Convergence, capital accumulation and the nominal exchange rate

Peter Benczur\textsuperscript{1} and Istvan Konya\textsuperscript{1}

September 2008

Abstract

This paper develops a flexible price, two-sector growth model with a nominal side in order to study the role of the exchange rate regime in transition dynamics (capital accumulation). We adopt a standard model of a small open economy with traded and nontraded goods, and enrich its structure with costly investment and a preference for real money holdings. We show analytically that (i) the choice of the exchange rate regime influences the transition dynamics of a small open economy, (ii) a one-sector model does not adequately capture the channels through which the nominal side interacts with real variables, and (iii) as a consequence, sectoral asymmetries are important for understanding the effects of the exchange rate regime on capital accumulation. We then calibrate the model to explore the quantitative significance of our results. We find that both the choice of the exchange rate regime and the level of the exchange rate in a currency board have significant and lasting effects on consumption, investment, sectoral allocations, and the composition of financial assets.

JEL Classification Numbers: F32, F41, F43

Keywords: two-sector growth model, small open economy, capital accumulation, household portfolios, real effects of nominal shocks.

1 Introduction

The nominal exchange rate is one of the most important prices for a small open economy. There are strong linkages among permanent or temporary exchange rate movements, the external position, the growth rate and fluctuations of the economy, the latter often showing sectoral asymmetries as well. In this paper we show that the exchange rate is not only important for

\textsuperscript{*}We are grateful to Andrew Blake, Ágnes Csermely, William Gavin, Michal Kejak, Miklós Koren, Balázs Világi, and participants of the conference celebrating the 50th anniversary of IE-HAS, seminars at MNB, St. Louis Fed, the 4th Workshop on Macroeconomic Research, the IE-HAS Summer Workshop, and the 2006 EEA Congress, for comments and suggestions. All the remaining errors are ours.

\textsuperscript{1}Magyar Nemzeti Bank and Central European University. Corresponding author, email: benczurp@mnb.hu

\textsuperscript{1}Magyar Nemzeti Bank and Central European University; email: konya@mnb.hu
the business cycle, but it can also significantly influence the growth process of a small open economy. We argue that the choice of the exchange rate regime is not neutral, and the capital accumulation path depends on the nominal regime.

As suggested by consumption smoothing, catching-up economies should be borrowing against their future income. As we document below, they also build up their asset holdings. A large fraction of these assets are local currency bank deposits and bonds, the value of which move together one in one with nominal exchange rates. This implies that the evolution of the nominal exchange rate will influence the asset accumulation process. Moreover, whether exchange rates are flexible, fixed or "frozen" (like in a currency board arrangement) also determines how much nominal asset accumulation can be achieved by nominal appreciation and how much requires household savings from income. Such a link then has repercussions for capital accumulation, growth and sectoral (tradables versus nontradables) reallocations. Our objective is to develop a simple but sufficiently rich framework, which is capable of addressing the aggregate and sectoral features of such a nominal growth process.

The structure of the model is the following. We consider a small open economy, with a traded and a nontraded sector. The source of growth is capital accumulation. We assume that the initial capital stock is below the steady state level, so the country experiences capital accumulation and excess growth along its transition towards the steady state.\(^1\) We adopt the now standard Tobin-q approach to capture gradual capital flows. We introduce an asset accumulation motif by assuming that households derive utility directly from holding (real) money balances (\textit{money in the utility}). As the income of consumers grows, they want to consume more and also to hold more money.

Small open economy models are subject to an indeterminacy problem in the sense that the net foreign asset position (and hence consumption) are not pinned down by the steady state conditions.\(^2\) This is particularly problematic for stochastic models that rely on log-linearization around the deterministic steady state. Since we work with a deterministic setup, we are able to solve our model without log-linearization. Our solution method explicitly accounts for the history dependent nature of steady state NFA. This also implies that in our setting revaluation effects have not only a temporary, but also a long-run impact.

After setting up the model we turn to the analysis of the growth process. We first show that in case of flexible exchange rates, the nominal economy behaves identically to an economy without money, in the sense that all real variables (most importantly, capital) are exactly the same as in a model where money has no role. The reason is that while convergence leads to a gradual

---

\(^1\)In what follows, we use the phrases ‘transition dynamics’ and convergence interchangingly, both referring to the trajectory leading towards the steady state.

\(^2\)See Schmitt-Grohé and Uribe (2003) for a discussion; they also propose various ways to deal with the problem.
increase in money holdings, it is simply implemented by an appreciating nominal exchange rate. This is a formal version of the popular phrase that FDI inflows put an appreciating pressure on nominal exchange rates.\(^3\) Equivalently, even when the exchange rate is fixed, the right amount of money creation by the central bank can implement the real path.

The nominal and the real paths differ, however, when both the exchange rate is fixed and money transfers are exogenous (thus they do not equal the demanded change in money holdings). This is the case, for example, when the country operates a currency board (fixed exchange rate and no money transfer from the central bank), or enters a currency union (at least when the allocation of unionwide seniorage revenues is exogenously determined). Historically, the gold standard shared the same features. Under these assumptions some of the increase in the domestic money stock must come from abroad. This necessitates either a trade surplus or foreign borrowing. Both require sacrificing real resources (consumption) for obtaining money, thus the growth path differs from that of an economy where money is not valued.

An application of our framework is the comparison of two nominal (currency board) paths which differ only in the level of the exchange rate. Different nominal exchange rates lead to persistent differences: from identical capital stocks, foreign bond and local currency holdings, a stronger nominal exchange rate means a higher foreign currency value of local currency holdings. As tradable prices are fixed in foreign currency, this is a positive shock to financial asset holdings.

The clearest case for such a comparison is when a country decides over its entry rate into a monetary union; but a realignment of a fixed exchange rate also shares these features as long as money supply is not completely flexible. An important application of our model is thus the choice of the euro conversion rate for EMU aspirants. As the role of money and bank deposits is larger in these economies than in previous EMU entrants, we can expect a stronger real impact of this choice. The historical episode of converting the East German currency into Deutschmarks also highlights the importance of the wealth effect of currency conversion and its persistent real effects; but one could also look back at the restoration of the gold standard in the UK after WWI.

To assess these differences, we first derive our main results analytically. Next, we calibrate the model using data from seven Central and Eastern European (CEE) countries. These countries are a good laboratory to evaluate our model for three reasons. First, they are on a convergence path towards the rich economies of the European Union. Second, they have different monetary arrangements: the Baltic countries operate a currency board, while the other four nations (the Czech Republic, Hungary, Slovakia and Poland) have a nominally floating regime. Finally, apart from the exchange rate regime, these countries are quite similar to each other in their institutions.

\(^3\)Strictly speaking, our benchmark model does not have FDI; instead, domestic investment is financed by foreign borrowing.
and level of development.

The model predicts that the currency regime has a strong impact on the level and composition of financial assets. We test this prediction by comparing data on net foreign assets and money holdings to model simulations under (i) flexible exchange rates and (ii) currency board. We find that the regime that corresponds to the actual exchange rate arrangement greatly outperforms the "wrong" scenario. In addition, we examine the convergence properties, and find that the model performs reasonably well.

Next we turn to the analysis of counterfactuals. We simulate two related scenarios. The first scenario is when a flexible exchange rate economy unexpectedly introduces a currency board (or joins a monetary union), while the second compares two different choices for the exchange rate in the currency board. Our results imply quantitatively significant and persistent effects for both scenarios, especially for consumption, investment, sectoral allocations, and the composition of financial assets. As mentioned above, some of these differences persist even in the long run, since the steady state NFA position depends on initial conditions.

The paper is organized as follows. The next section contains a literature review and presents some facts that underscore the importance of the nominal asset accumulation motive. Section 3 describes the model. Section 4 contains our analytical results concerning the choice of the exchange rate regime and the level of the exchange rate in a currency board. Section 5 presents the quantitative policy simulations, and Section 6 concludes.

2 The context of the model

2.1 Previous literature

Usual explanations for nominal shocks having lasting real effects build on staggered price or wage contracts. An early example is Taylor (1980). Recently, state- or time-dependent pricing models constitute the workhorse for analyzing nominal scenarios (see chapter 3 of Woodford (2003) for a general discussion). While pricing problems are clearly important to understand business cycle frequency developments, we believe that they should have limited impact over the growth horizon. Motivated by this, we depart from this literature by focusing on the effect of nominal shocks through nominal wealth accumulation (captured by money-in-the-utility). The major building blocks of our model are money-in-the-utility (a nominal effect), costly investment (a real friction) and sectoral technology differences (capital-labor intensities).

We use money-in-the-utility to capture the fact that some assets are denominated in local

---

4Devereux and Sutherland (2006) consider a somewhat similar mechanism: under incomplete asset markets, monetary policy (or nominal shocks in general) can influence the return structure of nominal bonds, thus yielding real effects.
currency (see section 2.2 for details). As nominal exchange rate movements revalue this stock, our approach is closely related to the recent literature on the revaluation channel of external adjustment (Lane and Milesi-Ferretti, 2005, Gourinchas and Rey, 2005). Tille (2005) also analyzes the real effects of such a revaluation. In our case, this revaluation happens automatically as the price of tradable goods is fixed in foreign currency.

Many current papers point to the importance of costly investment in shaping business cycle properties, inflation or real exchange rate behavior. Eichenbaum and Fisher (2006) argue that the empirical fit of a Calvo-style sticky price model substantially improves with firm-specific capital (and a nonconstant demand elasticity). Christiano et al (2001) present a model in which moderate amounts of nominal rigidities are sufficient to account for observed output and inflation persistence, after introducing variable capital utilization, habit formation and capital adjustment costs. Chapter 4 of the Obstfeld and Rogoff (1996) textbook contains an exposition of a two-sector growth model (the standard Balassa-Samuelson framework), with gradual investment in some of the sectors. We depart from these approaches by dropping staggered price setting, but – unlike Obstfeld and Rogoff – still allowing for a nominal side of the economy.

The presence of a traded and a nontraded sector allows us to merge trade theory insights with a monetary framework: for example, the presence of nontraded goods means that a redistribution of income between countries will affect their relative wages (the classical transfer problem, like in Krugman, 1987), or the Stolper-Samuelson theorem, linking changes in goods prices with movements in factor rewards. Having two sectors is also essential to introduce the relative price channel described in the introduction.

The growth literature also employs multisector models, but the two sectors there differ in the investment good they produce (physical versus human capital). Examples include Rebelo (1991) and Lucas (1988). Ventura (1997) is an example of a multisector growth model with an explicit trade framework. His model of growth in interdependent economies clearly illustrates the importance of merging trade and growth theory. The implications of a nontraded sector, however, are not addressed by that paper.

Our framework is closely related to that of Fernandez de Cordoba and Kehoe (2000), Bems and Hartelius (2006), and Rebelo and Végh (1995). The first two papers use a two-sector real model to study the current account and real exchange rate implications of trade and financial openings. Similarly to our framework, Rebelo and Végh (1995) add a nominal side by introducing money, which in their model serves to lower transaction costs. They use the model to examine the effects of exchange rate-based stabilizations (moving from a floating to a fixed exchange rate regime).

Our contribution relative to Rebelo and Végh (1995) is threefold. First, we want to work with as little direct interaction between nominal and real factors as possible. In Rebelo and
Végh (1995) money lowers real transaction costs, and thus influences intertemporal decisions unless the nominal interest rate is zero. This means that unlike in our model, even perfectly flexible prices and a floating exchange rate do not implement the nonmonetary economy. Since in our model money has a less central role, its influence on real variables does not follow from a single assumption, but rather from the interplay of various factors. We thus believe that our framework delivers novel insights into the linkages between the nominal and real sides of the economy.

Second, we view the motive for nominal asset accumulation as more general than just lowering transaction costs. While this distinction is not very important methodologically, it makes the interpretation of the stylized facts presented below much easier. In particular, we think that transaction costs alone cannot explain the fact that households keep a large fraction of their wealth in nominal, local currency denominated assets. Although money-in-the-utility does not explain why this is the case, it serves as a useful device to condense the various roles of money into a single assumption.

Finally, the Rebelo and Végh (1995) model is subject to the indeterminacy problem we discussed above. This makes their linear approximation method imprecise and potentially unreliable (as acknowledged by the authors themselves). As discussed in the Introduction, we avoid the problem by solving the non-linear system directly and taking care of the path-dependence of the NFA position.

2.2 Evidence on nominal asset accumulation

Here we document the specifics of EU and OECD household financial balance sheets which demonstrate (i) the asset accumulation motive in development, and (ii) the importance of nominal (local currency) assets in the overall portfolio. Figure 1 plots the three-year average household asset per GDP position for 27 countries, for years 2002-04. It is immediate from the graph that new member states exhibit much lower asset holdings. This is somewhat less true for previous catching-up countries like Spain, Portugal, or Korea.

Since the cross-section behavior of asset holdings might be driven by country-specific fixed factors, we also look at the time series picture. In general, there is an increasing trend across all OECD countries. To check whether new member states exhibit higher asset or asset per GDP growth rates, we create a synthetic "old Europe" entity, by adding up the ECU/euro value of household assets and GDP of fourteen European countries for years 1995-2004, and a "new

---

5 The countries are: Australia, Canada, Japan, Korea and the US; Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Portugal, Spain, Sweden and the UK; Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania (data exists only for 1999), Slovakia and Slovenia. Data are from the Eurostat and OECD.

6 The countries are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, the Netherlands,
Figure 1: Cross-section of household assets per GDP

Europe” entity of the Czech Republic, Hungary, Lithuania, Poland, and Slovakia.7

Figure 2 plots the evolution of per capita household assets in euro, where new Europe indeed exhibits a faster increase than old Europe. To put it into numbers, the nine-year average growth rate is 6.17% for old Europe and 12.45% for new Europe. The evolution of household assets per GDP shows a similar pattern (Figure 3): the nine-year average growth rate is 1.98% for old and 3.48% for new Europe.

Switching now to the composition of household balance sheets, Figure 4 shows that apart from Estonia, new member states have at least 40% share of currency, bank deposits and bonds (securities other than shares) in their asset holdings. Spain and Portugal also have such high numbers; while Austria, Japan, Korea and to a smaller extent, Belgium, Germany and Italy are more surprising examples of industrialized countries with a very high share. All other developed countries have substantially smaller shares, though it always exceeds 20%.

This distinction remains true if one looks at the entire nineties: with the above exceptions (plus Finland for the early nineties), developed economies rarely had a share higher than 40%, while new member states (with the exception of Estonia and Lithuania) never had a share below 40%. A similar pattern emerges when we look at the ratio of net deposit-type holdings (net currency, deposit and bond holdings minus bank loans) to net wealth (Figure 5): apart from Estonia, new members states are at the high end of the distribution, together with Austria,

---

7We did not have sufficiently long time series for Bulgaria, Estonia, Latvia, Romania or Slovenia.

---

7Norway, Portugal, Spain, Sweden, and the United Kingdom. Ireland did not have comparably long time series on its household balance sheets.
Figure 2: Household assets per head in euro

Figure 3: Household assets per GDP
Belgium, Italy, Japan and Korea.\footnote{8} 

Unfortunately, the currency composition of balance sheet data for new member states is not readily reported by either standard data sources or central bank websites. The data is posted for Hungary from 1990 to 2006,\footnote{9} and it shows that the share of foreign currency denominated assets never exceeded 20% of total household assets, and its time average is around 15%. For Poland, the currency composition is available for the flow but not for the stock data,\footnote{10} for years 2000-2004. Though cumulated changes cover only around 20% of the total asset stock, it is still indicative that within those, the contribution of foreign currency assets is less than 15%. Though the conclusion might not generalize to all new EU member states, it seems that nominal assets are overwhelmingly denominated in local currency.

To summarize, there is robust evidence that the nominal asset accumulation motive is present in the data. The households of new member states hold significantly less financial assets than those of the advanced EU countries, but they are catching up fast. We have also found evidence that a large fraction of these financial assets are nominal. Based on detailed data from Hungary and Poland, these nominal assets are dominantly local currency denominated, although the lack of comprehensive data prevents us from making a general statement.

\footnotetext[8]{These observations remain valid if we exclude bond holdings (item 3 of financial accounts statistics), and consider cash, bank deposits and loans only. In fact, the pattern is even more clear-cut; with Austria, Japan and Korea being the sole set of exceptions among industrial countries.}

\footnotetext[9]{http://english.mnb.hu/Engine.aspx?page=mnben_statistzikai_idosorok&ContentID=7008}

\footnotetext[10]{http://www.stat.gov.pl./gus/45_1533_ENG_HTML.htm}
3 The model

3.1 Production

We distinguish final and intermediate products to make clear the role of tradables \((T)\) and non-tradables \((NT)\) in final consumption and investment expenditure. Thus the final composite investment and consumption goods are assembled from tradable and non-tradable intermediate inputs. Tradables are either imported, or produced domestically using capital and labor. Non-tradable must be produced domestically, also with capital and labor. Capital is specific to a sector, and investment is subject to adjustment costs (see below at the household section). These assumptions are similar to those used by Bems and Hartelius (2006), who assume sector-specific capital adjustment costs. They serve to make the short-run transformation curve non-linear, which in turn implies that non-tradable production can only be changed incrementally. While not central to our qualitative results, sector specific capital with investment adjustment costs does have an important quantitative impact in the model.

3.1.1 Final goods

Final consumption \(C_t\) and investment in sectors \(T\) and \(NT\) are aggregates of tradable \((T)\) and non-tradable \((N)\) goods, and are assembled by competitive firms using Cobb-Douglas technologies. When describing the production technology for investment, it is important to account for the quadratic adjustment costs. Using \(i_{j,t}\) for investment in sector \(j\) net of adjustment costs, we
can write the production functions as follows\textsuperscript{11}:

\[
c_t = \lambda^\alpha (1 - \lambda)^{1-\lambda} (c_t^T)^\lambda (c_t^N)^{1-\lambda} \left(1 + \frac{\phi}{2} \frac{i_{j,t}}{k_{j,t-1}} \right) i_{j,t} = \lambda_i^\alpha (1 - \lambda_i)^{1-\lambda_i} (i_{j,t}^T)^{\lambda_i} (i_{j,t}^N)^{1-\lambda_i},
\]

where \(\phi\) measures the extent of investment adjustment costs. Because we lack data on the tradable intensity of investment at the sectoral level, we assume that this intensity is not sector specific (\(\lambda_i\)). Essentially the same assumption is made in Bems and Hartelius (2006), and in Bems (2008).

Cost-minimization and free entry (zero profits) can be used to calculate the demand functions for the tradable and non-tradable components of consumption and investment, and the price indexes for the final goods. We assume that the law of one price holds for tradables, and we normalize the foreign tradable price to unity, so that \(P_t^T = s_t\), where \(s_t\) is the nominal exchange rate. We prefer to work with the relative price of non-tradables, \(p = P_t^N / s\), which then yields

\[
P_t = s_t p_t^{1-\lambda} \\
P_t^I = s_t p_t^{1-\lambda_i}
\]

for the price indexes. Then demand for tradables and non-tradables in consumption and investment can be written as:

\[
c_t^T = \lambda p_t^{1-\lambda} c_t \\
c_t^N = (1 - \lambda) p_t^{-\lambda} c_t \\
i_{j,t}^T = \lambda_i p_t^{1-\lambda_i} i_{j,t} \left(1 + \frac{\phi}{2} \frac{i_{j,t}}{k_{j,t-1}} \right) i_{j,t} \\
i_{j,t}^N = (1 - \lambda_i) p_t^{-\lambda_i} \left(1 + \frac{\phi}{2} \frac{i_{j,t}}{k_{j,t-1}} \right) i_{j,t}
\]

for the demand functions.

\subsection*{3.1.2 Intermediate goods}

Tradables and nontradables are produced using and labor. For simplicity, we assume that labor is mobile across sectors, which implies that there is an economy-wide wage rate. As discussed previously, capital is sector specific, so the rental rates on capital are not equalized.

\textsuperscript{11}Note that the subscript \(j\) indexes investment targeted towards the accumulation of capital in sector \(j\), while the superscripts indicate the tradable and non-tradable components of these investments.
The production functions in both sectors are Cobb-Douglas:

\[
Y_t^T = k_{T,t}^{\alpha_T} l_t^{1-\alpha_T} \tag{5}
\]
\[
Y_t^N = A_N k_{N,t}^{\alpha_N} (1 - l_t)^{1-\alpha_N}, \tag{6}
\]

where \(l_t\) is the share of labor employed in the tradable sector (total labor supply is normalized to unity), and \(A_N\) is the (relative) total factor productivity in the NT sector. We allow for sectoral differences in the capital share \(\alpha_j\).

Perfect competition ensures that factors are paid their marginal products:

\[
w_t = (1 - \alpha_T) \left( \frac{k_{T,t}}{l_t} \right)^{\alpha_T} = p_t (1 - \alpha_N) A_N \left( \frac{k_{N,t}}{1 - l_t} \right)^{\alpha_N} \tag{7}
\]
\[
r_{T,t}^k = \alpha_T \left( \frac{k_{T,t}}{l_t} \right)^{\alpha_T - 1} \tag{8}
\]
\[
r_{N,t}^k = p_t \alpha_N A_N \left( \frac{k_{N,t}}{1 - l_t} \right)^{\alpha_N}, \tag{9}
\]

where we measure factor prices \(w_t\) and \(r_{j,t}^k\) in tradable units (or, alternatively, in foreign currency).

### 3.2 Households

Households can hold three types of assets: capital, interest bearing foreign bonds and non interest bearing domestic money. We assume that domestic money is not accepted by the rest of the world. For simplicity we also assume that domestic currency denominated bonds are not issued.\(^{12}\) Households can freely adjust their portfolios between money and bonds within a period, which implies that their sum (financial assets) is the appropriate state variable. In addition, households accumulate capital for both the T and NT sectors. As discussed above, investment is subject to quadratic adjustment costs.

Households draw income from (i) supplying labor, (ii) renting out capital to firms, and (iii) holding foreign bonds and domestic money. They allocate some of their income towards consumption and investment, and carry the remaining amount over to the next period in terms of financial assets. Although money does not pay interest, it is valued by households as it enters the utility function directly (money-in-the-utility).

\(^{12}\)Equivalently, we could assume that domestic currency denominated bonds are not accepted by foreigners but are viewed as perfect substitutes to foreign bonds by domestic households. In our representative household framework such bonds would then be in zero net supply, and the corresponding Euler equation would simply define a domestic currency interest rate, which is linked to the foreign currency interest rate \(i_t\) by a basic UIP condition. All other first-order conditions would remain unaffected.
Households thus solve the following problem:

\[
\max_{\{c_t\}} \sum_{t=1}^{\infty} \beta^t \left( \log c_t + \gamma \log \frac{H_t}{P_t} \right) \\
\text{s.t.} \quad \frac{b_t}{R_t} - b_{t-1} + \frac{H_t - H_{t-1}}{s_t} = w_t + \sum_{j=T,N} r^k_{j,t} k_{j,t-1} - \frac{P_t}{s_t} c_t \\
- \frac{P^I_t}{s_t} \sum_{j=T,N} \left( 1 + \frac{\phi}{2} \frac{i_{j,t}}{k_{j,t-1}} \right) i_{j,t} + \frac{\tau_t H_{t-1}}{s_t} \\
k_{j,t} = (1 - \delta) k_{j,t-1} + i_{j,t}
\]

where \( R_t \) is the discount rate on foreign currency denominated bonds \( b_t \), \( s_t \) is the nominal exchange rate, \( w_t \) is the wage rate, \( r^k_{j,t} \) is the real rental rate of capital in sector \( j \) (we measure factor prices in foreign currency), \( k_{j,t} \) is the stock of capital in sector \( j \), \( P_t \) is the consumption price index, \( c_t \) is the consumption aggregate, \( i_{t,j} \) is investment in sector \( j \), and \( P^I_t \) is the investment price index. \( H_t \) is the stock of domestic money, and \( \tau_t H_t \) is a government transfer.

Substituting the relative price of nontradables \( p_t \) into the consumption and investment price indexes, and after some simplification, the first-order conditions are written as follows:

\[
\frac{p_t^{1-\lambda} c_{t+1}}{p_t^{-\lambda} c_t} = \beta R_t \\
\frac{\gamma}{H_t} = \frac{1}{s_t p_t^{1-\lambda} c_t} - \frac{\beta (1 + \tau_{t+1})}{s_{t+1} p_{t+1}^{1-\lambda} c_{t+1}} \\
q_{j,t} = 1 + \frac{\phi}{2} \frac{i_{j,t}}{k_{j,t-1}} \\
q_{j,t} = \left[ p_{t+1}^{\lambda-1} r^k_{j,t+1} + (1 - \delta) q_{j,t+1} + \frac{\phi}{2} \left( \frac{i_{j,t+1}}{k_{j,t}} \right)^2 \right] \frac{1}{R_t} \left( \frac{p_{t+1}}{p_t} \right)^{1-\lambda} \\
k_{j,t} = (1 - \delta) k_{j,t-1} + i_{j,t}
\]

The first equation is the consumption Euler equation, the second is money demand, the third is the investment equation where \( q_{j,t} \) is Tobin’s \( q \), the fourth is the arbitrage condition between investment and bonds, and the last is the capital accumulation equation (restated for convenience). Note that the last three equations must hold separately for \( j = T, N \).

### 3.3 Equilibrium

To close the model, we need market clearing conditions for non-tradables and tradables, and we also have to specify the path of the discount rate on foreign bonds, \( R_t \). In accordance with the small open economy assumption, we take \( R_t \) to be exogenous. In steady state, we assume that it equals the domestic (and world) subjective discount rate, \( \bar{R} = 1/\beta \). Along the transition path to the steady state, we allow for an exogenous premium that we choose through our calibration
A consequence of the exogenous interest rate assumption is that the steady state net foreign asset position is not pinned down uniquely by the steady state conditions, i.e. it is history dependent. In models that rely on log-linear approximations around the steady state this is obviously problematic. To ensure the existence of a well-defined steady state, the literature has used various short-cuts, summarized in Schmitt-Grohe and Uribe (2003).\textsuperscript{13}

We deviate from this literature for two reasons. First, as our model is deterministic, we do not need to log-linearize around the steady state, and hence we do not face the indeterminacy problem. Instead, we solve for the full transition path using the non-linear system of equations defined by the equilibrium conditions, and hence we are able to explicitly account for the dependence of the steady state on initial conditions. Second, empirical evidence does not support the existence of a unique steady state NFA level. Using data from Lane and Milesi-Ferretti (2007), we find both a large cross-section and time series variation in NFA positions. Even among advanced industrial countries, NFA positions range from -50% to +50% of GDP, while emerging countries produce swings even bigger than this. Finally, our main results are similar (except for the NFA path), both qualitatively and quantitatively, even if we assume a debt-dependent interest rate.\textsuperscript{14}

Non-tradable market clearing requires that production equals consumption plus investment:

\begin{equation}
A_N k_{N,t}^0 (1 - l_t)^{1-\alpha} = c_{N,t} + i_{T,t}^N + i_{N,t}^N
\end{equation}

Tradables can be imported, thus we write the market clearing condition (the current account) as

\begin{equation}
\frac{b_t}{R_t} - b_{t-1} + \frac{H_t - (1 + \tau_t) H_{t-1}}{s_t} = k_{T,t}^0 (1 - l_t)^{1-\alpha} - c_{T,t} - i_{T,t}^T - i_{T,t}^T.
\end{equation}

Equations (1), (2), (3), (4), (7), (8), (9), (10), (12), (13), (14), (15) and (16) determine most endogenous variables. The remaining variables \( H, s, \) and \( \tau \) are determined by equation (11) and the monetary regime. In what follows, we consider three alternative regimes: \textit{flexible exchange rates} (and fixed money supply: \( \tau \equiv 0, H_t \equiv H_0 \)), \textit{perfectly elastic money supply} (and fixed exchange rates: \( S_t \equiv \bar{S}, H_t/H_{t-1} = 1 + \tau_t \)), and a \textit{currency board} (fixed exchange rates and no money transfers: \( S_t \equiv \bar{S}, \tau_t \equiv 0 \)).

Before we turn to these various cases, it is worth discussing how the steady state NFA position is determined in the model. Let us define consumption spending measured in tradables as \( x_t = p_t^{1-\lambda} c_t \). From (10) we can see that given \( x_1 \), the path of \( \{x_t\}_{t=2}^{\infty} \) is determined exogenously, since \( R_t \) is exogenous. Thus, in particular, the steady state value \( \bar{x} \) is also a function only of \( x_1 \).

\textsuperscript{13}Perhaps the most widespread assumption is to make the interest rate dependent on foreign debt. This essentially amounts to selecting a level for the steady state NFA, at which the interest premium is zero.

\textsuperscript{14}This is precisely what we did in previous versions of the paper.
It is also easy to see from the other equilibrium conditions that all other steady state variables are uniquely determined given \( x_1 \). Most importantly, the steady state current account (16) can be solved for \( b \), again as a function of \( x_1 \). Thus to compute the value of \( b \), we need to find the initial level of foreign currency denominated consumption expenditure.

The current account for period \( t = 1 \) can be rewritten as

\[
b_0 = \frac{b_1}{R_1} + \frac{H_1 - (1 + \tau_1) H_0}{s_1} + p_1^{1-\lambda_c} + p_1^{1-\lambda_f} \sum_{j=T,N} \left( 1 + \phi \frac{i_{j,1}}{k_{j,0}} \right) i_{j,1} - w_1 - \sum_{j=T,N} r_{j,t}^k k_{j,t}.\]

Iterate forward the right-hand side and evaluate at \( t \rightarrow \infty \) to get:

\[
b_0 = \frac{b_T}{R_1 R_2 \ldots R_T} + \sum_{t=1}^{T} \left( \prod_{j=1}^{t-1} R_j \right)^{-1} \left[ p_1^{1-\lambda_c} + \frac{H_t - (1 + \tau_1) H_{t-1}}{s_t} \right] \\
\quad + p_1^{1-\lambda_f} \sum_{j=T,N} \left( 1 + \phi \frac{i_{j,t}}{k_{j,t-1}} \right) i_{j,t} - w_t - \sum_{j=T,N} r_{j,t}^k k_{j,t-1} \right], \tag{17}
\]

where the second equality follows from utility maximization and the no Ponzi game condition that we impose.\(^{15}\) The right hand side is pinned down by \( x_1 \) and \( k_{j,0} \). As our true initial condition is for \( b_0 \) and not for \( x_1 \), we can use this equation to determine the unique value of \( x_1 \) which yields the given \( b_0 \).

This procedure also guides our numerical solution method. We first solve the system for an arbitrary value of \( x_1 \), as a well-behaved system of equations with the initial conditions \( x_1, k_{j,0} \). Next, we utilize equation (17) to calculate the value of \( b_0 \) that is consistent with the assumed \( x_1 \). If it differs from our true initial condition on \( b_0 \), we modify our choice for \( x_1 \). We iterate this way until the implied and actual \( b_0 \) are the same.\(^{16}\)

Notice that \( x_1 \) feeds back into the other equilibrium conditions, and thus influences the dynamics (and also the steady state) of the system. The main channel of this influence is through the non-tradable market clearing condition (15), which yields the relative price \( p_t \) as a function of consumption expenditure \( x_t \). The relative price, in turn, effects investment behavior and capital accumulation (see [13]), which then impacts factor prices and income. Thus, in

\( ^{15} \) The condition states that the present value of assets in the limit \( t \rightarrow \infty \) cannot be negative, i.e. the economy cannot borrow at a faster rate than the rate of interest.

\( ^{16} \) More precisely, we use the built in solvers of Matlab to find the \( x_1 \) that equates the implied and actual initial wealth levels.
general, changes in initial consumption expenditure are propagated through the whole dynamic system. We return to this issue later.

4 The role of the exchange rate

In this section we provide an analytical argument why different exchange rate regimes lead to differences in the real convergence path. We prove two important results. First, we show that under the flexible exchange rate and the perfectly elastic money supply regimes money is neutral. More precisely, in these economies money is determined residually, and the path of all real variables is identical to what they would be in a model with no money (the real model). Second, we prove that this is not the case under the currency board: the convergence path of the currency board economy is different from the real model (and hence from the flexible regimes). We also highlight the conditions under which our non-neutrality result holds. While this section focuses on the analytical arguments, in later section we calibrate the model to quantify the importance of the mechanism we describe.

4.1 Flexible monetary regimes

This section develops the flexible exchange rate and the elastic money supply regimes in detail. We show that the path of real variables is identical to a model where money has no role ($\gamma = 0$). In other words, when either the exchange rate or the money supply are allowed to adjust freely, the nominal asset accumulation motive does not distort the transition path of capital.

We start with the case when the exchange rate is flexible, but no money transfer takes place. Setting $\tau_t \equiv 0$, $H_t \equiv H$, the current account (16) becomes

$$\frac{b_t}{R_{t}} - b_{t-1} = k^a_{T,t} (1 - l_t)^{1-\alpha} - c_{T,t} - i_{T,t} - i_{N,t},$$

i.e. it is independent from the money stock and the nominal exchange rate. It is easy to check that the same holds for the other equilibrium conditions, except for the money demand condition (11). This implies that all variables except for the nominal exchange rate are determined in system which is independent of any nominal variable. Thus the flexible exchange rate economy is identical to the real model.

The path of the nominal exchange rate is determined by (11). More precisely, we can prove the following result (see Appendix B):

\[\text{In this case the exchange rate is fixed, but the government fully accomodates changes in money demand through the money transfers } \tau_t.\]
\[ \frac{s_{t+1}}{s_t} = \frac{1}{\beta R_t} \]  

(19)

Our calibration will feature a declining interest rate path, which implies a nominal appreciation over the convergence path. It is important to stress, however, that the appreciation is not a consequence of convergence, but results purely from the assumption on the exogenous interest rate path. Of course, convergence may lead to this interest rate behavior endogenously, but this is outside the scope of our model.

In the perfectly elastic money supply case \( H_t = (1 + \tau_t) H_{t-1} \) by definition, thus again the current account becomes (18). Analogously to the flexible exchange rate case, all other conditions are independent of nominal variables, hence the perfectly elastic money supply economy is equivalent to the real model. The path of nominal money balances is determined by the money demand equation residually, and it is given by (see Appendix B):

\[ H_t = \frac{\gamma \bar{x}_t}{1 - \beta} \]  

(20)

We summarize these arguments in the following proposition.

**Proposition 1** Both the flexible exchange rate and the elastic money supply economies implement the real version of the model.

### 4.2 The currency board

Now we turn to the case of a currency board, and show formally that the evolution of the economy is different from the real model, and hence also from the flexible regimes. We have the following result, where for convenience we compare the currency board to the flexible exchange rate case:

**Proposition 2** The currency board dynamic system is different from the flexible exchange rate economy for an arbitrary currency board exchange rate.

**Proof.** Note that the only differences between the two systems of equations are (i) the current account and (ii) the money demand equation, which we rewrite as

\[ \frac{H_t}{s_t} = \frac{\gamma x_t}{1 - s_t/(s_{t+1} R_t)} \]

We show that (17) implies a different expenditure path for \( \{x_t\}_{t=1}^\infty \), which in turn feeds back into the rest of the equations. Notice that the initial condition given by (17) has to hold not just for \( t = 1 \), but for all future time periods. Let \( \Theta_t \) indicate the infinite sum of the money terms \( \Delta H/s \) on the right-hand side of (17) starting at time \( t \). In the flexible regime case, the
money sum is simply zero, since $\Delta H \equiv 0$. Utilizing the money demand equation and the Euler equation (10), the money sum in the CB case becomes

$$
\Theta^{CB}_t = \sum_{j=1}^{\infty} \left( \prod_{i=1}^{j-1} R_t \right)^{-1} \left( H_j - H_{j-1} \right) = \frac{H_t}{s_{CB}} - \frac{H_{t-1}}{s_{CB}} + \frac{H_{t+1}}{s_{CB} R_t} - \frac{H_t}{s_{CB} R_t} + \frac{H_{t+2}}{s_{CB} R_t R_{t+1}} - \frac{H_{t+1}}{s_{CB} R_t R_{t+2}} + \ldots
$$

$$
= \frac{H_t}{s_{CB}} \left( 1 - \frac{1}{R_t} \right) + \frac{H_{t+1}}{s_{CB}} \left( 1 - \frac{1}{R_{t+1}} \right) + \ldots - \frac{H_{t-1}}{s_{CB}}
$$

$$
= \gamma x_t + \gamma x_{t+1} \frac{R_t}{R_t} + \gamma x_{t+2} \frac{R_t R_{t+1}}{R_t R_{t+1}} + \ldots - \frac{H_{t-1}}{s_{CB}}
$$

$$
= \gamma x_t + \gamma x_{t+1} \frac{1}{1 - \beta} - \frac{H_{t-1}}{s_{CB}}
$$

Now suppose that the paths of all variables are identical between the flexible exchange rate and the currency board regimes up to $t - 1$. Then $H_{t-1}$ is also common. Under flexible exchange rates, we have $H_t = H_{t-1}$. Using the flexible exchange rate version of the money demand condition to substitute for $H^\text{flex}_t = H_{t-1}$, we can rewrite $\Theta^{CB}_t$ as

$$
\Theta^{CB}_t = \frac{\gamma}{1 - \beta} \left( x^\text{CB}_t - \frac{s^\text{flex}_t}{s_{CB}} x^\text{flex}_t \right).
$$

Recall that $x^\text{CB}_t = x^\text{flex}_t$ if and only if $\Theta^{CB}_t = 0$, which in turn can be satisfied if and only if $s^\text{flex}_t = s_{CB}$. Moreover, this condition has to hold for all $t = 1, \ldots, \infty$, since we picked $t$ arbitrarily.

Thus the currency board and flexible exchange rate paths coincide if and only if the flexible exchange rate is identically fixed at the currency board level. From (19), however, we can see that this is impossible if $R_t \neq 1/\beta$. Even if there is no interest premium and the flexible exchange rate is endogenously fixed, the two economies coincide only if the currency board exchange rate is set at exactly the equilibrium floating level.

The intuition behind these results is simple. Under flexible exchange rates, households desire to accumulate real money balances is realized through the movements of the exchange rate. Under a currency board, this is not possible (printing the required amount of money is also ruled out by assumption). Thus households have to increase (nominal) money holdings either through running a trade surplus or through borrowing from abroad, both of which entail a real cost as long as the interest rate is above zero. The only exception is given in the last sentence of the proof, which we restate in the following corollary:

**Corollary 3** The currency board economy implements the real model if and only if (i) there is no interest premium ($R_t \equiv 1/\beta$), and (ii) the exchange rate is set at $s_{CB} = (1 - \beta) H_0 / \left( \gamma x^\text{flex} \right)$,
where $\bar{x}^{\text{flex}}$ is the constant expenditure level that corresponds to the flexible exchange rate economy.

An additional result that follows from Proposition 1 concerns the level of the currency board exchange rate. Let us assume that an economy previously under a flexible exchange rate regime unexpectedly introduces a currency board at time $t$. Just before the regime change, consumers hold $B_{t-1}$ foreign bonds and $H_{t-1}$ units of local currency. Evaluated at exchange rate $S$, household money holding equals $H_{t-1}/S$; while for a stronger parity $S' < S$, it would become $H_{t-1}/S' > H_{t-1}/S$. Since money is part of overall household wealth, a stronger exchange rate has a regular wealth effect, leading to a different initial consumption expenditure $x_t$, which in turn implies a different path for the other endogenous variables.

In a perfectly elastic money supply regime, the same initial asset shock is immediately neutralized by a change in the per period money transfer;\(^{18}\) while if a central bank of a flexible exchange rate economy prints money, that is immediately offset by a currency depreciation. This is summarized in our next result:

**Proposition 4** The level of the exchange rate or the size of the money stock has a real effect in a currency board regime; while it is neutral in the nominal implementation of the real model.

An interesting and obvious example is the conversion rate around German unification. As most East Germans had their savings in local currency (cash or bank deposits), this was purely a transfer/wealth effect, exactly in the spirit of our model. Not surprisingly, the East German economy showed strong symptoms of overvaluation, in response to a very strong conversion rate. The return of the UK to the gold standard after WWI and the euro conversion rate are similar examples.

Let us stress that one cannot use this framework to calculate an optimal conversion rate. In terms of consumer welfare (no matter whether we take into account the money part of it or not), the stronger the entry rate, the better. Again, this is due to the pure wealth transfer. In reality, there should be constraints on how much foreign currency the rest of the world is willing to give for a local currency, but such considerations are not part of our framework. Besides, governments might care for certain subgroups (like exporters), which would again limit the case for a strong entry rate. Nevertheless, as we demonstrate later in our quantitative exercise, our model does produce lasting and sizable real consequences of different entry rates.

\(^{18}\)In case of a revaluation, it means a negative transfer. One way to implement it is to levy a tax on money holdings. Alternatively, one can think of a "negative helicopter drop", which is in fact a "helicopter vacuum cleaner".
4.3 Role of certain assumptions

From Corollary 3 it is clear that our main result holds under extremely general conditions. In particular, neither the existence of an interest premium nor the presence of two sectors are necessary for the currency board/real model difference. Our quantitative results, however, do depend on the various assumptions, and in section we provide a brief overview of the likely effects.

An important feature of our model is that its steady state is not invariant to initial conditions. Because the steady state net foreign asset position depends on its initial level, so does steady state consumption spending. In the Appendix, we derive the steady state level of all other endogenous variables, and show that (i) $\bar{q}, \bar{v}, \bar{w}$, and $\bar{k}_j/\bar{I}_j$ are independent of initial conditions, while (ii) $\bar{c}_T, \bar{c}_N, \bar{\bar{I}}_j, \bar{\bar{k}}_j, \bar{\bar{i}}_j$ depend on $\bar{x}$, and hence on initial assets. We also show that the aggregate capital stock, $\bar{k} = \bar{k}_T + \bar{k}_N$ is a function of $\bar{x}$ if and only if the sectoral intensities $\alpha_T$ and $\alpha_N$ are different. To summarize, our model predicts that the currency board economy is different from the flexible regimes not only along the transition path but also in steady state.

Along the transition path, differences in the initial consumption expenditure $x_t$ are propagated through the production and capital accumulation blocks by the relative price of non-tradables, $p_t$. Indeed, in a model where capital is not sector-specific and sectoral intensities are equal ($\alpha_T = \alpha_N$), capital accumulation is independent from movements in $x_t$. This can be seen from equations (12), (13) and (14), which in this case fully determine investment behavior. The currency board and the flexible exchange rate economy still differ in this one-sector model, but only to the extent that consumption expenditure and money holdings evolve differently.

Thus we see that for capital paths to depend on the monetary regime, our two-sector assumption is crucial. The short-run transformation curve between tradables and non-tradables must be non-linear, either because of sector-specific capital or because of differing capital intensities. Thus different sectoral intensities are not required to produce relative price movements, but they naturally influence the quantitative predictions of the model.

5 Numerical exercises

In this section we look at quantitative predictions of our model. First, we explain our calibration strategy. Next, we present the numerical implementation of Proposition 2. As our main test of the model’s performance, we show that the model performs well in replicating the evolution of the net foreign asset position (NFA) of selected Central and Eastern European Countries (Hungary, Poland, Czech Republic, Slovakia, Lithuania, Latvia and Estonia), once we allow for differences in their respective currency regimes. In addition, we present how the model is able to match the observed behavior of other macro variables in the period 2001-2007.
After evaluating the model, we turn to the analysis of counterfactuals. We perform these exercises for the case of Hungary, but it is equally easy to carry them out for the other countries. In our first exercise, we compare what would have happened to the main macro variables if Hungary had had a fully flexible exchange rate regime starting from our initial year, 2001. In the second exercise we compare the effect of choosing exchange rates in a currency board. Our model predicts that the choice of the exchange rate regime has a sizable, nontrivial effect on the convergence path.

It is important to emphasize that we look at economies along their transition paths to the steady state in a deterministic context. We are comparing convergence paths (we use the term "transition" and "convergence" interchangeably) that start from the same initial conditions, but differ in the monetary arrangements. This is conceptually different from more typical exercises in stochastic models where the system is "shocked" around the deterministic steady state.

5.1 Calibration

5.1.1 Static parameters

The model allows for several normalizations by an appropriate choice of units. We set the steady state level of the relative non-tradable price at \( \hat{p} = 1 \), which determines the value of non-tradable productivity \( A_N \) through the steady state conditions. We also set the initial exchange rate to unity, regardless of the monetary regime. The discount rate \( \beta \) is chosen to yield a steady state interest rate of \( \bar{R} = 1/\beta = 1.04 \). This is the same value as used by Bems and Hartelius (2006), and corresponds to the average German interest rate on government bonds between 1975-2001.\(^{19}\) We also follow Bems and Hartelius (2006) and use German data to set the depreciation rate at \( \delta = 0.08 \).

To find the coefficient of real money balances in the utility function, we match the money-to-gdp ratio in steady state \( \bar{H}/\bar{Y} \) to the Euro area average of \( M2/Y = 0.7 \) between 1998-2005. This yields a value of \( \gamma = \). To be more precise, we perform this exercise in the Hungarian case, and use the implied \( \gamma \) in all the other simulations.

The capital adjustment cost \( \phi \) is taken from the literature on investment behavior. Cummins, Hassett and Hubbard (1996) estimate the effect of tax reform on investment across several countries, and find that the cross-country average of \( \phi \) is around 2. Cummins, Hassett and Oliner (2006), on the other hand, find values for the United States around 7.5. In our baseline calibration we use an intermediate value of \( \phi = 5 \), but we perform sensitivity analysis with \( \phi = 2 \) and \( \phi = 7.5 \).

\(^{19}\)We thus implicitly assume that Germany is in steady state in the period of interest.
The sectoral consumption, investment and production share parameters $\alpha_j$ and $\lambda_i$ are country specific. Since our production functions are Cobb-Douglas, we can use sectoral income share to calibrate the $\alpha_j$. A difficulty is to have the gross operating surplus-labor income-mixed income distinction at an industry level. For Hungary (1995-2005), Poland (2000-2005) and the Czech Republic (1995-2006), the required data is posted on the local CSO website. For the Baltic countries, we take numbers from Bems and Hartelius (2006) for Latvia and Estonia, year 1997. LITHUANIA???

To calculate capital’s share, we assume that the proportion of capital and labor compensation is the same in mixed income than without it. Gollin (2002) argues that this adjustment is reasonable relative to allocating all mixed income to capital. Herrendorf and Valentinyi (2008) show that for US data at least, the procedure leads to results that are similar enough to the actual mixed income shares imputed by the BLS (Herrendorf and Valentinyi 2008, Table 3).

Consumption and investment expenditure shares are calculated using input-output tables from the Eurostat, year 2000, except for Latvia where we use the year 1998 (the last available). Consumption includes household and government consumption. We classify sectors A,B,C,D and I as tradable, and sectors E-H and J,K,M,N as nontradable. OF WHAT?

5.1.2 Initial conditions

Since capital in our model is sector specific, we need initial conditions for capital in the tradable and nontradable sectors. In general, comparable cross-country data on capital stocks does not exist, so we choose initial conditions as follows. First we compute the relative GDP of the 7 countries (compared to Germany) at purchasing power standard from the Eurostat for the year 2001. CEE countries are poorer than Germany for two main reasons: their capital stock is lower, and they are less productive. Our model only captures the first element, so we correct the data by estimates of relative TFP for 2001. For this purpose, we utilize calculations by Carone et. al. (2006), Table 13. Their TFP projections imply that in 2001 CEE countries have an approximately 20% productivity lag relative to rich EU member states. Our benchmark calibration then assumes that the initial level of GDP in the model relative to each country’s respective steady state is given by the observed relative GDP adjusted by the relative TFP levels. For example, of the observed 2001 relative GDP is 0.5 and relative productivity 0.8, we calibrate the model initial GDP to $0.5/0.8=0.625$. A final issue concerns the appropriate aggregation of sectoral output: since PPP relative GDP is a constant price measure, we calculate initial model GDP by using the steady state relative price.

---

20 The EU-KLEMS project (http://www.euklems.net/) contains capital stock data for a selected set of countries. Unfortunately, among the countries we study they only have capital data for the Czech Republic.

21 Since relative TFP is difficult to measure, we later carry out sensitivity where we set relative productivity at 0.7 and 0.9 instead of the baseline assumption of 0.8.
Second, we use data for capital stocks to calibrate the initial allocation of capital across sectors. Thus while we are wary of using capital stock level data, we are more confident that relative sectoral distributions (which are independent of measurement units) are better measured. Sectoral capital stock data is available from Eurostat for 2001 for the Czech Republic and Lithuania. For Hungary, we utilize the capital stock estimates by Pula (2003). We correct the Pula (2003) numbers by adding residential capital stock based on data from the Hungarian statistical office. To summarize: we use the two observables detailed above (adjusted relative GDP and capital distribution across sectors) to compute the initial level of capital in tradables and nontradables.

Choosing initial conditions for wealth is simpler. For the initial money over GDP ratio we use 2001 M2 and nominal GDP data (in local currency) from the International Financial Statistics (IMF). Net foreign asset positions are taken from Eurostat, except for Latvia where we use IFS data.

5.1.3 Interest premium

The last issue in our calibration exercise is setting the exogenous path for the interest premium. We faced several problems in this exercise. First, interest rates and premia differ depending on the purpose of the loan and the identity of the borrower. Households and firms face quite different interest rates, and the same is true for collateralized vs. non-collateralized loans. Second, we have only limited data for the countries in our sample, some if which are for non-overlapping periods. Third, in the period where we evaluate the model’s performance (2001-2008), interest rates and risk premia saw a dramatic decline with a partial reversal towards the end. While our model is deterministic, it is hard to argue that the recent swings in risk premia were completely predictable, casting series doubt on the usage of actual premia, even if we solve the problems of finding comprehensive and representative data. Finally, and most importantly, our model is sensitive to small changes in the risk premium path, because the NFA path and its steady state level is history dependent.

To circumvent these problems, we assume that the risk premium is common across the 7 countries and follows a monotonic path, converging to zero. We calibrate the initial level to the cross-country average of the 2001 interest rate differential for corporate loans relative to the German level. The speed of convergence is picked by fitting the model generated NFA path to its observed equivalent for the case of Hungary, where the goodness of the fit is determined by (i) values between 2001-2008, (ii) the steady state NFA per GDP level, and (iii) the maximum level of indebtedness along the transition path. For the letter two, we consider paths where debt

---

22 We use three different procedures for the correction, all of which yield essentially the same result. Further details are available from the authors upon request.
does not exceed 150% of GDP, and steady state NFA is between -0.5 and 0.5. These are the extreme values in the dataset compiled by Lane and Milesi-Ferretti (2007), for EU15 countries (see Figure A1 for details). Finally, we assume that the risk premium path calibrated to the case of Hungary is the same across all seven countries. With this in hand, we evaluate our model by examining its ability to fit the NFA developments in the countries other than Hungary.

Figure 6 plots a "fanchart" of the effect of the risk premium adjustment speed ($\psi$) on the evolution of the NFA position of our baseline Hungarian calibration. The figure plots the simulated NFA paths for periods 1-20 and the steady state, and also the actual evolution of Hungarian NFA between 2001-2006 (periods 1-6). In our baseline calibration, we pick $\psi = 13$ (the solid thick line) as a good compromise between empirical fit and long-term behavior. We also check other values to check the robustness of our results in Section ?.

---

23 The single exception is Finland, whose NFA per GDP ratio was below -1.5 in 1999, but even in this case it stayed below -1 only in 1999-2000.
5.2 Model fit

In this section we evaluate the model against data from the sample of CEE countries discussed above. First, we look at the evolution of financial assets in the data and in our model. Second, we explore the model’s performance for a selected set of real variables.

5.2.1 Money and NFA

![Figure 7: Exchange rates against the euro (€/national currency, 1996=100)](image)

The model’s starkest implications are for the paths of financial assets: money balances and the net foreign asset position. More precisely, the accumulation of these assets depends on the currency regime. The Baltic countries have operated a currency board during our whole sample period, thus we classify them as fixed regimes. The other four countries were, in principle, floaters. Monetary policy, however, strongly influenced the extent of exchange rate movements. Figure 7 plots the evolution of the exchange rate for the Czech Republic, Hungary, Poland and Slovakia. Recall that the model predicts nominal appreciation under a truly flexible exchange rate. We clearly see this in the Czech Republic and Slovakia, thus we classify them as flexible regimes. Hungary, on the other hand, had a basically flat exchange rate, which indicates an implicit targeting by its central bank. Thus we classify Hungary as having an (implicitly)
fixed regime, although this is obviously a more qualified statement than in the case of the
Baltic countries. The case of Poland is more difficult, as it experienced a marked depreciation,
followed by a strong appreciation. Given that the model is forward looking, and we see nominal
appreciation in the second half of the sample period, we classify Poland as floater.

We run the following experiment. Given our calibration detailed above, we simulate the
model under two alternative scenarios: flexible exchange rate and currency board. We test the
model by comparing its predictions for money holdings and NFA under both scenarios with
what we see in the data. If the mechanism we emphasize is important, the flexible regime model
simulation should perform better for countries we classified as floaters, while the currency board
simulation should be superior for countries we classified as having a fixed regime. Given the
relative simplicity of our model, we do not expect either match to be perfect, especially for the
NFA which is very sensitive to the premium calibration. We do expect, however, a significant
difference (in the right direction) in the relative performance of the model under the alternative
exchange rate regimes.

Figure 8 present our results. In almost all cases the model produces a significantly better
fit when we use the model version that corresponds to our ex ante qualification. The only
exception is Poland, where the currency board model is preferred according to the data. Given
the ambiguity about de facto Polish exchange rate regime, this could actually be viewed as
favorable to our model, but it is safer to conclude that in the Polish case the test is inconclusive.

Also, since we calibrate the risk premium path using Hungarian NFA data the model’s
good fit there has no testing value. Interestingly, the test for money holdings in Hungary
is somewhat inconclusive: at the beginning of the period the currency board version performs
better, while towards the end of the period the flexible version is preferred. This may be because
the Hungarian regime became more flexible over time; we do not have firm evidence to prove
this point.

In terms of levels, generally we do a better job in matching the $M_2/Y$ ratio than the NFA
path. Since the NFA is very sensitive to the risk premia, this is not surprising. Recall that we
impose the same risk premia (based on a Hungarian calibration) for all countries, from which
the true values may differ. Of course, we could calibrate each country premia to their NFA path,
but then our test would only be based on $M_2/Y$ observations. We prefer a less than perfect
NFA fit to retain more degrees of freedom for model evaluation.

To summarize, we conclude that financial asset data strongly support the model. In partic-
ular, the model performs quite well in matching these data, and its performance is significantly
better if we use the correct exchange rate regime for the simulations. Our next step is to look
at the model’s performance for other, non-financial variables.
Figure 8: Financial asset evolution under alternative model specifications

27
5.2.2 Convergence

As we show below, the differences in the convergence process under a currency board and flexible exchange rates are economically meaningful and significant, but not big enough to pick up from noisy data. The exceptions are financial assets, which we examined in the previous section. For all other variables, we only present the model simulations under our ex ante regime specification. In case of Poland and Hungary, we keep our original assumptions of float and currency board respectively.

Figure ? shows the results for (i) GDP convergence, (ii) the relative price, and (iii) the sectoral allocation of capital. Since we use (i) and (iii) as initial conditions, the model and data paths coincide at $t = 0$; we chose data units so that the same holds for the relative price. Given our earlier discussion of productivity growth, we remove the predicted TFP growth using Carone et al. (2003) to get our relative GDP measure.

The first column contains GDP simulations for the seven countries. In most cases the model does a very good job, which also highlights the importance of removing the productivity trend. In four (LATVIA!!!) out of seven countries we match the speed of convergence almost exactly. In Hungary, we do not explain the slowdown that started in 2003, and in Poland we predict a much faster convergence than what we see in the data. These two failures may be due to cyclical factors: the Hungarian slowdown was a result of large fiscal imbalances, while Poland was just emerging from a recession in the second half of the time period.

Except for the Czech Republic, we are not able to explain the developments in the relative price and we have limited success for the relative capital stock. Relative capital is roughly constant in the data, while our model predicts increases in several cases. In the case of relative prices, we may have more success over a longer time period: recall that $p$ is normalized to 1 in the steady state. This implies that even though we predict an initial fall for some countries, eventually $p$ rises back towards 1. Notice also that in the data the relative price does seem to tend towards unity, given our normalization for the initial value.

Another reason that our model does not capture sectoral allocations and the relative price better is that we do not include asymmetric (or indeed any) technological progress. Since we systematically underpredict $p$, and faster TFP growth in non-tradables would lead to a sustained rise in the relative price of non-tradables, adding the Balassa-Samuelson to the model would improve its performance in this respect. Since our primary goal was not to match sectoral allocations, we opted for simplicity and left TFP growth completely out of the model.

Overall, we conclude that the model is successful in explaining movements in aggregate variables, such as GDP, the net foreign asset position, and the money/GDP ratio, but it has some difficulties in replication sectoral allocations. In addition to what was already discussed, this may simply reflect a discrepancy between model and measurement: the tradable/nontradable...
Figure 9: The real convergence properties of the model
breakdown is difficult to measure properly, and the reliability of relative capital stocks is also questionable.

5.3 Counterfactual analysis

After evaluating the models performance, we turn to the analysis of counterfactual scenarios. We are interested in the extent of differences between (i) a flexible and a currency regime, (ii) two currency board regimes under different exchange rates. In other words, we want to quantify the "real effects" of having a fixed exchange rate. We perform these experiments only for the Czech Republic: they can easily be repeated for the other countries.

5.3.1 Comparing exchange rate regimes

We first look at the impact of introducing a currency board in a flexible exchange rate regime. For this purpose, we use our calibration for the Czech Republic, which was one of the two countries with a truly floating exchange rate in our sample period. We choose the fixing period to be 2006, since this is the last year we have data for initial conditions, and here we are primarily interested in making projections for the future. The clear motivation for this exercise is Euro entry: our experiment can be interpreted as an examination of the effects of Czech Euro entry in 2006.

Technically, we compare the model’s predictions under the two alternative exchange rate regimes, assuming they both start from the same initial conditions (the actual 2006 Czech values), and keeping all parameters the same. We measure the difference between the two regimes in percentages: for any variable $z$, the numbers on the figure are $100 \cdot \left( \frac{z^{CB} - z^{FLEX}}{z^{FLEX}} \right)$.

Figure 10 shows the results. Most differences are small: GDP is lower by 0.02 percentage points on impact, but converges to a slightly higher level (0.05 percentage points). The magnitudes are similar for all real variables, with a slightly higher effect in case of sectoral reallocation. As we demonstrated before, the exceptions are the financial variables, the money stock and foreign assets, where changing the exchange rate regime would lead to a very significant reallocation of financial assets. The model predicts that the Czech Republic would have a higher NFA (less foreign debt) and less money under a currency board regime than under its actual flexible exchange rate regime, by about 20% in each case.

Our results thus suggest that the exchange rate regime, while having a "real effect", is quantitatively not very important for convergence and capital accumulation. The reason for this "almost neutrality" is twofold. First, since the interest premium is exogenous, a changing NFA position does not feed back to the cost of borrowing. In a model where the interest premium depends on the net foreign asset position (a common assumption in recent small open economy models), the choice of the exchange rate regime would have a bigger impact. Second,
Figure 10: Differences between the flexible regime and the currency board
the importance of sectoral reallocation is muted by the fact that the sectoral factor shares are not very different in the countries we analyze. Since the transformation curve is close to linear, shifts in demand lead to only small movements in the relative price. This, in turn, induces only limited changes in the incentives to accumulate capital.\textsuperscript{24}

5.3.2 Choosing the fixing rate

TO BE COMPLETED

5.4 Sensitivity analysis

TO BE COMPLETED

6 Some concluding comments

TO BE COMPLETED

References


\textsuperscript{24}In a version of our model where the interest premium is endogenous and factor intensity differences are more substantial, we find that the effects of the exchange rate regime are a magnitude larger. These results are available from the authors upon request.


Appendix

A Net foreign asset positions in selected countries

Figure 11: NFA per GDP positions in selected EU countries
B Derivations under flexible regimes

In this section we present the steps that lead to equations (19) and (20). Under a flexible exchange rate, the money stock is fixed, so we can rewrite the money demand equation (11) as

\[
\frac{\gamma}{H} = \frac{1}{s_t x_t} - \frac{\beta}{s_{t+1} x_{t+1}}.
\]

In steady state, the equation implies

\[
\bar{H} = \frac{\gamma \bar{x}}{1 - \beta}.
\]

Let us define spending in local currency as \(s_t x_t\). Using this definition and substituting for \(H\) in the money demand equation, we get:

\[
\frac{1 - \beta}{\bar{x}} = \frac{1 - \beta}{\chi_t} - \frac{1 - \beta}{\chi_{t+1}} \quad \downarrow \quad \frac{1 - \beta}{\beta} \left( \frac{1}{\bar{x}} - \frac{1}{\chi_t} \right).
\]

The last equation implies that if \(\chi_t > \bar{x}\), then \(\chi_{t+1} > \chi_t\) and vice versa. Hence the only way the system can reach the steady state is that \(\chi_t = \bar{x}\) for all \(t\). Thus we have \(s_t x_t = s_{t+1} x_{t+1}\), which together with (10) yields (19).

The proof is similar in the perfectly elastic money supply case. Using the fact that \(1 + \tau_t = H_t / H_{t-1}\) and that the exchange rate is fixed, we rewrite the money demand equation in and out of steady state as follows:

\[
\gamma \bar{s} = \frac{H_t}{x_t} - \frac{\beta H_{t+1}}{x_{t+1}}
\]

\[
\bar{s} = \frac{(1 - \beta) \bar{H}}{\gamma \bar{x}}.
\]

Substituting for \(\bar{s}\) and introducing the notation \(\xi_t = H_t / x_t\), the money demand equation becomes

\[
(1 - \beta) \bar{\xi} = \xi_t - \beta \xi_{t+1} \quad \downarrow \quad \xi_t - \xi_{t+1} = \frac{1 - \beta}{\beta} (\bar{\xi} - \xi_t).
\]

The only way the system can reach the steady state is that \(\xi_t\) is constant, i.e. \(H_t = x_t \bar{H} / \bar{x}\). Substituting for \(\bar{H} / \bar{x}\) yields (20).
C Steady state results

Here we present the steady state conditions and derive which variable’s steady state value is dependent on initial conditions. We normalize $\tilde{p} = 1$, and choose $A_N$ accordingly, which lead to the list of equations below. Notice that the capital rental rates are equalized in steady state, so we can drop the $j$ index:

\[
\begin{align*}
\tilde{c}_T &= \eta \tilde{x} \\
\tilde{c}_N &= (1 - \eta) \tilde{x} \\
\tilde{i}_j &= \delta \tilde{k}_j \\
\tilde{i}_j^T &= \eta_I \left(1 + \frac{\phi}{2}\right) \delta \tilde{k}_j \\
\tilde{i}_j^N &= (1 - \eta_I) \left(1 + \frac{\phi}{2}\right) \delta \tilde{k}_j \\
\tilde{q} &= 1 + \delta \phi \\
\tilde{r}^k &= (1 + \delta \phi) \left(\frac{1}{\beta} - 1\right) + \delta + \frac{\delta^2 \phi}{2} \\
\frac{\tilde{k}_T}{\tilde{l}} &= \left(\frac{\alpha_T}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_T}} \\
\frac{\tilde{k}_N}{1 - \tilde{l}} &= \left(\frac{A_N \alpha_N}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_N}} \\
\tilde{w} &= (1 - \alpha_T) \left(\frac{\tilde{k}_T}{\tilde{l}}\right)^{\alpha_T} \\
(1 - \alpha_T) \left(\frac{\alpha_T}{\tilde{r}^k}\right)^{\frac{\alpha_T}{1-\alpha_T}} &= (1 - \alpha_N) A_N \left(\frac{A_N \alpha_N}{\tilde{r}^k}\right)^{\frac{\alpha_N}{1-\alpha_N}} \to A_N
\end{align*}
\]

We still need to solve for $\tilde{l}$, for which we use the non-tradable market clearing condition:

\[
A_N \left(1 - \tilde{l}\right) \left(A_N \alpha_N \frac{\tilde{r}^k}{\tilde{r}_N}\right)^{\frac{\alpha_N}{1-\alpha_N}} = (1 - \eta) \tilde{x} + (1 - \lambda_I) \left(1 + \frac{\phi}{2}\delta\right) \delta \left[\tilde{i} \left(\frac{\alpha_T}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_T}} + (1 - \tilde{l}) \left(\frac{A_N \alpha_N}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_N}}\right]
\]

This equation implicitly defines $\tilde{l}$ as a function $\tilde{x}$. Collecting terms, the multiplier of $\tilde{l}$ can be written as

\[
\delta_l = A_N \left(\frac{A_N \alpha_N}{\tilde{r}_N}\right)^{\frac{\alpha_N}{1-\alpha_N}} + (1 - \lambda_I) \left(1 + \frac{\phi}{2}\delta\right) \delta \left[\left(\frac{\alpha_T}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_T}} - \left(\frac{A_N \alpha_N}{\tilde{r}^k}\right)^{\frac{1}{1-\alpha_N}}\right].
\]

37
Using the equation that defines $A_N$, we can sign this expression as follows:

\[
\delta_l = \frac{1 - \alpha_T}{1 - \alpha_N} \left( \frac{\alpha_T}{\mu} \right) \delta^T \left( 1 - \eta_l \right) \left( 1 + \frac{\phi}{2} \right) \delta \left( \frac{\alpha_T}{\mu} \right) \delta^T \left( 1 - \frac{1 - \alpha_T}{\alpha_T} \frac{\alpha_N}{1 - \alpha_N} \right)
\]

\[
\sim \frac{\bar{r}^k}{\alpha_N} \left( 1 - \alpha_T \frac{\alpha_N}{\mu} \right) \left( 1 - \eta_l \right) \left( 1 + \frac{\phi}{2} \right) \delta \left( 1 - \alpha_T \frac{\alpha_N}{\mu} \right) 1 - \alpha_N
\]

\[
> \frac{1 - \alpha_T}{\alpha_T} \left( 1 - \alpha_N \right) - \left( 1 - \eta_l \right) \left( 1 + \frac{\phi}{2} \right) \delta \frac{\alpha_N}{1 - \alpha_N}
\]

\[
> \frac{1 - \alpha_T}{\alpha_T} \left( 1 - \alpha_N \right) - \left( 1 + \frac{\phi}{2} \right) \delta \frac{1}{1 - \alpha_N}
\]

\[
= \frac{1 - \alpha_T}{\alpha_T} \left( 1 - \alpha_N \right) \left[ \bar{r}^k \left( 1 + \frac{\phi}{2} \right) \delta \right]
\]

\[
= \frac{1 - \alpha_T}{\alpha_T} \left( 1 - \alpha_N \right) \left( 1 + \delta \phi \left( \frac{1}{\beta} - 1 \right) \right) > 0.
\]

This implies that $\bar{l}$ depends negatively on $\bar{x}$. Using the steady state conditions, we can group all steady state variables regarding their relation to $\bar{x}$:

- does not depend on $\bar{x}$: $\bar{q}, \bar{r}^k, \bar{w}, A_N, \xi, \bar{\xi}_T$
- depends on $\bar{x}$: $\bar{c}, \bar{c}_N, \bar{l}, \bar{k}_j, \bar{k}$
- may depend on $\bar{x}$ (if $\alpha_T \neq \alpha_N$): $\bar{k}$

The last step is to relate the steady state NFA, $\bar{b}$ to $\bar{x}$.

\[
\left( \frac{1}{R} - 1 \right) \bar{b} = \bar{w} + \bar{r}^k \bar{k} - \bar{x} - \left( 1 + \frac{\phi}{2} \right) \delta \bar{k}
\]

\[
\bar{b} = \frac{1}{\beta - 1} \left( \bar{w} - \bar{x} + \bar{k} \left( \bar{r}^k - \left( 1 + \frac{\phi}{2} \right) \delta \right) \right)
\]

\[
= \frac{1}{1 - \beta} \left( -\bar{w} + \bar{x} - \bar{k} \left( \left( 1 + \delta \phi \right) \left( \frac{1}{\beta} - 1 \right) \right) \right).
\]

Here

\[
\bar{k} = \bar{\xi}_T \left( 1 - \frac{\alpha_T}{\alpha_N} \frac{\alpha_N}{1 - \alpha_N} \right) + \bar{l} \left( 1 - \frac{1 - \alpha_T}{\alpha_T} \frac{\alpha_N}{1 - \alpha_N} \right)
\]

\[
\bar{l} = A - B \bar{x}
\]

\[
\bar{k} = C - B \bar{\xi}_T \left( 1 - \frac{1 - \alpha_T}{\alpha_T} \frac{\alpha_N}{1 - \alpha_N} \right) \bar{x},
\]

so

\[
\bar{b} = \frac{1}{1 - \beta} \left( -\bar{w} + \bar{x} - \bar{k} \left( \left( 1 + \delta \phi \right) \left( \frac{1}{\beta} - 1 \right) \right) \right)
\]

\[
= D + \frac{1}{1 - \beta} \left( 1 + \left( 1 + \delta \phi \right) \left( \frac{1}{\beta} - 1 \right) \right) B \bar{\xi}_T \left( 1 - \frac{1 - \alpha_T}{\alpha_T} \frac{\alpha_N}{1 - \alpha_N} \right) \bar{x}.
\]
Notice that the relation between $\bar{b}$ and $\bar{x}$ again depends on relative intensities. For at most moderate differences (which is in line with existing evidence), a higher $\bar{x}$ implies a higher $\bar{b}$: higher expenditure is financed from higher financial investment income. For extreme differences, it might go the other way, in which case physical capital income finances the extra expenditure.

The intuition is clear. All prices are independent from expenditures (they are set by technology). A higher steady state expenditure level (which is financed by higher investment income from $\bar{b}$) leads to a higher NT sector (lower $\bar{l}$), and higher consumption of both goods. A higher $\bar{l}$ leads to a different $\bar{k}$ if the two sectors are different in their capital intensities. In particular, a higher $\bar{x}$ leads to a lower $\bar{k}$ if the NT sector is more labor intensive, a higher $\bar{k}$ if the NT sector is more capital intensive, and the same $\bar{k}$ if $\alpha_T = \alpha_N$. 

39