous discussion we assumed the monopolist could separate markets perfectly. What if that’s not the case? “Dia del espectador”. In practical terms this means that $p_i(x_1, x_2)$. The monopolist problem is now,

$$\max_{x_1, x_2} p_1(x_1, x_2)x_1 + p_2(x_2, x_2)x_2 - cx_1 - cx_2$$

The first order conditions now can be written as

$$p_i + \frac{\partial p_i(x_1, x_2)}{\partial x_1} + \frac{\partial p_i(x_1, x_2)}{\partial x_2} = c$$

In terms of elasticity the condition becomes now

$$p_1[1 - \frac{1}{\epsilon_1}] + \frac{\partial p_2}{\partial x_1 x_2} = c$$

$$p_2[1 - \frac{1}{\epsilon_2}] + \frac{\partial p_1}{\partial x_2 x_1} = c$$
Putting these two together (assuming QL preferences so, \( \frac{\partial x_1}{\partial p_2} = \frac{\partial x_2}{\partial p_1} \)),

\[
p_1\left[1 - \frac{1}{\epsilon_1}\right] - p_2\left[1 - \frac{1}{\epsilon_2}\right] = [x_1 - x_2] \frac{\partial p_2}{\partial x_1}
\]

What is the sign of this expression? \( \frac{\partial p_2}{\partial x_1} \) is probably positive. Why? After all a movie on Thursday is likely to be a substitute of a movie on Wednesday.

Assume (WOLOG) that \( x_1 > x_2 \), then \( p_1\left[1 - \frac{1}{\epsilon_1}\right] - p_2\left[1 - \frac{1}{\epsilon_2}\right] > 0 \)

So \( \frac{p_1}{p_2} > \frac{1 - \frac{1}{\epsilon_2}}{1 - \frac{1}{\epsilon_1}} \)

It follows that if \( \epsilon_2 > \epsilon_1 \) then \( p_1 > p_2 \). So if the smaller market has the more elastic demand, the initial intuition remains.
Welfare considerations of third-order price discrimination

- Idea: Are consumers better off if the monopolist can use price discrimination?
- Under quite general conditions \(^2\) the necessary and sufficient conditions for welfare improvement are:
  - Necessary condition: The total quantities sold increase when price discrimination is allowed.
  - Sufficient condition: The weighted sum of quantities are positive. Where each quantity is weighted by the difference between price and marginal cost. Imagine this set up: \(MC = 5, p_1 = 16, p_2 = 9\), \(\Delta x_1 = -4\) and \(\Delta x_2 = 10\). The initial price was 10. The total quantity increased \(\Delta x_1 + \Delta x_2 = 6\) but the weighted quantity is \((16 - 5)(-4) + (9 - 5)(10) = -4\). In this case, welfare decreases.

- Some special cases: (1) Linear demand implies that no change in overall quantities. It is not welfare increasing; (2) If discrimination opens new markets (Ryanair!) then it is welfare increasing.

\(^2\)Quasilinear utility function with \(u(.)\) strictly concave, constant marginal cost