Procrastination Markets*

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Abstract

We develop models of markets with procrastinating consumers when competition operates — or is supposed to operate — both through the initial selection of providers and through the possibility of switching providers. As in other work, consumers fail to switch to better options after signing up with a firm, so at that stage they exert little downward pressure on the prices they pay. Unlike in other work, however, consumers — falsely expecting to do still better in the future — are not keen on starting with the best available offer, so at this stage they do not generate much price competition either. In fact, a competition paradox results: an increase in the number of firms or the intensity of marketing increases the frequency with which a consumer receives switching offers, so it facilitates procrastination and thereby potentially raises prices. By implication, continuous changes in the environment can, through a self-reinforcing entry or marketing process, lead to discontinuous changes in market outcomes. Sign-up deals serve their classically hypothesized role of returning ex-post profits to consumers extremely poorly, while in other senses they exacerbate the failure of price competition.

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1 Introduction

A large and growing literature recognizes that individuals procrastinate — repeatedly put off tasks they know they should perform — in a variety of settings, and the welfare consequences can be disastrous.[1] In markets, perhaps the most important potential manifestation of procrastination occurs when a consumer fails to switch away from an unfavorable status-quo contract for energy (Hortaçsu et al., 2017, Ito et al., 2017, Office of Gas and Electricity Markets, 2018, 2019a), health insurance (Handel, 2013, Polyakova, 2016), auto insurance (Kiss, 2019), a credit card (Shui and Ausubel, 2005), or another essential product or service. Our goal is to understand the implications of these consumer tendencies for the patterns and degree of competition between firms.

To obtain our insights, we develop and analyze models of markets with procrastinating consumers in which we allow competition to operate both through the initial selection of providers and through the possibility of switching providers. Paralleling classical work on switching costs as well as more recent work on naive consumers starting from DellaVigna and Malmendier (2004), we predict that a procrastinating consumer often fails to switch to better options after signing up with a firm, so at that stage she exerts little downward pressure on the prices she pays. Indeed, the observation that consumer inertia softens competition at the switching stage is widely recognized by regulators (Competition & Markets Authority, 2016b, Canadian Radio-television and Telecommunications Commission, 2017, Financial Conduct Authority, 2017, 2018, Australian Competition & Consumer Commission, 2019). But existing theories — variously nicknamed bargains-and-ripoffs, invest-then-harvest, and the waterbed effect — imply that competition shifts to when consumers choose providers initially, and overall prices may well remain unchanged or even decrease (Taylor, 2003, Cabral, 2016, Ericson, 2020). In contrast, our model says that because a consumer expects to engage in optimal switching behavior in the future and is therefore not keen on starting with the best available offer, price competition at the initial stage is soft or non-existent as well. What is more, a competition paradox results: an increase in the number of firms or the intensity of marketing increases the frequency with which a consumer receives switching offers, so it facilitates procrastination and thereby potentially lowers competition and raises prices. By implication, entry or marketing can be self-reinforcing, generating discontinuous jumps in market patterns in response to continuous changes in the environment. Sign-up deals, which in existing theories provide a vehi-

[1] We discuss the literature on procrastination in Section 2.2.

cle for returning ex-post profits to consumers, serve this purpose extremely poorly, while in other senses they exacerbate the failure of competition. Finally, although the consumer is prone to sticking with a bad deal she accepts initially, when she does switch, she might switch to an offer that is as unfavorable as her previous one.

We present our basic model in Section 2. On the demand side, a consumer needs a service from time 0 to time T, and pays for it per unit of time. At time -1, she is assigned to an initial offer, but she can switch providers at evenly spaced switching opportunities beginning at time 0. Shortly before a switching opportunity, she receives the relevant switching offer, and she can exchange her contract for the new one by paying an immediate effort cost of s. Being present-biased and naive, the consumer discounts payments and future switching costs by a factor $\beta < 1$, but believes that she will not do the same in the future. Turning to the supply side, the offers the consumer receives come from firms with a production cost of zero that simultaneously choose non-negative price pairs consisting of an initial offer capped at v and a switching offer. From all initial offers, one is randomly chosen to be the consumer's initial offer above. The switching offers of the other firms are randomly allocated to the switching opportunities above, with the wait time between opportunities decreasing in the number of firms and the intensity of their marketing activities. We investigate the symmetric pure-strategy Nash equilibria of the game played between firms, assuming that the firms correctly understand consumer behavior.

Our model captures, in a stylized way, the competitive landscape of some important service industries. In one application, the consumer is looking to make a purchase by borrowing a fixed amount that she will be able to repay at time T. Credit-card issuers make offers consisting of an interest rate on purchases as well as an interest rate on balance transfers. The consumer gets a card to make her purchase, but then receives many solicitations to transfer her balance to another card. And in another application, the consumer is a new resident — or even a continuing resident with an expiring contract — who would like to arrange for an essential service such as electricity or gas. She is initially assigned one provider — in the case of a continuing resident her existing one — but she may have many opportunities to switch providers.

We explore implications of our basic model in Section 3. To begin, we show that the consumer never switches, and she is charged the highest initial price that she sticks with if all switching offers are for a price of zero. That she fails to switch in turn means that at least one of two conditions is satisfied. First, under the "no-incentive-to-switch condition," she thinks that switching is not worth the cost s. Second, under the "incentive-to-procrastinate condition," she thinks that switching at the next opportunity dominates switching immediately, so that she perpetually procrastinates in switching. For few firms and low-intensity marketing, the equilibrium price is determined by the no-incentive-to-switch condition, and by standard logic the total profit an initial firm can make is constrained by the switching cost augmented by the discount factor (s/β). For more firms or high-intensity marketing, however, the incentive-to-procrastinate condition determines the equilibrium price, and the competition paradox obtains. The higher is the number of firms or the more intense is marketing, the more frequently the consumer receives switching offers, so the more tempted she is to procrastinate and therefore the higher is the price an initial firm can charge. In this region, a policy that increases competition at the switching stage may or may not be beneficial. To the extent that the policy lowers the switching cost s, it lowers prices. To the extent that it provides access to more offers at a switching opportunity, it has no effect. And to the extent that it expands the set of switching opportunities, it exacerbates procrastination and thereby raises prices.

Having established that competition from switching offers cannot discipline markets, we ask whether initial competition can. To do so, we assume that multiple firms make simultaneous initial offers to the consumer (all of which she can take for free), but we also allow for products to be slightly differentiated. We show that if the ratio of firms reaching the consumer initially is sufficiently low, then the above remains the unique equilibrium for an arbitrarily small amount of differentiation, and it remains an equilibrium even with no differentiation at all. Intuitively, the consumer would not respond to small price cuts because she would reason that she can switch to a still better deal in the future. And instead of offering a deep price cut, a firm prefers to gamble that the consumer chooses it randomly, reducing initial competition.

The competition paradox can generate stark comparative statics when we endogenize the number of firms and the intensity of marketing in the industry. We assume that entry has fixed cost F, and increasing marketing intensity — which increases the firm's chances of reaching the consumer at each decision point — has constant marginal cost c. We show that the c-F space can then be divided into two regions with qualitatively different equilibrium outcomes. On one side, the no-incentive-to-switch condition determines the price, and the consumer pays a total price of s/β. On the other side, the incentive-to-procrastinate condition is central, the consumer pays a total price of Tv, and both the number of firms and the intensity of marketing each firm chooses are discontinuously higher. Intuitively, at the point where procrastination starts playing a role, entry increases the equilibrium price. The resulting increase in profits lures further entry, and this self-reinforcing mechanism continues until prices reach the monopoly level.

In Section 4, we consider competition when firms use sign-up deals — one-time rewards or temporary price reductions — rather than permanently low prices to attract consumers. We assume that two firms make initial offers, the consumer can switch to the non-chosen offer later, and she can only take advantage of a firm's sign-up deal once. Because the consumer wants to benefit from both deals and end up with the cheaper unit price eventually, she strictly prefers to take up the *more expensive* deal — just to end up procrastinating switching away from it. Hence, even more robustly than in our basic model, initial competition in unit prices completely fails. Furthermore, because the consumer does not think she needs to take the larger bonus first, competition in bonuses fails as well. Hence, the sign-up bonuses do not serve their classically hypothesized role of channeling initial competition and returning ex-post profits to consumers, but they do serve the novel role of stunting competition in unit prices.

In our analysis of the main models and in Section 5, we argue that the mechanism limiting competition is robust to many natural modifications. If the consumer has some rare periods in her life in which her switching cost is zero, then she does switch, but often to a deal that is as bad as her previous one. If the consumer is partially rather than fully naive as modeled by O'Donoghue and Rabin (2001), then prices often remain unchanged. Unless there are many of them, the presence of time-consistent consumers may not change equilibrium prices at all. If the consumer can choose to go without the service, then competing firms facing no price caps may offer a unit price that is both above her valuation and as high as or higher than what a monopolist would offer — a terrible deal she accepts and later fails to switch away from or cancel. And most importantly, alternative psychological reasons for procrastination, such as underestimation of future switching costs (Tasoff and Letzler, 2014) or overconfidence about memory (e.g., Ericson, 2011), give rise to the same basic predictions, although they may drive some subtle specific pricing patterns.

We emphasize that to illustrate our novel points in the clearest possible way, we have developed models in which competition fails completely, and does so even for undifferentiated products. As we point out, there are natural variants of our models in which consumers do care somewhat about the initial offer, so for undifferentiated products competition ensues. But even then, the logic we have identified implies that competition is weak, so a little product differentiation or another weak competition-inhibiting force can eliminate it.

In Section 6, we discuss theoretical research most closely related to our paper. While previous work has not studied the market effects of procrastination in switching, researchers have identified obstacles to competition due to limitations on sign-up deals, adverse selection, or the inability of consumers to understand or compare prices. Because our mechanism is operational even when none of these obstacles apply, it helps explain important examples of high prices for which previous work does not appear to provide a complete account. For credit cards and utilities, for instance, firms can commit to future prices, and they already offer sign-up deals in which they set low prices for a limited period. It is not obvious why profitable consumers would be relatively unresponsive to these deals (which is required by models of adverse selection) or that consumers do not understand them (which is required by models of consumer confusion). Hence, firms could compete more fiercely by extending these deals for longer — a strategy that faces no constraint.

We conclude in Section 7 with some questions for further research. While our model explains broad features of some markets with switching offers, we have not explored how policy should respond to the problems we identify. We suspect that disciplining markets by improving consumer switching behavior is exceedingly difficult, so systems that work without active consumer engagement may be necessary. We also point out that the logic of the competition paradox might apply not only to situations in which the consumer buys something expensive, but also to situations in which she fails to buy something valuable.

2 Basic Model of Switching Markets

We first model "switching markets" — i.e., markets with the possibility of switching — in which an offer takes the simplest possible form, a linear price.

2.1 Setup

Consumer's Problem (Figure 1). From time 0 until time T, the consumer needs a service that she pays for per unit of time. At time -1, she is automatically assigned to a price p^{-1}. Shortly before times 0, T_w, $2T_w$, $\ldots$, $KT_w < T$, she receives switching offers for prices p^0, p^1, p^2, $\ldots$, p^K,

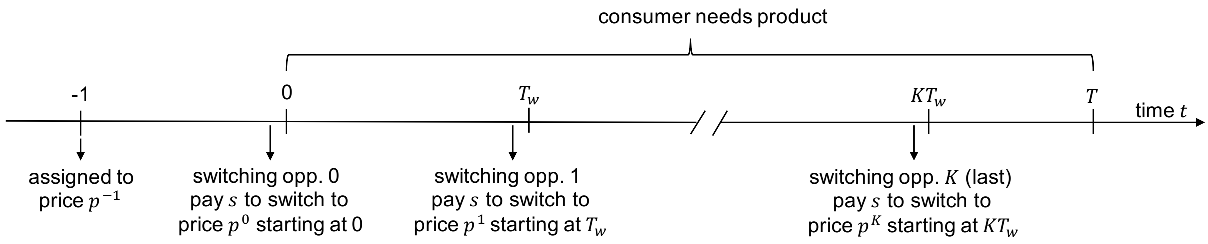

Figure 1: Illustration of Consumer's Problem

respectively, all of which she observes at the beginning. If she takes up offer $\kappa \in \{0, \ldots, K\}$, then she pays an instantaneous effort cost $s > 0$, and starting subsequently at time κT_w she pays price p^κ instead of the last price she accepted or was assigned to. We call T_w the wait time between switching opportunities, and let $K = \lceil T/T_w \rceil - 1$;[2] this ensures that the last contract period, $T - KT_w$, is no longer than T_w either.

Consumer Behavior. The consumer is present-biased as introduced by Laibson (1997) and naive as defined by O'Donoghue and Rabin (1999a). To specify her behavior formally, we denote her switching decision at opportunity κ by $d_\kappa \in \{0, 1\}$, where $d_\kappa = 1$ stands for switching, and the *total* price she pays between switching opportunities κ and $\kappa+1$ — or, for $\kappa = K$, after opportunity κ — by p_κ.[3] At each opportunity κ, the consumer aims to solve

$$\min_{d_\kappa, \ldots, d_K} \quad d_\kappa s + \beta \cdot \sum_{\kappa'=\kappa+1}^{K} d_{\kappa'} s + \beta \cdot \sum_{\kappa'=\kappa}^{K} p_{\kappa'}, \tag{1}$$

where $\beta \in (0, 1]$ is her short-run discount factor. Naively, the consumer believes that whatever plan she makes today, she will carry it out in the future. Hence, at each decision point $\kappa = 0,\ 1,\ \ldots,$ K she solves problem (1) and chooses the d_κ in her solution. We impose the tie-breaking rule that the consumer switches only if she strictly prefers to.

Game between Firms. The offers the consumer receives come from $N \geq 2$ firms that can all provide the service at a cost of zero. Firms simultaneously choose price pairs (p_I^n, p_S^n), where $p_I^n \in [0, v]$ is firm n's initial offer and $p_S^n \in [0, v]$ is firm n's switching offer. Then, one initial offer p_I^n is randomly (and with equal probability) chosen to be the initial offer p^{-1} above. For each switching opportunity, one switching offer $p_S^{n'}$ is randomly (and with equal probability) chosen

[2] For $x \in \mathbb{R}$, $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

[3] Precisely, we consider the cases (i) $\kappa \leq K - 1$ and (ii) $\kappa = K$ separately. In case (i), if $d_{\kappa'} = 0$ for all $\kappa' \leq \kappa$, then $p_\kappa = T_w \cdot p^{-1}$; otherwise, $p_\kappa = T_w \cdot p^j$ for $j = \arg\max_{\kappa' \leq \kappa} d_{\kappa'} = 1$. In case (ii), if $d_{\kappa'} = 0$ for all $\kappa' \leq K$, then $p_K = (T - KT_w) \cdot p^{-1}$; otherwise, $p_K = (T - KT_w) \cdot p^j$ for $j = \arg\max_{\kappa' \leq K} d_{\kappa'} = 1$.

from the competitors $n' \neq n$ of the initial firm. The consumer's wait time is $T_w = T/(m(N-1))$, where $m > 0$ is a firm's marketing intensity defined as the frequency with which its offers reach consumers. We investigate the symmetric pure-strategy Nash-equilibrium outcomes of the game played between firms, assuming that the firms correctly understand consumer behavior.

2.2 Applications and Discussion of Assumptions

Our simple model captures the essential elements of many switching markets. As a first example, consider credit cards. The consumer needs to buy a good on credit that she can pay off at time T. She has access to one credit card on which she can charge the purchase, but she receives balance-transfer offers from other issuers. Each issuer can specify the interest rate on its card.

Another example is utilities. The consumer moves into a new home and seeks an essential service such as electricity, gas, landline, or broadband. When she moves in, she is randomly assigned an initial supplier. She is, however, in a liberalized market with a number of alternative suppliers, and these suppliers make offers to her to switch. Alternatively, our model applies when the consumer is not moving, but her current contract period is expiring and therefore her provider can change its price. The consumer can switch away before time 0 and receive a competitor's contract, do nothing and just get the current provider's new contract, or go with her current provider for a while and switch later.

In many applications, it is plausible that firms distinguish between new consumers and switching consumers. In the credit-card context, issuers regularly set different terms for purchases and balance transfers. In subscription markets, providers often charge "loyalty penalties" — different prices for continuing (loyal) and new consumers.[4] If instead firms cannot distinguish the two consumer types, then in our basic model intractable mixed-strategy equilibria arise, but the economic mechanisms we identify do not disappear. Substantiating this view, below we identify variants of our model in which the indistinguishability of the two consumer types does not affect outcomes. Furthermore, in Section 4 we analyze situations in which firms offer sign-up bonuses rather than permanently low prices as an inducement to switch, and cannot distinguish the two consumer types. In that context,

[4] For instance, the Competition & Markets Authority (2018) discusses evidence for loyalty penalties in the UK markets for retail domestic energy, home and motor insurance, broadband, mobile tariffs, cash savings, and mortgages, and suggests that loyalty penalties may also arise in other auto-renewal, roll-over or subscription products or services. We can capture loyalty penalties by thinking of p_I^n and p_S^n as the prices that firm n offers to continuing and new consumers, respectively, with a consumer only paying a switching cost if she switches providers.

we provide reasons why a firm may not want to offer different contracts to the two consumer types even if it can distinguish them.

Our model assumes that the consumer must accept the initial contract, and there is a cap v on the price firms can charge. These assumptions are consistent with a situation in which — as with utilities — the consumer cannot realistically go without the service, and partly for this reason prices are regulated. We show in Section 5.1 that even worse outcomes may obtain if the consumer has the option not to buy, and there is no binding price cap. Relatedly, we assume that there is only one offer the consumer can accept for free. This assumption is plausible if the consumer is — again as in the case of utilities — defaulted into a contract if she does nothing, while she must exert effort to find alternative providers. Nevertheless, we make this assumption mainly to identify outcomes when competition derives solely from the possibility of switching. Below, we extend our model to allow for competition in initial offers as well.

Our specification of consumer behavior in terms of naive present bias is motivated both by direct evidence on discounting and beliefs,[5] and by indirect evidence on behavior consistent with the framework. Most importantly, there is overwhelming evidence from both academic and policy circles that consumers often fail to switch to more favorable options in the markets for energy (Competition & Markets Authority, 2016a, Hortaçsu et al., 2017, Ito et al., 2017), health insurance (Handel, 2013, Handel and Kolstad, 2015, Polyakova, 2016), credit cards (Shui and Ausubel, 2005), paid TV (Shcherbakov, 2016), mobile-phone services (Shy, 2002), auto insurance (Kiss, 2019), and mortgages (Keys et al., 2016, Andersen et al., 2020), and attempts to make switching costs very low do not result in high switching rates (Office of Gas and Electricity Markets, 2019a,b). Since a model of rational consumers predicts high switching rates under reasonable levels of switching costs and discount factors, it is inconsistent with these facts.[6] But because models of naive present

[5] In particular, papers by DellaVigna and Paserman (2005), Paserman (2008), Fang and Silverman (2009), Meier and Sprenger (2010), Carter et al. (2017), and Laibson et al. (2018) document a taste for immediate gratification, while Skiba and Tobacman (2008), Acland and Levy (2015), Fedyk (2018), Augenblick and Rabin (2019), Chaloupka et al. (2019), Carrera et al. (2020), Bai et al. (forthcoming), and Kuchler and Pagel (2021) find that individuals are at least partially naive about this taste. On the other hand, Allcott et al. (2020) find that only the least experienced quartile of payday-loan borrowers underestimate their likelihood of future borrowing, while others predict future borrowing correctly on average, suggesting that a model of sophisticated rather than naive present bias better describes these borrowers.

[6] For instance, Handel (2013) estimates that inertia in health-insurance choices costs employees \$2,032 on average; Kiss (2019) estimates that in a standard setting, a switching cost of \$373 is necessary to explain Hungarian drivers' failure to switch auto-insurance providers; and Andersen et al. (2020) estimate a psychological mortgage-refinancing cost of \$1,716. Relatedly, Chetty et al. (2014) document that defaults have large effects on the retirement savings of Danish households, and estimate that at least 85% are passive in that they do not adjust their own contributions in

bias — including those of O'Donoghue and Rabin (1999b,c, 2001, 2008) and, as we explain below, ours — often predict severely costly procrastination, they can account for the failure to switch with much lower switching costs, so they are consistent with the evidence. In Section 5.3, we argue that alternative models of procrastination can — within the settings we study — be seen as reinterpretations of our present-bias-based model, so they generate equivalent or similar predictions.

Firms' marketing intensity m is most straightforwardly interpreted as direct marketing aimed at individual consumers. As a simple example, m could be the number of mail credit-card solicitations an issuer sends, with the envelope the consumer happens to receive initially and at a switching opportunity randomly determined. Then, since there are $m(N-1)$ switching offers that the initial firm's competitors send in total, there are $m(N-1)$ switching opportunities, so — assuming the opportunities are distributed evenly before time T — the consumer's wait time is $T_w = T/(m(N-1))$. More generally, m can capture any marketing or advertising that attracts the consumer's attention and induces her to consider the firm for her initial purchase or for switching. Such alternatives could include online solicitations and marketing as well as general advertising.

Our model exogenously assumes that total marketing intensity (mN) and total marketing intensity for switching ($m(N-1)$) are increasing in the number of firms (N). This property will also arise endogenously when firms choose m strategically (Section 3.2). More subtly, our model assumes that when a firm markets more intensely to initial consumers, it also markets more intensely to switchers. In some cases, this is true by the nature of the market; e.g., a credit-card solicitation specifies interest rates for both purchases and balance transfers, so it includes an initial offer as well as a switching offer. More generally, it is plausible that marketing activities aimed at initial purchases also induce consumers to consider the firm for switching.

The main reason for our restrictive solution concept, symmetric pure-strategy equilibrium, is tractability. We are unaware of general analytic methods that would enable us to solve for the behavior of a naive consumer facing an arbitrary sequence of prices. Based on this and our own attempts, mixed-strategy equilibria appear intractable, and even the analysis of asymmetric pure-strategy equilibria involves many tedious and non-transparent case distinctions.[7] We conjecture, however, that the equilibrium outcome we identify does emerge as the unique pure-strategy equi-

response to changes in automatic contributions.

[7] The unrealistic assumption that the consumer observes all prices at the beginning is for similar reasons of tractability.

librium in some natural modifications of our model that simplify consumer or firm behavior. This is the case if at any switching opportunity the consumer can take up any earlier offer, so that her decisions cannot be driven by the risk of losing a good offer at hand. The same is the case if multiple firms make switching offers at each opportunity and there is an (arbitrarily small) share of time-consistent consumers, so that perfect competition for switchers necessarily results. In addition, we do not impose symmetry in our model in Section 4.

3 The Competition-Destroying Effect of Competition

3.1 Basic Effect

To illustrate the logic of equilibrium formally, we solve for the maximum initial price p_I that the consumer sticks with if all firms choose a switching price of 0. For the consumer not to switch away at opportunity 0, at least one of two conditions must be satisfied. First, she may prefer never switching to switching at opportunity 0. Second, she may prefer switching at opportunity 1 to switching at opportunity 0, so that she delays switching — she procrastinates — naively thinking that she will switch next time. We analyze each condition in turn.

No Incentive to Switch. Switching at opportunity 0 rather than never saves $\beta T p_I$ in discounted future payments, and has immediate cost s. Hence, the consumer prefers never switching if

$$p_I \leq p^{NIS} \equiv \frac{1}{\beta} \cdot \frac{s}{T}. \tag{NIS}$$

Note that if Condition (NIS) is satisfied, then — given that the saving from switching is highest at opportunity 0 — the consumer never benefits from switching, so she never switches.

Incentive to Procrastinate. If the consumer switches at opportunity 0, she must pay an immediate effort cost of s. If she switches at opportunity 1, then she must pay p_I until then, lowering her discounted utility by $\beta p_I T_w$; and she must pay the switching cost then, lowering her discounted utility by βs. Hence, she prefers switching at opportunity 1 if $s \geq \beta p_I T_w + \beta s$, or

$$p_I \leq p^{IP} \equiv \frac{1-\beta}{\beta} \cdot \frac{1}{T_w} \cdot s = \frac{1-\beta}{\beta} \cdot m(N-1) \cdot \frac{s}{T}. \tag{IP}$$

Note that if Condition (IP) holds, then — by the exact same logic — the consumer procrastinates on switching at all opportunities before the last one. And because Condition (IP) implies that

$s > \beta p_I T_w \geq \beta p_I(T - KT_w)$, the consumer strictly prefers not to switch at the last opportunity, so she never switches.

It is now easy to argue that all firms charging $p_I^n = p_I^* = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ and $p_S^n = 0$ is an equilibrium. The initial firm keeps the consumer forever if $p_I^n \leq p_I^*$ and loses the consumer immediately if $v \geq p_I^n > p_I^*$, so firms have no incentive to deviate on the initial price. And since the initial price is p_I^*, attracting the consumer at a switching opportunity is impossible, so firms have no incentive to deviate on the switching price either. In fact, this fully describes the possible equilibrium outcomes:

Proposition 1 (Equilibrium Outcomes)**.**

(i) There exists an equilibrium in which $p_I^n = p_I^* = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ *and* $p_S^n = p_S^* = 0$.

(ii) In any symmetric pure-strategy equilibrium, $p_I^n = p_I^*$*, and the consumer never switches.*

We illustrate the key implications of Proposition 1 under the assumption that v is large, which ensures that all the cases below exist. First, suppose that $\beta = 1$, i.e., the consumer is a classical time-consistent decisionmaker. Then, $v > p^{NIS} > p^{IP} = 0$, so $p_I^* = p^{NIS}$, and the consumer pays a total price of $Tp^{NIS} = s$. In other words, the consumer's switching cost determines the initial firm's market power.

Now suppose that $\beta < 1$, i.e., the consumer is time-inconsistent. If $m(N-1)$ is small, then $v > p^{NIS} > p^{IP}$, so $p_I^* = p^{NIS}$, and the consumer pays a total price of $Tp^{NIS} = s/\beta$. While β enters the formula, a logic similar to that in the time-consistent case is operational: the switching cost, now augmented by the consumer's discount factor, determines the initial firm's market power.

If $m(N-1)$ is larger, then $v > p^{IP} > p^{NIS}$, so $p_I^* = p^{IP}$, and the consumer pays a total price of $Tp^{IP} = (1-\beta)m(N-1)s/\beta$. In what we call the competition paradox, this total price is greater than the switching cost augmented by the discount factor, and increases in the number of firms and the intensity of marketing. With either increase in competition, the consumer receives offers more often, so she can get out of a high-price deal sooner in the future. As a result, she is more prone to procrastination, allowing the initial firm to charge a higher price. From a strategic perspective, (anticipated) competition destroys (actual) competition: by triggering procrastination, competition from the many other switching offers hinders each switching offer's ability to compete with the initial offer. Taking this logic to its conclusion, if $m(N-1)$ is sufficiently large — i.e., the market is sufficiently crowded or firms' efforts to reach consumers are sufficiently intense — then the consumer pays the monopoly price v.

For existing estimates of β (Augenblick et al., 2015, Laibson et al., 2018, Augenblick and Rabin, 2019, Chaloupka et al., 2019), the total price the consumer pays in the first region above with low $m(N-1)$ is between s and $2s$. In the second region with high $m(N-1)$, however, the total price can be an arbitrarily large multiple of s. Hence, our model — like other models of procrastination — is consistent with evidence discussed in Section 2.2 that some consumers fail to switch for gains that are far larger than any realistic switching cost. Furthermore, the evidence suggests that the markets in question must be in the second region, in which our novel effects obtain.

While in our model the frequency of switching opportunities is increasing in $m(N-1)$, in some markets the frequency is set by government regulation. This does not affect the main prediction that consumer procrastination can lead to high prices, but in our framework it invalidates the comparative-static prediction that prices increase in the number of firms or the intensity of marketing. In natural variants of our model, however, the same comparative static re-emerges even with a fixed frequency of switching opportunities. If products are differentiated, for instance, an increase in the number of competitors means that the consumer can find a better product for her tastes or needs next time. And if — as we discuss briefly in Section 5.3 — the consumer has imperfect memory, then an increase in the number of marketing messages may mean that she expects more reminders to switch next time. In either case, she sees less of a cost in delaying, again exacerbating her procrastination and thereby increasing prices.

Due to the competition paradox, policies aimed at increasing competition at the switching stage — typical in the regulation of switching markets[8] — have ambiguous effects on consumer welfare. As an example, consider the introduction of an online tool that helps a consumer find and switch to competing offers. This tool likely lowers the switching cost s, which as intended can lower the price the consumer pays (unless $p_I^* = v$, p_I^* is increasing in s). The tool may also allow a person to consider multiple switching options simultaneously, which we can model by assuming that at every switching opportunity, the consumer can choose between the switching offers of all competitors to the initial firm. Clearly, this leaves the equilibrium in Proposition 1 unchanged. And by giving easy

[8] For example, the Office of Gas and Electricity Markets (2019a) investigates in a number of trials how to best activate consumers to consider alternative contracts. Similarly, the Competition & Markets Authority (2018) is concerned about cases in which firms make switching unnecessarily costly by charging high exit fees, requiring customers to repeatedly contact the provider, and requiring cancelation through lengthy phonecalls (while sign-up is much easier).

access to switching offers, the tool might lower the wait time T_w between switching opportunities. This exacerbates the consumer's tendency to procrastinate, so it can increase prices.

Policy solutions, however, may also be aimed at initial competition, to which we now turn. We suppose that at time -1, the consumer chooses between the initial offers of $N_I \geq 2$ firms, and once she makes her choice, the switching offers of the other $N_S = N - N_I \geq 1$ firms are allocated to the switching opportunities as above. At the same time, the products are slightly differentiated: there is one randomly chosen initial firm for which the consumer gets a one-time sign-up benefit with present value $V \geq 0$. She may, for instance, prefer to use one credit card for a purchase because of a specific perk (e.g., airline miles) the card offers; but independently of which card she uses, she can pay off the debt with any available card. Alternatively, she may have a slight convenience benefit from signing up with the particular firm, for instance because she has its offer at hand and must look for the other offers. We define p^{IP} as in Equation (IP), but with $N - 1$ replaced by N_S. To avoid tedious case distinctions that do not affect the economic point, we assume that $v > p^{IP} > p^{NIS}$, so that in Proposition 1 we have $p_I^* = p^{IP}$.

As a benchmark, note that if $\beta = 1$, then profits approach zero as V approaches zero, and they must be zero for $V = 0$. If $\beta < 1$, then the equilibrium is often far from this:

Proposition 2 (Initial Competition with Slight Differentiation). *Suppose $v > p^{IP} > p^{NIS}$ and*

$$\frac{N_I}{N_S} < \frac{(1-\beta)m}{\beta}. \tag{2}$$

(i) For any $V \geq 0$, there exists an equilibrium in which all firms offer $(p_I^n, p_S^n) = (p^{IP}, 0)$, and the consumer never switches.

(ii) If $V > 0$, then there is no other symmetric pure-strategy equilibrium.

(iii) If $V = 0$, then there is also an equilibrium in which $(p_I^n, p_S^n) = (0, 0)$.

Parts (i) and (ii) say that if the number of firms making initial offers is sufficiently small relative to the number of firms making switching offers, then an *arbitrarily small* amount of differentiation leads to the same outcomes as in Proposition 1, and these outcomes can obtain in equilibrium even with no differentiation at all. Intuitively, the consumer reasons that she will switch away from any high price, so — defeating the standard Bertrand logic — she does not care about small differences between high prices. Perversely, therefore, in this region competition does not operate exactly because prices are too high. A price cut is attractive only if the consumer sees no reason to switch in the future (i.e., if the price satisfies $Tp_I \leq s$). Under Condition (2), such a price cut is unprofitable: a firm would rather charge p_I^* and hope that it is randomly chosen by the consumer.

Part (iii) says that if $V = 0$, then the competitive outcome is also an equilibrium. The multiplicity of equilibria highlights an interesting dichotomy in procrastination markets. If initial prices are low, then the consumer does not plan to switch away from the offer she first takes. She therefore looks for the best initial offer, creating competitive pressure that results in marginal-cost pricing. But if initial prices are high, then the consumer plans to switch away from the offer she first takes. She therefore does not care about initial prices, and competition in initial prices — which, because she does not switch away, she ultimately pays — is eliminated.

To our knowledge, no previous model generates a failure of competition in the kind of straightforward market setting as ours: where homogeneous consumers who know they will be using a service meet largely undifferentiated providers who commit to observable linear prices. In models of switching costs with classical rational consumers, outcomes are close to competitive if firms commit to a single linear price and consumers choose between multiple offers when signing up. In models with boundedly rational consumers, a failure of competition in linear prices can only occur if consumers are unable to compare prices or believe they will not use the service.[9]

Furthermore, the logic of Proposition 2 allows us to identify several natural modifications under which competition keeps failing. First, because the proposition is meant to address situations in which V is vanishingly small or zero, it requires Condition (2). But if $V > \beta s$, then for any N_I/N_S, the equilibrium in Proposition 1(i) exists. The consumer knows that by paying a switching cost s in the future, she can switch to a zero-price option. Hence, if she has an added benefit of at least βs from starting with a specific product, she takes and sticks with that product.

Second, consider our assumption that the consumer can switch away from the initial firm before ever paying its price. In some situations, there may be a lag before the first switching opportunity. This *decrease* in competition from switching has the potential to *lower* prices by inducing some competition in initial offers. Still, if the consumer can switch away relatively quickly, then initial competition is mild, so a small amount of differentiation eliminates it.

Third, one might wonder how the presence of rational time-consistent consumers (for whom $\beta = 1$) affects market prices. For simplicity, we use the variant of our model in which the consumer

[9] We discuss related research in more detail in Section 6.

receives offers from all non-initial firms at every switching opportunity. Then, an equilibrium with the same prices survives under a modified Condition (2) in which the right-hand side is multiplied by the share of time-inconsistent consumers. Hence, the equilibrium may be robust to the presence of a non-trivial share of time-consistent consumers. This prediction contrasts with a large class of models starting with Varian (1980) in which any positive share of rational consumers induces some competition. Intuitively, a small price cut at time -1 fails to attract time-consistent consumers for the same reason it fails to attract naive consumers: because they plan to switch to a still better deal in the future.

Fourth, some combinations of assumptions lead to even starker failures of competition. As an example, suppose that in Proposition 1 we have $p_I^* = v$, $V > \beta s$, and each firm can make only a single price offer that does not distinguish between initial and switching consumers. Furthermore, *all* firms make an initial offer to the consumer, and all non-chosen firms make switching offers at every switching opportunity. Then, there is an equilibrium in which all firms charge a price of v. Unlike above, where consumers fail to take up good deals, now firms do not even offer them. Intuitively, a consumer is not responsive to a price cut initially because she thinks she will take up the same deal later — which she does not. As a result, a firm only loses from competing in price.

Fifth, while we have interpreted time -1 as the time at which the consumer first buys the service, it could also be a time at which she switches providers because her switching cost happens to be zero. The consumer may, for instance, already have high-interest credit-card debt, and be in an exceptional period of her life in which she has time and motivation to put her finances in order.[10] Since her switching cost is not observable, firms cannot distinguish her from a consumer who is at a costly switching opportunity, so the variant of our model in the previous paragraph applies. We can therefore conclude that when the consumer does finally switch providers, she may switch to a deal that is just as bad as her existing one.

Our basic model fits the US credit-card industry in the 1980's, before many features familiar today were invented. At that time, there were no teaser periods — these were introduced in the early 90's (Evans and Schmalensee, 2005) — and annual fees appear to have been largely

[10] Consistent with this perspective, Shui and Ausubel (2005) document that consumers who are induced to accept a credit-card offer are not prone to switching later, despite receiving many offers. The authors estimate that consumers' switching cost must be stochastic, with it being low only in rare periods. Similarly, Brot-Goldberg et al. (2021) find that attention to switching is random and rare.

standardized at a level of \$20, creating an industry with "mass-marketed, straightforward loans."[11] In this environment, a single price that must be disclosed to the consumer — the interest rate — mostly describes an offer. Many credit-card issuers solicited business nationwide, including for balance transfers (Ausubel, 1991). Furthermore, many consumers having borrowed or about to borrow large sums must have known that they will have outstanding balances at least for a while, so they knew they will be using the service. Consistent with our predictions, interest rates were still very high (Ausubel, 1991).

3.2 Endogenous Marketing and Entry

A crucial primitive of our basic model is the wait time $T_w = T/(m(N-1))$ between switching opportunities. We now endogenize this number in two steps. We first endogenize firms' marketing intensity m, holding the number of firms N fixed. Then, we endogenize both m and N.

Endogenizing m. We modify our basic model in the following ways. Simultaneously with its pricing decision, firm n chooses its marketing intensity $m^n > 0$ at constant marginal cost $c > 0$. These marketing decisions determine the consumer's wait time between offers and a firm's chances of reaching the consumer at each decision node. Specifically, firm n's initial offer is assigned to the consumer at time -1 with probability $m^n/(M^{-n} + m^n)$, where $M^{-n} = \sum_{n' \neq n} m^{n'}$. If firm n is selected initially, then the wait time is $T_w = T/M^{-n}$, and at each switching opportunity, firm $n' \neq n$ makes the switching offer with probability $m^{n'}/M^{-n}$. We assume that $Tv > s/\beta$; otherwise, the initial price is always equal to v. We look for equilibria in which $p_S^n = 0$ for all n.

Proposition 3 (Endogenous Marketing Intensity)**.** *Suppose that $Tv > s/\beta$, and let*

$$\bar{c} = \frac{(N-1)^2(1-\beta)s}{N^2\beta}.$$

(i) If $c > \bar{c}$, then

$$p_I^n = \frac{s}{\beta T}, \qquad p_S^n = 0, \quad \textit{and} \quad m^n = \frac{(N-1)s}{N^2\beta c}.$$

(ii) If $c < \bar{c}$, then

$$p_I^n = v, \qquad p_S^n = 0, \quad \textit{and} \quad m^n = \frac{(N-1)Tv}{N^2 c}.$$

[11] See Robin Stein (2004), "The Ascendancy of the Credit Card Industry," Secret History of the Credit Card series, PBS/Frontline. Available at `https://www.pbs.org/wgbh/pages/frontline/shows/credit/more/rise.html`, accessed September 30, 2020.

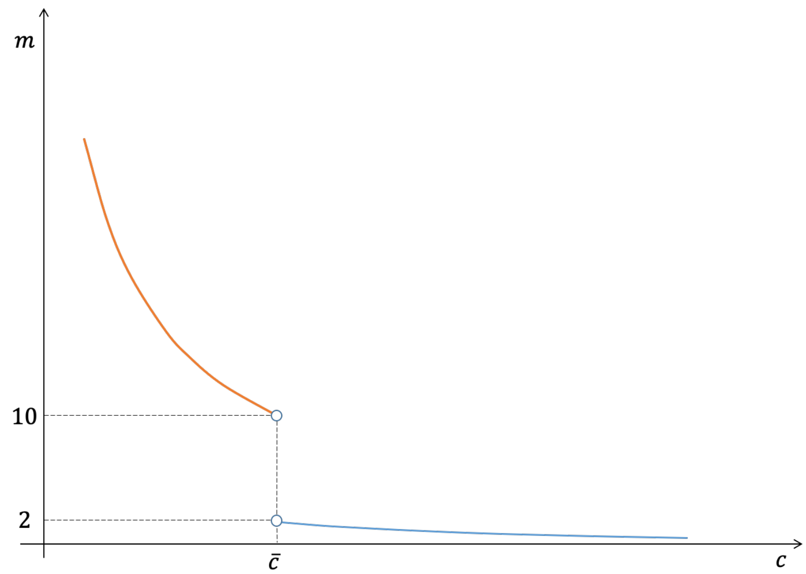

Figure 2: The Number of Offers (m) as a Function of the Cost (c)
Parameters: $\beta = 0.5$, $N = 2$, $s = 1$, $v = 0.1$, $T = 100$. A decrease in c in the high range leads to a slow, steady increase in m. At the critical $\bar{c}$, m jumps from 2 to 10. Further decreases in c lead to continuous, but steep increases in m.

To help understand Proposition 3, we describe industry dynamics as the cost c decreases. For an illustration in a particular case, see Figure 2. So long as c is high, firms do not send many offers, so the no-incentive-to-switch condition determines the initial price. Hence, initial prices remain constant at $p_I^n = s/(\beta T)$, and a decrease in c simply leads each firm to send more offers chasing the same available gross industry profits.

When the cost is lower than the threshold $\bar{c}$, however, the price and the number of offers each firm sends both jump discontinuously. At this point, procrastination enters the picture, and similarly to Proposition 1, the initial price p_I^n that firm n can charge equals $p_I(M^{-n}) = \min\{v, (1-\beta)sM^{-n}/(\beta T)\}$. Since firm n's probability of being chosen as the initial firm is $m^n/(M^{-n} + m^n)$, its expected profit is

$$\frac{m^n}{M^{-n} + m^n} \cdot Tp_I(M^{-n}) - cm^n.$$

Unless the price is constrained by v, firm n's marginal profit from increasing m^n starting from a symmetric situation is

$$\frac{M^{-n}}{(M^{-n} + m^n)^2} \cdot Tp_I(M^{-n}) - c = \frac{M^{-n}}{(M^{-n} + m^n)^2} \cdot \frac{1-\beta}{\beta} \cdot M^{-n}s - c = \frac{(N-1)^2}{N^2} \cdot \frac{1-\beta}{\beta} \cdot s - c > 0.$$

Hence, there cannot be an equilibrium in which $p_I^n < v$, so we must have $p_I^n = v$. Intuitively, when there are so many offers that consumer procrastination becomes relevant, firms can raise their initial prices. But this leads each firm to send yet more offers, allowing all of them to increase their initial prices further. The self-reinforcing process operates until initial prices reach v.

A straightforward implication of Proposition 3 is that in equilibrium, $m^n N$ and $m^n(N-1)$ are increasing in N, confirming the exogenous assumption of our basic model that the consumer's wait time is decreasing in N. With more firms, each firm has a lower market share. This implies that when a firm increases its marketing effort, it is less likely to cannibalize its own pre-existing marketing, and more likely to steal the consumer from the competition. Even holding prices constant, therefore, the total incentive for marketing is greater. In addition, an increase in N raises $\bar{c}$, which may result in a jump in prices and hence a further increase in marketing intensity.

It is worth comparing a firm's profit in this model to that in more standard models. From Proposition 3, the expected profit of a firm is

$$\frac{1}{N} \cdot Tp_I^n - cm^n = \frac{1}{N} \cdot Tp_I^n - \frac{(N-1)Tp_I^n}{N^2} = \frac{Tp_I^n}{N^2}. \tag{3}$$

This implies that even in the second region above, where the price consumers pay per period is constant at v, firms' profits are declining with the number of firms at the rate $1/N^2$. Arguably, therefore, firms do not earn larger net profits than in many other settings. For instance, in the classical model of Salop (1979), profits are also proportional to $1/N^2$. But unlike in the Salop model, where entry leads to stiffer price competition that benefits consumers, here competition works through trying to get to consumers first, which does not benefit consumers at all.

The prediction that firms burn the profits from high prices with heavy expenditure on marketing is consistent with the widely recognized fact that credit-card issuers spend a tremendous amount on marketing relative to other firms (e.g., Evans and Schmalensee, 2005).[12] While other models are consistent with high prices and heavy marketing, they typically require multiple disparate assumptions to explain why firms compete hard in the latter but not in the former. In particular, if high prices result from strong brand preferences, then unless it influences brand preferences, marketing should not be very effective in attracting consumers. If high prices result from adverse selection, then marketing is heavy only if the consumers it attracts are not similarly adversely

[12] For instance, despite a heavy shift toward online marketing, issuers sent 341 million direct mail solicitations *per month* in 2017-2018 (Consumer Financial Protection Bureau, 2019, page 73). And to take a specific issuer, American Express spent \$2.9 billion on marketing and advertising in 2019, which is 10.5% of its total interest and non-interest income (FDIC Consolidated Report, December 31, 2019).

selected. And if high prices are due to consumers' inability to compare prices, then it is not immediately clear why consumers would respond to marketing. In our model, the two parts of the story derive from a single basic assumption and are complements: high prices not only cause, but are also caused by, heavy marketing.

Endogenizing Both N and m. We now endogenize the number of firms N as well. Suppose that at the beginning of the game, a firm must pay a fixed cost $F > 0$ to enter the market. Then, the firms that entered play the game with endogenous m above. We restrict attention to the region where $F < s/\beta$, which ensures that when p_I is determined by Condition (NIS), more than one firm enters. We ignore integer constraints on the number of firms.[13] In case there are multiple equilibria in N, we assume — consistent with an industry gradually growing up from a small number of firms — that the equilibrium with the lowest N is selected; this does not affect the economic point we make.

Proposition 4 (Endogenous Entry and Marketing Intensity)**.** *Suppose that $Tv > s/\beta > F$, and let $\bar{c}(F) = (1-\beta)(\sqrt{s/\beta} - \sqrt{F})^2$.*

(i) If $c > \bar{c}(F)$, then

$$p_I^n = \frac{s}{\beta T}, \qquad p_S^n = 0, \qquad N = \sqrt{\frac{s}{\beta F}}, \quad \text{and} \quad m^n = \frac{\sqrt{\frac{sF}{\beta}} - F}{c}.$$

(ii) If $c < \bar{c}(F)$, then

$$p_I^n = v, \qquad p_S^n = 0, \qquad N = \sqrt{\frac{Tv}{F}}, \quad \text{and} \quad m^n = \frac{\sqrt{TvF} - F}{c}.$$

Analogously to Proposition 3, one way to explain Proposition 4 is to consider the effect of a continuous decrease in c and F on market outcomes. When these costs are high, a decrease in F has the unsurprising effect of continuously drawing more firms into the market, and a decrease in c has the unsurprising effect of inducing firms to increase their marketing. When *either* F or c reaches a critical level, however, both the number of firms and each firm's marketing intensity jump discontinuously. The logic is the familiar self-reinforcing nature of contacting consumers when Condition (IP) binds: an increase in either N or m increases the equilibrium price, leading yet more firms to enter and to increase overall marketing. Interpreting N as the number of firms that contact

[13] Formally, we assume that prices and profits are determined by the formulas in Proposition 3 even for non-integer N, and we look for levels of N for which a firm's expected profit equals F.

an individual consumer, the discontinuity does not necessarily involve entry into the industry — it could be that existing firms vastly expand their breadth of marketing by both reaching out to more consumers and contacting each consumer more often.

The prediction of discontinuities in market outcomes is not robust to plausible modifications of our model. For instance, if marketing has increasing marginal cost or if F and c are heterogeneous, then the discontinuities often disappear. Nevertheless, these results crisply illustrate the self-reinforcing nature of entry, which is a general feature of our framework's logic.

4 Sign-Up Deals

Received wisdom says that introductory offers — common in many switching markets — are pro-competitive practices, and one primary channel through which firms return ex-post profits to consumers.[14] We reevaluate this conclusion in the context of procrastinating consumers.

We start from our basic model with exogenous m and N, focusing for simplicity on a duopoly ($N = 2$). We assume that the offers are undifferentiated ($V = 0$), the consumer can choose either offer at time -1, she can switch to the non-chosen offer at any switching opportunity, and $K \geq 1$. In this version of our model with no additional competition from switching, the unique symmetric pure-strategy equilibrium with linear prices involves marginal-cost pricing.[15]

Crucially, however, we modify our setup by assuming that firms offer a sign-up bonus instead of a switching price: firm n's offer takes the form (B^n, p^n), where $B^n \in \mathbb{R}$ is a one-time transfer to the consumer and $p^n \in [0, v]$ is the price of the service per unit of time. Since the inducement to switch now comes from the bonus, (B^n, p^n) serves as firm n's initial as well as switching offer; below, we discuss what happens when firms can offer different bonus contracts to initial consumers and switchers. Consistent with the fact that it is very difficult to deliver immediate utility to the consumer at the time of contracting, B^n and p^n accrue in the future. The consumer can only

[14] Gehrig and Stenbacka (2002) find in their classical switching-cost model that "the strategic use of introductory offers should be promoted, not banned, in environments where firms are able to discriminate across different vintages of customers," and Farrell and Klemperer (2007) explain that "[f]irms compete ex ante for [...] ex post power, using penetration pricing, introductory offers, and price wars." Such "competition for the market [...] can even be fiercer than compatible competition by weakening differentiation."

[15] Suppose, toward a contradiction, that $p_I^* > 0$. Arguments akin to those in our previous results imply that the consumer never switches. Now consider the deviation $p_I^1 = p_S^1 = p_I^* - \epsilon$. Then, the consumer does not want to end up with price $p_I^2 = p_I^*$. Independently of whether she plans to end up with p_S^2 or $p_I^1 = p_S^1$, she strictly prefers to start with firm 1's offer, so she takes it with probability 1. Once she takes it, she never switches. For a sufficiently small $\epsilon > 0$, therefore, the deviation is profitable. Finally, $p_I^n = p_S^n = 0$ is clearly an equilibrium.

collect the sign-up bonus of a firm once. We impose two tie-breaking rules for time $t = -1$: (i) if the consumer is indifferent whether to plan on collecting a bonus, she does plan to do so; and given this, (ii) if she is indifferent as to which offer to take, she randomizes with equal probability. Finally, we impose a condition that guarantees the existence of a pure-strategy equilibrium,

$$\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v, \tag{4}$$

as well as the condition $Tv > s$, which guarantees positive profits at the identified prices.

To build intuition for the market outcome, we illustrate the consumer's behavior in an off-equilibrium situation. Suppose that $\beta = 1/2$, $s = 100$, $v = 20$, $T_w = 1$, and the two firms offer $B^1 = 150$, $p^1 = 20$ and $B^2 = 180$, $p^2 = 1$. The comparison is crystal clear: firm 2 offers the strictly better deal both in terms of the sign-up bonus and in terms of the unit price. So what does the consumer do? She signs up for *firm 1's* offer, and sticks with it at each switching opportunity. Given that the sign-up bonuses are above s, the consumer plans to collect both of them. Furthermore, she does not care about the order in which she collects the bonuses and wants to end up with the lower unit price, so she strictly prefers to take up the worse deal first. Then, when she ponders whether to switch immediately or next time, she sees no harm in collecting the other bonus later, so the bonus plays absolutely no role in her decision. Because a price difference of $p^1 - p^2 = 19$ is also not sufficient to prevent her from delaying, she procrastinates until she reaches the last switching opportunity. And parameters are such that once there, she no longer thinks switching is worth it.[16]

The consumer's preference for the worse deal is harmful to competition:

Proposition 5 (Sign-Up Deals)**.** *Suppose $Tv > s$ and Condition* (4) *holds. There is a unique pure-strategy equilibrium, and in this equilibrium both firms offer $(B^*, p^*) = (s, v)$. The consumer takes one of the offers at $t = -1$ and never switches.*

Despite being engaged in Bertrand competition with identical costs and products, firms charge the monopoly unit price. The consumer does (once) collect a sign-up bonus s, but this could be negligible relative to the total price she pays, so the bonus does extremely little in the way of returning profits to her. Worse, since without sign-up deals marginal-cost pricing obtains, it is the deals that give rise to the uncompetitive outcome in the first place.

[16] For a formal argument, see Appendix A.

The logic follows the insights from the example above. The consumer plans to take advantage of both sign-up bonuses, but then never switches away from the firm she first signs up with. We argue that in this situation, no firm has a profitable deviation. First, firm n might try to attract the consumer at time -1 by lowering p^n — but this has the perverse consequence that the consumer now strictly prefers the other offer. Second, firm n might try to induce the consumer to switch to it by lowering p^n — but by Condition (4), preventing consumer procrastination is not possible even with $p^n = 0$.[17] Third, firm n might try to attract the consumer at time -1 by increasing B^n — but this leaves the consumer indifferent, so it is merely a waste of money. Fourth, firm n might try to induce the consumer to switch to it by raising B^n — but this has no effect at switching opportunities before the last one, and is too expensive if the consumer takes it at the last opportunity. Finally, if firm n lowers B^n to below s, then the consumer strictly prefers to take advantage of only the other firm's deal, so she chooses the other firm.

It is worth considering again how time-consistent consumers (for whom $\beta = 1$) affects the equilibrium we have identified. These consumers would follow through on an optimal plan to end up with a cheaper unit price, so they create an incentive to compete. Yet if firm n lowers p_n, it is not only guaranteed to attract time-consistent consumers, but it is also guaranteed to lose time-inconsistent consumers. Hence, the uncompetitive outcome is robust to the presence of a non-trivial share of time-consistent consumers.

Similarly to the case of our basic model, we are unaware of previous work that predicts such a stark failure of competition in the simplest of circumstances: when consumers are homogeneous and therefore no adverse selection operates, firms commit to prices that consumers observe, and there is no constraint on competition in sign-up deals. In models with rational consumers, competition ensues if firms can commit to future prices. In models with rational consumers when firms are unable to commit to future prices, and in models with naive consumers, limits to competition arise only if firms face adverse selection or explicit constraints in initial competition.

In practice, service providers often hand out the sign-up bonuses in the form of temporary price

[17] For a complete picture, we explain what happens if a firm can choose $p^n < 0$. In our model in which the consumer is assumed to exit the market at time T, a profitable deviation arises: firm n can choose a p^n that is sufficiently negative to prevent procrastination, and make profits through a negative B^n. To avoid arbitrage, the firm can even impose a limit on the total consumption to which the negative price applies. Plausibly, however, a consumer faced with a negative price would not cancel her service at time T, and not knowing the consumer's precise circumstances, the firm cannot require her to cancel it at that time either. In this case, the consumer can collect the same amount of payments from the negative price if she switches now and if she switches next time. Hence, her incentive to switch now is no greater than with a price of zero, so she procrastinates.

reductions, such as a low-interest (or interest-free) period in the credit-card market or a temporary low price in the energy market. In fact, the loyalty penalties we have mentioned in Section 2.2 often result from such teaser deals. We can formalize these deals by assuming that B^n is paid through giving the product for free for some amount of time. This leads to an equivalent logic, and in our setting the same equilibrium survives. Going slightly beyond our setting, firms will prefer to offer sign-up bonuses in the form of price reductions rather than transfers if there are consumers who would take advantage of a sign-up bonus but then not stay with the firm for long.[18]

Our predictions can contribute to explaining limited competition in some subscription-type markets with little differentiation in products and offers that feature sign-up deals facing no obvious limit to competition. To continue with our credit-card example, the interest rates consumers paid in the U.S. credit-card market remained very high despite the introduction of teaser periods in the 1990's (e.g., Ausubel, 1997, DellaVigna and Malmendier, 2004).[19] Similarly, multiple researchers have observed that predictions about how competition would lower retail electricity prices proved too optimistic.[20] Furthermore, firms in these markets have not exhausted the possibilities for ex-ante competition: for instance, there is no obvious reason why a credit-card issuer or electricity provider cannot commit to a teaser period that is longer than the usual few months.

The prediction that there is no competition in bonuses at all relies on our assumption that the long-term discount factor, usually denoted by δ, equals 1. With $\delta < 1$, the consumer prefers to receive future payments earlier, and to exert future switching effort later. While this induces competition in the sign-up bonuses, the competition can be extremely weak. To illustrate, suppose that the sign-up bonus is paid by temporarily reducing the price of the service to zero. Even if firm n increases the zero-price period to T — i.e., it promises the good for free for the entire time — the

[18] First, there may be a proportion of consumers — such as time-consistent consumers with switching cost less than s — who take advantage of the offers in Proposition 5 by signing up with a firm and then switching away. This imposes a loss on the initial firm in our model, but not when the teaser involves giving out the good. Second, in the presence of procrastinators with a lower switching cost, the disincentive to give money is even greater. To see this, recall that if the consumer does not choose firm n initially, then her strongest inducement to switch to firm n is at the last possible opportunity. If she does switch at this point, she imposes a loss on firm n.

[19] This has not changed: the balance-weighted credit-card interest rate for balances accruing interest, according to the Federal Reserve's latest release, is 16.98% (`https://www.federalreserve.gov/releases/g19/current/`, accessed September 14, 2020).

[20] Joskow (2003) notes that "there is a growing perception that ... [US] retail competition programs have had disappointing results," which he attributes partly to consumer behavior. In the UK's deregulated energy market, the Competition & Markets Authority (2016a) found that suppliers earn high profits, estimating an overcharge of £1.4 billion paid by UK customers. See also Agency for the Cooperation of Energy Regulators (2018). These observations lead the UK's Domestic Gas and Electricity (Tariff Cap) Act of 2018 to reimpose a price cap on standard variable tariffs (Hinson, 2018).

consumer perceives the benefit of starting with firm n to be only the discounted cost of switching between the offers (βs). Hence, for instance, a small convenience reason for picking the other firm at time -1 prevents even a huge bonus from attracting the consumer.[21]

To explore the possibility that the consumer sometimes does manage to switch providers, we consider a modification of our model. We assume that the consumer needs the service until time $2T$, and at time T' shortly after switching opportunity K and before time T, her switching cost is zero. At time T', two firms make offers to provide the service from time T. One of these is the firm that was not chosen by the consumer at time -1, and the other is a new firm. Analogously to competition between the initial firms, if the consumer takes up a firm's offer at time T', then subsequently the other firm makes $K+1$ evenly spaced switching offers to provide the service from times $T, T+T_w, \ldots, T+KT_w$. If the consumer does not take up an offer at time T', then she remains on her existing contract, and the new firm makes the switching offers. The consumer's switching cost after time T' is again s. She can collect the bonus once from each firm before time T', and once from each firm starting at time T'. We impose the same tie-breaking rules for time T' as for time -1. At the beginning of the game, firms are randomly assigned to either time -1 or time T', and then choose their offers simultaneously.

Proposition 6 (Switching Behavior)**.** *Suppose $Tv > s$ and Condition* (4) *holds. There is a unique pure-strategy equilibrium, and in this equilibrium all three firms offer $(B^*, p^*) = (s, v)$. The consumer signs up for an offer at time -1, switches at time T', and never switches again.*

In equilibrium, the initial firms not only fail to compete with each other, but they also fail to compete with the new firm appearing later. Intuitively, a firm can only compete with a later offer by lowering its unit price p^n, but this guarantees that it loses the initial competition. Furthermore, competition at time T' fails for the same reason as at time -1. Hence, while the consumer does switch, she merely exchanges one bad deal for another. Although we have not found systematic evidence, casual observation suggests that there is some truth in this prediction. Many or most

[21] The equilibrium we have found is supported by the consumer's plan to cycle through the two deals at the beginning, first accepting the more expensive unit price and immediately switching to the cheaper unit price. An alternative plan she might make is to sign up for the cheaper option first, and then take advantage of the more expensive option's sign-up bonus at the very end. In our model, this strategy is strictly inferior because if she switches at the end, then she must pay the more expensive unit price for the period of $T - KT_w$. In our deterministic model, such an asymmetry in her ability to avoid the expensive price at the beginning versus the end is arbitrary. But the asymmetry does capture a realistic aspect of a world with uncertainty: if the consumer does not know how long she needs the service, then it is impossible for her to precisely time switching to the expensive price at the end, but it is possible for her to precisely time switching away from the expensive price at the beginning.

credit cards that successfully attract consumers with balance transfers, for instance, seem to be just average cards with typical high interest rates.

We conclude the section by discussing what happens when a firm can make different bonus offers to initial and switching consumers. In our first model, this gives rise to a profitable deviation: a firm can announce that its bonus is only available to initial consumers, inducing the consumer to start — and stick with — its offer.[22] In realistic settings, however, a promise not to give bonuses to switchers is not credible. To see this, consider a modification of our second model in which the consumer does not observe a switching offer until the relevant switching opportunity. Then, the same equilibrium outcome as in Proposition 6 survives: a firm that does not win the consumer initially wants to attract her at time T', so it cannot credibly promise not to pay a bonus to switchers.[23] Indeed, while sellers often claim that their promotions are temporary, the same promotions tend to arrive again in the future.

5 Extensions and Variants

5.1 Markets with an Option to Cancel

We modify our model with sign-up bonuses at the beginning of Section 4 to consider situations in which the consumer does not have to buy the service, and the price is not capped. We assume that the consumer's value is v per unit of time, and that in addition to being able to switch, she can cancel a contract at any switching opportunity $\kappa = 0, \ldots, K$. Switching to, taking up, or canceling a contract at a switching opportunity costs s, and the consumer can do any combination of these (including canceling a contract just acquired) at a single opportunity. She cannot hold two contracts at the same time. To focus on the case in which the incentive-to-procrastinate condition determines prices, we assume that $(1-\beta)m > 1$. And to guarantee the existence of a pure-strategy equilibrium, we impose a slightly stronger condition than Condition (4):

$$\left(\frac{1-\beta}{\beta} - \frac{1}{m}\right) \cdot \frac{m}{T} \cdot s > v. \tag{5}$$

[22] A related potential deviation is one in which the firm commits to a low marketing intensity m, in effect telling the consumer that it will not send switching offers. The logic below applies to this deviation as well.

[23] The equilibrium can be supported by off-equilibrium beliefs in which the consumer assumes that a firm's switching offer is the same as its initial offer. Such a belief appears plausible: the consumer reasons that if the firm thinks it can attract her with the off-equilibrium initial contract at time -1, then the firm must think it can attract her with the same contract at time T'.

Proposition 7. *Suppose that Condition* (5) *holds, and* $(1-\beta)m > 1$. *There is an equilibrium in which both firms offer*

$$B^* = s, \quad p^* = v + \frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s.$$

The consumer signs up with one of the firms, and never switches or cancels.

In the equilibrium of our Bertrand pricing game identified in Proposition 7, the consumer pays a price *above* her value. To appreciate the result, consider first what a monopolist would do. The monopolist could offer the straightforward contract $p = v$ and $B = 0$, which the consumer would accept. Alternatively, the monopolist could design a tricky contract by (i) choosing $p > v$ such that the consumer expects to but does not cancel the contract, and (ii) setting $B = s$ to compensate the consumer for her cancellation cost and thereby inducing her to accept. Since canceling next time lowers the discounted cost of canceling from s to βs but also imposes a loss of $p - v$ for a time interval of length $T_w = T/m$, the monopolist can charge exactly the p^* identified in the proposition. It is easy to check that for this p^*, the monopolist prefers the tricky contract.

In light of the above, Proposition 7 says that despite being engaged in Bertrand competition, the firms behave as if they were monopolists. In a sense, this failure of competition is even more extreme than in the setting of Section 4, where a monopolist would not offer a sign-up bonus, and hence competition increases consumer welfare at least by a bit. Note also that the consumer would never accept a unit price $p^n > v$ without a sign-up bonus — in fact, without the bonus the unique equilibrium has $p^n = 0$ — so the bonuses are again essential for the failure of competition.

The intuition is in several parts. First, the smallest bonus that induces the consumer to participate still equals the cost of cancelling an overly expensive contract. Second, once the consumer has accepted an offer, she is indifferent to taking up the other offer, so her incentive to procrastinate is driven by when to cancel her existing contract. Third, a firm cannot profitably attract the consumer by decreasing p^n. A small price cut leaves the consumer indifferent, as she thinks she would cancel even the lower-price contract immediately. Similarly to Proposition 2, therefore, in this region competition does not operate exactly because prices are too high. In addition, Condition (5) ensures that a deep price cut is not profitable for a firm. Fourth, increasing B^n is not profitable for the same reason as in Proposition 5: it can only be used to attract the consumer at the last switching opportunity, which is too late to be profitable.

Despite implying monopolist behavior, however, Proposition 7 does not fully convey the potential failure of competition when there is an option to cancel. To illustrate, suppose for simplicity that bonuses are exogenously fixed at a level $\bar{B}$ satisfying $s < \bar{B} < \min\{2s - Tv/m, s/\beta\}$, and m is an integer.[24] Under a monopoly, the logic behind the choice of p^n remains unchanged, so the same p^n remains optimal. But under duopoly, there is now a symmetric pure-strategy equilibrium with a *higher* p^n, so that Bertrand competition can strictly *increase* prices.[25] Intuitively, competition introduces more opportunities, so the consumer plans to do more: she plans not only to cancel her current contract, but also to take up and cancel the competitor's contract. Since doing more entails a higher immediate cost, she is more prone to procrastination, allowing firms to raise prices.

As a potential example, the consumer might sign up for a renewal service such as Netflix or Audible that she values below the price, and then keep her membership forever. Based on previous work on procrastination, such as DellaVigna and Malmendier (2004), it is unsurprising that a procrastinating consumer might not cancel a bad deal. But the possibility that procrastination induces undifferentiated firms engaged in price competition to charge monopoly or even higher prices has not been pointed out in previous research.

5.2 Partial Naivete

We discuss what happens in our basic model when the consumer underestimates, but is not fully naive about, her present bias. To do so, we modify the model using O'Donoghue and Rabin's (2001) specification of partial naivete: we assume that the consumer has a point belief $\hat{\beta}$ about her future short-term discount factor, and $\beta \leq \hat{\beta} \leq 1$. In this formulation, $\hat{\beta} = \beta$ corresponds to sophistication, $\hat{\beta} = 1$ corresponds to the full naivete we have assumed so far, and $\beta < \hat{\beta} < 1$ corresponds to partial naivete.

We use the notation from Section 3.1, and start by supposing that $p_I^* < v$, so $p_I^* = \max\{p^{NIS}, p^{IP}\}$. We argue that for an arbitrarily small amount of naivete — i.e., any $\hat{\beta} > \beta$ — Conditions (NIS) and (IP) are unaffected, so that the equilibria in Propositions 1 and 2 remain in place. The first claim is immediate since the consumer's beliefs about her future behavior play no role in Condition (NIS). Turning to the second claim, notice that for any initial price $p_I \geq p^{IP}$, a

[24] A situation where the bonus is higher than most consumers' switching costs can also arise endogenously when consumers are heterogeneous in s. We assume that m is an integer to ensure that the last contract period, $T - KT_w$, is of length T_w. Allowing this period to be much shorter would introduce the artificial possibility that if the consumer signs up with a firm at the last opportunity, then her contract is automatically and quickly cancelled.

[25] For a formal argument, see the discussion following the proof of Proposition 7 in the appendix.

person with a short-term discount factor $\hat{\beta} > \beta$ strictly prefers switching immediately to switching later. Hence, a consumer with belief $\hat{\beta}$ must think that if she does not switch now, then she will switch next time. She therefore makes the same comparison as in our basic model, and the same incentive-to-procrastinate condition results. Intuitively, since p^{IP} makes the consumer indifferent whether to switch now or next time, an arbitrarily small amount of naivete is sufficient for her to mispredict her behavior.[26]

We now suppose that $p_I^* = v$, and derive conditions under which the consumer procrastinates on switching away from this price. Given the above considerations, the equilibria in Propositions 1-4 survive under the same conditions. We define the consumer's "tolerance for delay" d as the number of opportunities she is willing to delay switching. If the consumer delays for d opportunities, then she loses $\beta d T_w v$ in payments and saves $(1-\beta)s$ by pushing the switching cost to the future, so her tolerance for delay is

$$d = \frac{1-\beta}{\beta} \cdot \frac{1}{T_w} \cdot \frac{s}{v}.$$

Analogously, we define $\hat{d}$ as the consumer's perceived future tolerance for delay, which is given by the same formula with β replaced by $\hat{\beta}$. The consumer believes that if she delays now and would delay more than $\hat{d}$ additional opportunities starting next time, then — this being outside her perceived tolerance — she would rather switch next time. Hence, she must believe that if she delays now, she will switch within $\lfloor \hat{d} \rfloor + 1$ opportunities.[27] If $\hat{d} < \lfloor d \rfloor$, then this is within her tolerance for delay d, so she procrastinates. The condition $\hat{d} < \lfloor d \rfloor$ can be satisfied for $\hat{\beta}$ close to β, especially in a crowded market: for any $\hat{\beta} > \beta$, the consumer procrastinates if T_w is sufficiently small.

5.3 Other Models of Procrastination

To model consumer behavior, we have assumed naive present bias, the most widely used microfoundation for procrastination. We argue in this section that other plausible microfouncations lead to similar insights, and might help account for some subtler patterns in firms' pricing behavior.

Underestimation of Switching Costs. Tasoff and Letzler (2014) document that subjects overestimate their probability of redeeming a rebate-like form by 49 percentage points, with additional evidence — e.g., that lowering transaction costs affects redemption but not beliefs — suggesting

[26] The conclusion that in a market setting an arbitrarily small amount of O'Donoghue-Rabin-type naivete can have large effects is reminiscent of previous work by DellaVigna and Malmendier (2004) and Heidhues and Kőszegi (2010).

[27] For $x \in \mathbb{R}$, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

that the overoptimism is due to an incomplete appreciation of future costs. Similarly, Rodemeier (2020) documents that online shoppers underestimate the hassle cost of claiming a rebate.

To model this, suppose that $\beta = 1$, but the consumer has incorrect views about her future switching cost: while the true switching cost at any time is $s' > s$, and she understands that her current switching cost is s', she believes with certainty that her switching cost at any point in the future will be s. This leads to the same consumer behavior, and hence the same firm behavior, as our model with $\beta = s/s'$. Intuitively, discounting the future benefits relative to the current switching cost s by a factor of β while believing that future selves will not do so, is equivalent to having a switching cost s/β while believing that future selves will have switching cost s.

Overconfidence about Memory or Attention. Consistent with the idea that a consumer is too optimistic about her memory or attention, Ericson (2011) documents that subjects overvalue a payment in six months that they have to remember to claim, and Rogers and Milkman (2016) and Bronchetti et al. (2020) find that people undervalue reminders.

To model this, suppose that the consumer's switching cost at a moment in time is either s_L or $s_H > s_L$. During any switching opportunity, she faces times with both switching costs, but in order to act, she must also recall the task. Crucially, the consumer never remembers when her switching cost is s_L; it might be the case, for instance, that at such times she is engaging in leisure activities and forgets chores. But when the consumer thinks about the future, she naively believes that she will remember the task for both switching-cost realizations. This is equivalent to the model above with $s' = s_H$ and $s = s_L$.[28]

Subtle Pricing Patterns. Alternative sources of procrastination not only have similar basic implications in our settings, but they might also help account for additional observations regarding the details of firms' pricing strategies. Consider, in particular, some patterns in how firms implement loyalty penalties. In the UK electricity market, customers are subject to price jumps after an initial contract period expires.[29] In the UK home-insurance and auto-insurance markets, companies

[28] The conditions under which the above models lead to the *exact same* consumer behavior as our present-bias-based model are arguably special. While less extreme versions of our assumptions do not lead to the exact same behavior, they are likely to satisfy the logic of consumer behavior underlying our results: that (i) a consumer overestimates her inclination to switch in the future, and, as a result, (ii) she is prone to not switching now, where (iii) these tendencies become more severe with more options.

[29] At the end of the initial "acquisition" contract, customers who do not actively look for a new contract are often moved to an expensive "default" or standard variable tariff, which is subject to a regulatory price cap (`https://www.ofgem.gov.uk/energy-price-caps/about-energy-price-caps/price-my-energy-bill-capped/default-tariff-price-cap` assessed on November 23, 2020). About 70% of UK energy customers are on such a default contract (Competition & Markets Authority, 2018, page 26).

engage in “price walking:” they increase margins renewal after renewal until a target margin is reached (Financial Conduct Authority, 2019, page 44). In the UK broadband market, in which there is a general downward pricing trend, companies engage in “legacy pricing:” they keep contract details unaltered for out-of-contract customers, while offering much better deals to new customers (Office of Communications, 2019). And German electricity and broadband providers often offer two-year contracts with monthly payments that start out low but revert to the “regular” level after six to twelve months, and then automatically renew at the latter price for passive consumers. We suspect that these specifics are at least in part motivated by firms’ attempts to avoid drawing consumer attention to price hikes. Firms that must (due to consumer-protection regulation) inform consumers of price changes and contract renewal, for example, may want to switch from the introductory price to the regular price well before renewal, and then simply say that terms are unchanged. By the same token, legacy pricing or price walking may be better at flying under a consumer’s radar than large price hikes. At the same time, price jumps may be easier to hide in the electricity market, where bills depend on variable usage and a price jump is not large in absolute terms.

The attempt to take advantage of consumers’ inattention also helps explain a pattern that our present-bias-based framework does not predict: that contracts sometimes lock consumers in for an extended period, such as two years. In our model, this is not optimal for a firm because the inability to get out of the contract lowers the consumer’s tendency to procrastinate, lowering the price the firm can charge. But a long-term contract may be optimal for a number of reasons. First, if prices are regulated, then even in our model a firm does not benefit from shortening the contract beyond the length that induces procrastination at the regulated price. Second, a longer-term contract may be better at avoiding consumer attention at the point of renewal (when she has not thought about it for a while), especially since, as discussed above, it involves a longer period in which the regular price is charged. To go further, if the consumer is prone to forgetting about switching, then it may be optimal to lock her in again for another long-term contract. Third, long-term contracts may protect firms from non-procrastinating consumers who would, with short-term contracts, take advantage of sign-up bonuses.

6 Related Literature

In this section, we discuss related research not covered elsewhere. While we point out other differences below, our paper is the first to study the market effects of procrastination in switching, the first to predict the procrastination-induced competition paradox, and the first to generate a failure of competition in transparent linear prices or sign-up deals for homogeneous consumers. These insights help account for some high observed prices for which previous work does not appear to provide a complete explanation.

Because procrastination can be seen in reduced form as making it more difficult for a consumer to switch, our theory is related to models of consumer inertia due to switching costs (e.g., Farrell and Klemperer, 2007) or default effects (Ericson, 2020); and because procrastination leads the consumer to underestimate the price she will pay, our theory is related to models of hidden prices (e.g., Gabaix and Laibson, 2006, Armstrong and Vickers, 2012). These models predict limited competition in price components a consumer cannot avoid or does not appreciate, but they also predict increased price competition when the consumer signs up initially or finally does switch. In our model, in contrast, even initial competition is compromised.[30]

Of course, although in the existing literature it is equally possible that ex-ante competition more than offsets the lack of ex-post competition (Farrell and Klemperer, 2007, Rhodes, 2014, Cabral, 2016), the literature identifies some possible reasons for the offset to be incomplete. A simple insight is that if a sign-up bonus takes the form of a reduction in the initial price or the price of a base good, then a price floor may limit how much ex-post profit can be handed out ex ante. Our model generates high profits without a price floor. Indeed, while some dimensions of prices — e.g., the annual fee for credit cards — are arguably at a floor, there are other dimensions — e.g., the length of a teaser period — on which there is no binding constraint.

More subtlely, adverse selection, whereby a firm offering a better deal disproportionately attracts less profitable consumers, may limit competition in some markets. But it is not clear that these accounts apply to all relevant settings.[31] Our theory generates high prices with homogeneous naive

[30] In addition, for competitive models with switching costs to generate high prices, firms must be unable to commit to future prices when consumers sign up initially. In our model, in contrast, high prices obtain even though they are announced and known to consumers.

[31] Ellison (2005) develops a model in which consumers are heterogeneous in their elasticities with respect to both base-good and add-on prices, with a positive correlation between the two elasticities. Then, cutting the base-good price attracts mostly consumers who will not buy the profitable add-on, discouraging competition. But paralleling the logic for switching costs, he shows that if consumers are rational and firms can commit to and advertise future prices,

consumers, and thereby shows that adverse selection is not necessary for high prices to obtain. What is more, due to the completely different logic the lack of competition is robust to the presence of a non-trivial share of rational consumers who can be profitably attracted. These features help explain high prices even where adverse-selection stories cannot provide a full account.[32]

More distantly related, a number of papers have considered the implications of partially or fully naive present bias on contracting in some of the same markets motivating our analysis. In all these papers, firms make high ex-post profits from naive consumers, but competition at the initial stage eliminates their net profits. A long history of work (e.g. DellaVigna and Malmendier, 2004, Murooka and Schwarz, 2018, Johnen, 2019) shows that auto-renewal contracts can be used to exploit a naive present-biased consumer's misperception about her probability of canceling. Eliaz and Spiegler (2006) and Heidhues and Kőszegi (2010) investigate how firms inefficiently distort contracts to exploit fully or partially naive time inconsistency in a model with two decision periods. In contrast, Gottlieb and Zhang (forthcoming) establish that in a long-horizon borrowing model in which firms commit to dynamic contingent contracts, the inefficiency disappears.[33]

In predicting that firms may charge high prices even in highly unconcentrated industries, our theory is reminiscent of the literature on strategic choice complexity (Carlin, 2009, Piccione and Spiegler, 2012, Chioveanu and Zhou, 2013, Spiegler, 2016). The mechanism we identify, however, bears no resemblance to that in this literature. In models of choice complexity, firms can take direct steps to make it difficult for consumers to compare prices, thereby dodging competition. In our model, firms cannot take such steps, and limits to competition arise from the logic of consumer behavior. Due to the difference in the mechanisms, there are numerous differences in implications. For instance, in models of choice complexity, fierce competition robustly results in widespread

then the adverse-selection logic does not hold. Hence, Ellison (2005) informally proposes a modification based on the presence of naive consumers who are less responsive to the sign-up deal than unprofitable sophisticated consumers. Without our model, it is not obvious why naive consumers would be relatively unresponsive; e.g., one might conjecture that naive consumers — often low-income consumers with debt — might be especially attracted by clearly visible or heavily advertised improvements in a credit card teaser deal. As a somewhat different story, Ausubel (1991) proposes that profitable credit-card consumers might not care about interest rates because they falsely expect not to borrow, whereas consumers who are sensitive to interest rates are likely to be bad credit risks. But a credit-card borrower who is about to buy something expensive or who has already accumulated substantial credit-card debt should — even if she is naive present-biased — realize that she will be carrying debt for a while, and she should value cheap options to carry debt for longer. When there are such consumers, the adverse selection with respect to credit risk must be extreme to generate high prices.

[32] For a completely different limitation on ex-ante competition based on naive consumers, see Johnen (2020).

[33] In this case, the firm sets up the consumer's sequence of choices so that the consumer repeatedly procrastinates in switching to a better consumption path. In doing so, the firm balances the consumer's concerns in each period, resulting in a nearly efficient consumption path.

obfuscation, but it does not robustly generate high prices and profits. In addition, these models are incompatible with an outcome in which firms charge a high linear price, such as a standard variable tariff, and it is easy to announce a price that consumers understand is lower. Even more simply, these models do not apply when there is a single key price, such as the APR in the credit-card market of the 80's, that is easy to see and compare across firms.

Relatedly, the literature on captive consumers starting from Varian (1980) posits that a share of consumers do not compare prices and therefore do not respond to price cuts, limiting competition in undifferentiated products. These models also predict that entry does not necessarily lower average prices (Janssen and Moraga-González, 2004, Armstrong and Vickers, 2018), although it does lower prices for some consumers. Since our consumers do not respond to price cuts, in simplistic terms they can be thought of as captive, and hence our model can be thought of as providing a microfoundation for the presence of captive consumers. This microfoundation, however, applies in circumstances when positing that consumers simply do not compare prices is implausible, such as when there are heavily marketed inducements to switch. Many predictions are also different. Unlike in our model, for instance, in models of captive consumers any share of rational shoppers induces some competition. As a result, these models typically feature mixed-strategy equilibria, so they cannot explain uniform high prices such as similar profit-making interest rates. Furthermore, search-cost-induced consumer inertia can only explain high prices if advertising prices is very costly, as otherwise a firm would like to set a low price and broadly advertise it.

7 Conclusion

Given the main message of our paper — that switching markets do not work well — it is natural to ask whether there are interventions that increase consumer welfare. In our model, any intervention that mitigates procrastination can help by lowering an initial firm's competitive advantage. But existing work suggests that overcoming procrastination through policy solutions is difficult.[34] To go further, a partially naive consumer may overestimate the effect of a consumer-based policy, which can have the perverse effect of exacerbating procrastination and hence lowering competition at the

[34] For instance, under a model of naive present bias, strict time windows for switching that are spread out over time can mitigate procrastination. But if procrastination is partly due to imperfect memory, such a policy can exacerbate procrastination. If both mechanisms are operational, then one must combine deadlines with reminders (Ericson, 2017). This raises other issues: it might be difficult to time reminders just right, and if the reminders work by drawing the consumer's attention, then she might pay less attention to other important things.

initial stage.

Although more research is clearly necessary, the above considerations suggest that relying on consumer engagement to achieve low prices in switching markets is exceedingly difficult. If so, more serious interventions might be necessary. A noteworthy example is Ohio's system described by Joskow (2003), where by default municipalities purchased power on behalf of consumers and switched them to a cheaper supplier when available. Consumers who wanted to make their own choices could opt out, but not many did. This system short-circuits procrastination because obtaining a lower price does not require consumers to make an active switching decision. Unsurprisingly, many more consumers were on low-price contracts than in other parts of the US.

Our paper focuses on the implications of procrastination on prices when the consumer buys. But the competition paradox might also give rise to important implications when the consumer does *not* buy even though she should. As has been recognized by many researchers, procrastination due to present bias or forgetting might delay a consumer's investment into retirement (e.g., Carroll et al., 2009) or preventive care (e.g., Baicker et al., 2015), and the same may be true of investment into the stock market or purchase of life insurance as well. The logic of our model says that relying on a large competitive market cannot solve such underparticipation by procrastinators: having many competitors with good options means that the consumer does not lose much by delaying entry for a little while, so that she is prone to procrastinating.

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Appendices

A Analysis of Consumer-Behavior Example in Section 4

We provide an analysis of the individual-decisionmaking example before Proposition 5: $\beta = 1/2$, $s = 100$, $v = 20$, $T_w = 1$, and two firms offer $B^1 = 150$, $p^1 = 20$ and $B^2 = 180$, $p^2 = 1$.

Because $B^1, B^2 > s$ and $p^2 < p^1$, the consumer cannot do better than collecting both bonuses and paying price p^2 for the entire time from 0 to T. Furthermore, she can achieve this by taking up firm 1's contract and then switching to firm 2 at opportunity 0, and there is no other way to achieve the same payoff. Hence, the consumer strictly prefers to take up firm 1's contract at $t = -1$.

At each switching opportunity except for the last one (i.e., at $\kappa = 0, 1, \cdots, 98$), the consumer's discounted utility if she switches to firm 2 immediately is

$$-s + \beta\left[-(T - \kappa T_w)p^2 + B^2\right] = -100 + \frac{1}{2}\left[-(100 - \kappa) + 180\right] = -60 + \frac{1}{2}\kappa,$$

whereas her discounted utility if she switches to firm 2 at the next opportunity is

$$\beta\left\{-s - T_w p^1 - [T - (\kappa+1)T_w]p^2 + B^2\right\} = \frac{1}{2}\left\{-100 - 20 - [100 - (\kappa+1)] + 180\right\} = -\frac{39}{2} + \frac{1}{2}\kappa.$$

Thus, the consumer procrastinates switching at each opportunity $\kappa = 0, 1, \cdots, K-1$.

At the last switching opportunity ($\kappa = K = 99$), the consumer's discounted utility if she switches to firm 2 is

$$-s + \beta\left[-(T - KT_w)p^2 + B^2\right] = -100 + \frac{1}{2}\left[-(100 - 99) + 180\right] = -\frac{21}{2},$$

whereas her discounted utility if she does not switch is

$$\beta\left[-(T - KT_w)p^1\right] = \frac{1}{2}(-20) = -10.$$

Thus, the consumer does not switch at the last switching opportunity, so she never switches from firm 1.

B Proofs

Proof of Proposition 1. We begin by establishing that (given the tie-breaking assumption), for any sequence of price offers $(p^{-1}, \cdots, p^K)$, there is a unique vector of switching decisions $d_0, \cdots, d_K$.

Let $p_{\geq\kappa} = (p^\kappa, \cdots, p^K)$ be the vector of switching prices the consumer faces from switching opportunity κ onwards.

We start by establishing this for a time consistent agent, i.e., an agent for whom $\beta = \hat{\beta} = 1$. We solve the game backwards. We let $e(p, \kappa; p_{\geq\kappa})$ denote the expenditure from switching opportunity $\kappa \in \{0, \dots, K\}$ onwards that a time-consistent consumer incurs for the service when the current price she pays (prior to the switching decision) is given by p. The expenditure at the last switching opportunity is $e(p, K; p_{\geq K}) = \min\{s + p^K(T - KT_w),\ p(T - KT_w)\}$. At switching opportunity $\kappa < K$, the consumer switches if and only if at the current price satisfies

$$pT_w + e(p, \kappa + 1; p_{\geq\kappa+1}) > s + p^\kappa T_w + e(p^\kappa, \kappa + 1; p_{\geq\kappa+1}). \tag{6}$$

Starting from the penultimate switching opportunity $K - 1$, we can thus recursively define the optimal expenditure from κ onwards as

$$e(p, \kappa; p_{\geq\kappa}) = \min\{pT_w + e(p, \kappa + 1; p_{\geq\kappa+1}), s + p^\kappa T_w + e(p^\kappa, \kappa + 1; p_{\geq\kappa+1})\}.$$

This completes the characterisation of a time-consistent consumer's switching behavior.

Since a naive consumer believes that her future selves behave as a time-consistent consumer would, self t who pays a current price p switches at opportunity $\kappa < K$ if and only if

$$\beta pT_w + \beta e(p, \kappa + 1; p_{\geq\kappa+1}) > s + \beta p^\kappa T_w + \beta e(p^\kappa, \kappa + 1; p_{\geq\kappa+1}), \tag{7}$$

and she switches at $\kappa = K$ if and only if $\beta p(T - KT_w) > s + \beta p^K(T - KT_w)$. This completes the characterization of a naive time-inconsistent agent's switching behavior.

(i). We establish that there exists a pure-strategy equilibrium with the properties stated in the proposition. Let every firm offer $p_I = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ and $p_S = 0$, and let the naive consumer behave as specified above. In this equilibrium, each firm makes positive profits if it is assigned to be the initial firm, and zero profits if it is not assigned to be the initial firm. We have established in the text that on the equilibrium path, for any $p_I^n \leq \min\{v, \max\{p^{NIS}, p^{IP}\}\}$, the consumer never switches. We next show that if a firm deviates and sets a price $p_S > 0$, the consumer will also not switch. Because a consumer cannot benefit from switching to a higher price if she does not benefit from switching to a price of zero, whenever $p_I \leq p^{NIS}$ a price $p_S > 0$ cannot attract the consumer. Hence, suppose $p^{NIS} < p_S \leq p^{IP}$, so the naive consumer procrastinates switching in the candidate equilibrium. Consider a realization of the switching order in which the

consumer is offered the deviant firm's switching price at time $\kappa < K$. Since the switching price of the deviant firm is $p_S > 0$ and all other firms offer a switching price of zero, we note that $p^\kappa \geq p^{\kappa+1}$. Hence, because a time-consistent consumer could always switch in the next period to a price weakly below p^κ at a cost of s, we have

$$e(p, \kappa + 1; p_{\geq \kappa+1}) \leq e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}) + s.$$

The naive consumer refrains from switching at any $\kappa < K$ if (7) holds for $p = p^{IP}$ and $p^\kappa = p_S > 0$, which is equivalent to

$$\beta p^{IP} T_w + \beta e(p^{IP}, \kappa + 1; p_{\geq \kappa+1}) \leq s + \beta p^\kappa T_w + \beta e(p^\kappa, \kappa + 1; p_{\geq \kappa+1}). \tag{8}$$

Using the above bound for $e(p, \kappa + 1; p_{\geq \kappa+1})$ with $p = p^{IP}$ and that $T_w = T/M$, our last equation holds because

$$p^{IP} \leq p^\kappa + \frac{1-\beta}{\beta}\frac{M}{T}s = p^\kappa + p^{IP}.$$

Hence, at $\kappa < K$, a firm cannot attract the naive consumer currently paying p^{IP} by deviating and setting a price $p_S > 0$. Next, consider a realization of the switching order in which the consumer is offered the deviant firm's switching price at time $\kappa = K$. At the last switching opportunity, the consumer refrains from switching if and only if $\beta p^{IP}(T - KT_w) \leq s + \beta p^K (T - KT_w)$ or

$$p^{IP} \leq p^K + \frac{s}{\beta}\frac{1}{T - KT_w},$$

which, using that $(T - KT_w) < T/M$, follows from the definition of p^{IP}. Obviously, the deviant firm setting $p_S > 0$ also cannot profitably attract a naive consumer currently paying $p = 0$ (in case her deviation would induce the consumer to switch prior to κ), and hence any deviation to $p_S > 0$ is unprofitable.

Given that rivals set a switching price of zero, we established in the text that the naive consumer does not switch away from the initial offer if and only if $p_I^n \leq \min\{v, \max\{p^{NIS}, p^{IP}\}\}$. Hence, setting $p_I^n = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$ is also optimal, and we established that a symmetric pure-strategy equilibrium with the properties stated in the proposition exists.

(ii). We now show that in any symmetric pure-strategy equilibrium, the consumer never switches and all firms set $p_I^n = \min\{v, \max\{p^{NIS}, p^{IP}\}\}$.

Let (p_I, p_S) be the common initial and switching prices in a candidate symmetric pure strategy equilibrium. In this equilibrium, the consumer refrains from switching at switching opportunity

$\kappa = 0$ in case either she has no incentive to switch or she has an incentive to procrastinate. Switching at opportunity 0 rather than never saves the consumer $\beta T(p_I - p_S)$ in discounted future payments, and has immediate cost s. Hence, Self 0 prefers never switching if

$$p_I \leq \frac{1}{\beta} \cdot \frac{s}{T} + p_S = p^{NIS} + p_S. \tag{9}$$

Now suppose the condition (9) is violated and consider the consumer's incentive to procrastinate. If it is suboptimal from self $\kappa = 0$'s perspective to switch at time $\kappa = 1$ to the price p_S, then it is also suboptimal to switch at a later point in time, so the naive consumer plans never to switch; but then (9) would have to be satisfied, contradicting the case we are considering. Hence, the consumer must plan to switch at $\kappa = 1$ in case she does not switch at $\kappa = 0$. If the consumer switches at $\kappa = 0$, she must pay an immediate effort cost of s. If she switches at $\kappa = 1$, then she must pay p_I until the next switching opportunity $\kappa = 1$, lowering her discounted utility by $\beta(p_I - p_S)T/M$; and in this case she must pay the switching cost next time, lowering her discounted utility by βs. Hence, she prefers to stay inactive in case

$$p_I \leq \frac{1-\beta}{\beta} \cdot M \cdot \frac{s}{T} + p_S = p^{IP} + p_S. \tag{10}$$

In case either (9) or (10) holds with a strict inequality, the consumer at time zero strictly prefers not to switch at opportunity $\kappa = 0$. Furthermore, we now show that in this case she also strictly prefers not to switch at any opportunity $\kappa > 0$. Switching at an opportunity $\kappa > 0$ saves the consumer strictly less than $\beta T(p_I - p_S)$, hence in such a subgame the non-switching condition (9) holds for a larger set of initial prices. The procrastination condition (10) remains the same for all $\kappa < K$; and a consumer who does not switch at $\kappa = K$ pays the higher price for a shorter time interval and does not have to pay the switching cost at time T making not switching even more attractive. It implies that if either 9 or 10 holds with a strict inequality, the consumer strictly prefers not to switch at every future switching opportunity. But then a firm can benefit from slightly raising the initial price p_I, a contradiction unless p_I is already at the price ceiling of v. We conclude that

$$p_I \geq \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\}.$$

In case the above inequality is strict, the consumer strictly prefers to switch at opportunity $\kappa = 0$ and an initial firm would earn zero profits from consumers assigned to it. The firm, however, could deviate and lower its initial price p_I^n so the above inequality holds with equality in which case its assigned consumers would not switch, and thus the deviant firm would earn positive profits from consumers assigned to it, a contradiction. We conclude that in a symmetric pure-strategy equilibrium,

$$p_I = \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\}, \tag{11}$$

and hence the consumer never switches in a symmetric pure-strategy equilibrium. Furthermore, in case $v \leq \max\{p^{NIS}, p^{IP}\}$, it establishes that $p_I = v$.

We finally rule out $p_S > 0$ in case $v > \max\{p^{NIS}, p^{IP}\}$. Suppose otherwise, i.e., a symmetric pure-strategy equilibrium with $p_S > 0$ exists. Then there exists an $\epsilon > 0$ so that

$$p_I = \min\{v, \max\{p^{NIS} + p_S, p^{IP} + p_S\}\} > \max\{p^{NIS} + \epsilon, p^{IP} + \epsilon\}.$$

Now consider a firm n that deviates and sets $p_S^n = \epsilon > 0$. With positive probability, the consumer is initially assigned to one of firm n's rivals and receives firm n's switching offer at $\kappa = 0$. We next argue that the consumer prefers switching in this case, and since this implies that the deviant firm n earns positive expected profits from a consumer assigned to its rivals rather than zero as in the candidate equilibrium, this establishes the desired contradiction. Because for $p_S^n = \epsilon$, $p_I > p^{NIS} + p_S^n$ at $\kappa = 0$, a consumer with an initial contract p_I strictly prefers switching to never switching. Now if all firms would have deviated to the lower switching price $p_S^n = \epsilon$, the consumer would also strictly prefer switching at $\kappa = 0$ to procrastinating and switching at the next opportunity $\kappa = 1$ because $p_I > p^{IP} + \epsilon$; starting from this point and raising firm n's rival switching prices makes procrastination (weakly) less desirable since the consumer now faces the same or higher prices at any future period. Hence, when seeing the deviant firm's switching price at $\kappa = 0$, the consumer will switch immediately and pay a strictly positive price, a contradiction. We conclude that $p_S = 0$. Thus, (11) implies that $p_I = \max\{p^{NIS}, p^{IP}\}$ when $v > \max\{p^{NIS}, p^{IP}\}$. □

Proof of Proposition 2. The characterization of the consumer behavior from time 0 onwards remains unchanged from that in the proof of Proposition 1 but for the minor adjustments given that now the rivals' average marketing intensity is $M = N_S m$ (rather than $M = (N-1)m$). This also implies that if rivals set $p_I = p^{IP}$, setting $p_S = 0$ is a best response.

(i). We first prove that for all $V \geq 0$ a symmetric pure-strategy equilibrium exists in which all firms charge $(p_I^n, p_S^n) = (p^{IP}, 0)$. We already established that setting $p_S = 0$ is part of a best response. We will now verify that setting $p_I = p^{IP}$ is also optimal. To do so, we first characterize the consumer's choice at time -1 when all firms in N_S set $p_S = 0$. For any offer with price p_I^n the consumer chooses at time -1, she anticipates switching to a price $p_S = 0$ at opportunity $\kappa = 0$ in case $Tp_I^n > s$. Therefore, in case $V > 0$ and all firms in N_I set prices above $p_I^n > \frac{s}{T}$, the naive consumer — thinking that she will end up switching and paying a price for the service of zero anyhow — selects the firm that gives it a convenience benefit of $V > 0$ in case such a firm exists. In case $V = 0$, we suppose the consumer randomizes between all firms with equal probability. For a price $p_I^n \leq \frac{s}{T}$ the consumer does not anticipate switching, and only in case $\beta T p_I^n \leq \beta s - V$ or equivalently $p_I^n \leq \frac{s}{T} - \frac{V}{\beta T}$ she prefers taking the low price offer and sticking with it to the offer from a rival firm that yields convenience benefit V today and requires switching tomorrow. She obviously also prefers the low-price offer if it yields the convenience benefit in addition.

In either case, given that rivals charge $p_I = p^{IP}$, for all $p_I^n \in \left(\frac{s}{T} - \frac{V}{\beta T},\, p^{IP}\right]$ a firm belonging to the set N_I earns from the consumer

$$\frac{p_I^n}{N_I} \leq \frac{p^{IP}}{N_I} = \frac{N_S}{N_I}\frac{1-\beta}{\beta} m \frac{s}{T},$$

hence over this range of initial prices $p_I^n = p^{IP}$ is optimal. A firm belonging to the set N_I earns zero from this consumer when $p_I^n > p^{IP}$, because then the consumer will switch at time 0, so this cannot be part of a profitable deviation. When $p_I^n \leq \frac{s}{T} - \frac{V}{\beta T}$, a firm belonging to the set N_I trivially earns at most p_I^n from attracting the consumer. Thus, there is no profitable deviation in an initial price in case

$$\underbrace{\frac{N_S}{N_I}\frac{1-\beta}{\beta} m}_{>1} \frac{s}{T} \geq \frac{s}{T} - \frac{V}{\beta T},$$

where the left hand side is greater than s/T by the assumption stated in the proposition. We conclude that for all $V \geq 0$, a symmetric pure-strategy equilibrium exists in which all firms charge $(p_I^n, p_S^n) = (p^{IP}, 0)$.

(ii). Let (p_I, p_S) be the common initial and switching prices in a candidate symmetric pure-strategy equilibrium. By the same argument as in the proof of Proposition 1, the consumer prefers never switching to switching in case $p_I \leq p^{NIS} + p_S$ and she procrastinates switching in case $p_I \leq p^{IP} + p_S$. By the assumption of this proposition, we have $v > p^{IP} > p^{NIS}$, so the consumer ends up not switching at every opportunity from the initial contract in our candidate equilibrium if and only if $p_I \leq p^{IP} + p_S$.

We now argue that in a candidate equilibrium $p_I = \min\{p^{IP} + p_S, v\}$. If firms charge an initial price $p_I > p^{IP} + p_S$, any consumer selecting it at -1 will switch at switching opportunity $\kappa = 0$, so firms earn zero profits when belong to the pool of initial firms N_I. But a firm that deviates to the lower price $p_I^n = p^{IP} + p_S$ continues to attract the consumer whenever the consumer gets the convenience benefit of $V > 0$ when buying from it. Furthermore, any consumer selecting firm n at -1 would not switch in this case, so setting such an initial price is part of a profitable deviation. Next, for the sake of contradiction, suppose $p_I < \min\{p^{IP} + p_S, v\}$. In this case, an initial firm $n \in N_I$ that deviated and increased its price by ϵ satisfying $0 < \epsilon < \min\left\{\frac{V}{\beta T}, \min\{p^{IP} + p_S, v\} - p_I\right\}$ would generate the same demand from consumers observing its offer at -1: the first condition in the latter inequality ensures that a consumer who gets an immediate convenience benefit of V when selecting firm n will continue to select firm n because the anticipated increase in future payments is bounded from above by $T\epsilon$; and the second condition ensures that a consumer who selected firm n's initial offer does not want to switch at any switching opportunity κ. Hence, there is a profitable deviation in this case. We conclude that $p_I = \min\{p^{IP} + p_S, v\}$.

The final paragraph in the proof of Proposition 1 applies unaltered, and implies that $p_S = 0$ since $v > \max\{p^{NIS}, p^{IP}\}$. Because $v > p^{IP} > p^{NIS}$ and $p_I = \min\{p^{IP} + p_S, v\}$, we conclude that in any symmetric pure-strategy equilibrium $(p_I, p_S) = (p^{IP}, 0)$.

(iii) To see that $(p_I, p_S) = (0, 0)$ is a symmetric pure-strategy equilibrium for the case in which $V = 0$, note first that it is impossible to induce a consumer whose initial contract specifies a price of $p_I = 0$ to switch, so setting $p_S = 0$ is a best response. Second, a consumer anticipates not switching and paying a price of zero when selecting an initial offer of $p_I = 0$, so a firm that deviates and charges a higher initial price $p_I^n > 0$ attracts no consumers. Thus, such a deviation is also unprofitable. □

Proof of Proposition 3. Suppose that $p_S^n = 0$ for all n. In a symmetric pure-strategy equilibrium (p_I, p_S, m), there cannot exist a profitable deviation for any firm. This implies that when fixing m at the equilibrium level, no firm can benefit from setting a different initial or switching price. Let M^{-n} denote the number of offers sent by firms other than n. Fixing M^{-n} and m^n at the equilibrium level, it thus follows from Proposition 1 that $p_I = \min\left\{v, \max\{\frac{s}{\beta T}, \frac{(1-\beta)M^{-n}s}{\beta T}\}\right\}$.

We now solve for the number of offers firm n sends (i.e., m^n) given M^{-n}. Since firm n's probability of being chosen as the initial firm is $m^n/(M^{-n}+m^n)$, its expected profit is

$$\frac{m^n}{M^{-n}+m^n}\cdot Tp_I(M^{-n})-cm^n.$$

The first-order condition with respect to m^n is

$$\frac{M^{-n}}{(M^{-n}+m^n)^2}\cdot Tp_I(M^{-n})-c=0, \tag{12}$$

which leads to $M^{-n}+m^n=\sqrt{M^{-n}Tp_I(M^{-n})/c}$. In any pure-strategy equilibrium, the total number of offers $M^{-n}+m^n$ is a constant, and since the right-hand side is strictly increasing in the number of rival offers M^{-n}, the number of offers is the same across all firms n. Thus, m^n must be the same in any pure-strategy equilibrium.

Setting $M^{-n}=m^n(N-1)$, m^n is uniquely determined by (12):

$$m^n=\frac{(N-1)T}{N^2c}p_I(m^n(N-1)).$$

Given $\beta Tv>s$, note that $p_I(M^{-n})=\frac{s}{\beta T}$ if $M^{-n}<\frac{1}{1-\beta}$, $p_I(M^{-n})=\frac{(1-\beta)M^{-n}s}{\beta T}$ if $M^{-n}\in\left[\frac{1}{1-\beta},\frac{\beta Tv}{(1-\beta)s}\right]$, and $p_I(M^{-n})=v$ if $M^{-n}>\frac{\beta Tv}{(1-\beta)s}$. Suppose first $M^{-n}<\frac{1}{1-\beta}$. Then substituting $p_I(M^{-n})$ into above yields $m^n=\frac{(N-1)s}{N^2\beta c}$. In this candidate equilibrium, the condition $M^{-n}=m^n(N-1)<\frac{1}{1-\beta}$ holds if and only if $c>\frac{(N-1)^2(1-\beta)s}{N^2\beta}=\bar{c}$. Suppose second $M^{-n}>\frac{\beta Tv}{(1-\beta)s}$. Then substituting $p_I(M^{-n})$ into above yields $m^n=\frac{(N-1)Tv}{N^2c}$. In this candidate equilibrium, the condition $M^{-n}=m^n(N-1)>\frac{\beta Tv}{(1-\beta)s}$ holds if and only if $c<\frac{(N-1)^2(1-\beta)s}{N^2\beta}=\bar{c}$. We thus established that, given $p_S=0$, for both case (i) $c>\bar{c}$ and case (ii) $c<\bar{c}$, m^n and p_I are uniquely pinned down as a candidate equilibrium.

Consider case (i) and consider potential deviations by firm n. Given $M^{-n}=\frac{(N-1)^2s}{N^2\beta c}$, because $p_I(M^{-n})$ does not depend on m^n, it follows from Proposition 1 that choosing $p_I^*=\frac{s}{\beta T}$ and $p_S^*=0$ is a best response for firm n regardless of m^n. Given $p_I^*=\frac{s}{\beta T}$ and $p_S^*=0$, (12) uniquely determines $m^n=\frac{(N-1)s}{N^2\beta c}$. Hence, for firm n, taking the strategy specified in Proposition 3 (i) is a best response.

Consider case (ii) and consider potential deviations by firm n. Given $M^{-n}=\frac{(N-1)^2Tv}{N^2c}$, because $p_I(M^{-n})$ is already at the cap v, it follows from Proposition 1 that choosing $p_I^*=v$ and $p_S^*=0$ is a best response for firm n regardless of m^n. Given $p_I^*=v$ and $p_S^*=0$, (12) uniquely determines $m^n=\frac{(N-1)Tv}{N^2c}$. Hence, for firm n, taking the strategy specified in Proposition 3 (ii) is a best response. $\square$

For the proof of Proposition 4, we first characterize the equilibria when $c=\bar{c}$.

Lemma 1. *Suppose $\beta T v > s$ and $c = \frac{(N-1)^2(1-\beta)s}{N^2\beta} = \bar{c}$. Then there exists a continuum of symmetric pure-strategy equilibria $(p_I^*, p_S^* = 0, m^*)$. The initial price p_I^* can be supported in such a symmetric pure-strategy equilibrium if and only if $p_I^* \in [\frac{s}{\beta T}, v]$.*

Proof of Lemma 1. Recall from the proof of Proposition 3 that

$$m^n = \frac{(N-1)T}{N^2 c} p_I(M^{-n}).$$

Suppose $M^{-n} \in \left[\frac{1}{1-\beta}, \frac{\beta T v}{(1-\beta)s}\right]$. Then substituting into $p_I(M^{-n})$ defined in the proof of Proposition 3 with using $M^{-n} = (N-1)m^n$ in a symmetric equilibrium implies that $c = \frac{(N-1)^2(1-\beta)s}{N^2\beta}$; hence, such equilibria exists for $c = \bar{c}$, and in this case for any $m \in \left[\frac{1}{(1-\beta)(N-1)}, \frac{\beta T v}{(1-\beta)(N-1)s}\right]$, $(p_I = \frac{(1-\beta)m(N-1)s}{\beta T}, p_S = 0, m)$ constitutes a symmetric pure-strategy equilibrium. This implies that any $p_I^* \in [\frac{s}{\beta T}, v]$ can be supported as a symmetric pure-strategy equilibrium. Because Proposition 1 implies that the equilibrium initial price is $p_I = \min\left\{v, \max\{\frac{s}{\beta T}, \frac{(1-\beta)M^{-n}s}{\beta T}\}\right\}$ and we consider the case in which $\beta T v > s$, any $p_I^* \notin [\frac{s}{\beta T}, v]$ cannot be an equilibrium initial price. $\square$

Proof of Proposition 4. Taken N as given, by (3) post-entry profits are equal to

$$\pi(N, c) = \frac{T p_I}{N^2}.$$

For any N such that $c \neq \frac{(N-1)^2(1-\beta)s}{N^2\beta}$, post-entry price and equilibrium profits are uniquely determined by Proposition 3. For the case of $c = \frac{(N-1)^2(1-\beta)s}{N^2\beta}$, monotonicity of post-entry profits in the initial price together with Lemma 1 implies that the lowest post-entry equilibrium profits are $s/(\beta N^2)$. As we look for the minimum number of firms that can be supported in equilibrium, we use that the lowest-post entry profits are

$$\pi(N, c) = \begin{cases} \frac{Tv}{N^2} & \text{if } c < \frac{(N-1)^2(1-\beta)s}{N^2\beta} \\ \frac{s}{\beta N^2} & \text{if } c \geq \frac{(N-1)^2(1-\beta)s}{N^2\beta} \end{cases}. \tag{13}$$

In equilibrium of the entry game, firms must make zero profit, i.e. $\pi(N, c) = F$. We first look for a candidate equilibrium in which $c < \frac{(N-1)^2(1-\beta)s}{N^2\beta}$. Substituting (13) into the zero-profit equation yields $N = \sqrt{\frac{Tv}{F}}$. Such a candidate equilibrium exist if and only if it satisfies the inequality $c < \underline{c}(F) \equiv (1-\beta)\frac{(\sqrt{Tv}-\sqrt{F})^2 s}{\beta T v}$. We next look for a candidate equilibrium in which $c \geq \frac{(N-1)^2(1-\beta)s}{N^2\beta}$. Substituting (13) into the zero-profit equation yields $N = \sqrt{\frac{s}{\beta F}}$. Such a candidate equilibrium exist if and only if it satisfies the inequality $c \geq \overline{c}(F) \equiv (1-\beta)\left(\sqrt{\frac{s}{\beta}} - \sqrt{F}\right)^2$.

Note that $\bar{c}(F) \leq \underline{c}(F)$ and hence an equilibrium exists for all parameters, because

$$\begin{aligned} & \bar{c}(F) = (1-\beta)\left(\sqrt{\frac{s}{\beta}} - \sqrt{F}\right)^2 \leq \frac{(1-\beta)(\sqrt{Tv}-\sqrt{F})^2 s}{\beta T v} = \underline{c}(F) \\ \iff \quad & \beta T v \left(\sqrt{\frac{s}{\beta}} - \sqrt{F}\right)^2 \leq s(\sqrt{Tv}-\sqrt{F})^2 \\ \iff \quad & \sqrt{\beta T v}\left(\sqrt{\frac{s}{\beta}} - \sqrt{F}\right) \leq \sqrt{s}(\sqrt{Tv}-\sqrt{F}) \\ \iff \quad & \sqrt{s} \leq \sqrt{\beta T v}, \end{aligned}$$

where the third line follows from the assumptions that $s/\beta \geq F$ and $Tv > s/\beta$, and the final line follows from $Tv > s/\beta$. □

Proof of Proposition 5. Consider the following candidate equilibrium in which both firms offer $(B^*, p^*) = (s, v)$. If the consumer is indifferent whether to plan on collecting a bonus at $t = -1$, she does plan to do so. If she is indifferent as to which initial offer to take at $t = -1$, she selects each firm with probability $1/2$. Between $t = 0$ and $t = T$, whenever the consumer is indifferent, she does not switch.

As a preliminary observation, note that because the consumer cannot collect the bonus twice and she always sees the same switching offer, she both plans to and actually switches at most once independently of whether firms choose equilibrium or non-equilibrium offers. We thus focus on the first time the consumer switches below. Also, since a naive consumer solves an optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we use the convention below that in such a case she believes her future self will switch immediately when indifferent.

(i). We first show that the above strategies constitute an equilibrium. Since $p^*T - B^* = Tv - s > 0$, each firm prefers the equilibrium offer to any offer which results in the firm having no sales. Note that the consumer is indifferent whether to plan to switch between firms in the candidate equilibrium, and by the above equilibrium specification, she keeps believing that her future self will switch at the next switching opportunity. Hence, the consumer thinks that she will receive B^* from both firms. Given this, a deviation to $B^i > s$ and $p^i = v$ does not induce a consumer's switching until the last opportunity. At the last opportunity, the consumer switches to firm i only if $-s + \beta B^i \geq 0$ or equivalently $B^i \geq \frac{s}{\beta}$, so firm i's profits from it are at most

$(T-K\frac{T}{m})p^i - B^i \leq \frac{T}{m}v - \frac{s}{\beta}$, which is negative by the assumption $\frac{1-\beta}{\beta}\cdot\frac{m}{T}\cdot s > v$. Hence, a deviation $B^i > s$ and $p^i = v$ merely decreases firm i's total profits. Also, each firm has no incentive to deviate to $B^i < s$ and $p^i = v$. To see this, suppose that the consumer takes up firm i's offer at $t = -1$. Because $p^i = p^*$ and $B^* = s$, the consumer is indifferent whether to plan to switch from firm i to firm j. Hence, given that the consumer takes up firm i's offer at $t = -1$, her anticipated total payoff is $-Tp^i + B^i < -Tv + s$. When the consumer takes up firm j's offer at $t = -1$ and would not switch thereafter, her anticipated total payoff is $-Tp^* + B^* = -Tv + s$. Therefore, the consumer would strictly prefer to take up firm j's offer at $t = -1$ and then not switch because switching leads to no savings per period and the switching cost is larger than i's bonus. Thus, $B^i < s$ and $p^i = v$ is not a profitable deviation.

Suppose the rival firm $j \neq i$ makes the candidate equilibrium offer $(B^*, p^*) = (s, v)$ and consider deviations by firm i in which $p^i < v$. If the consumer selects firm j's offer at $t = -1$, the consumer believes that she will switch to offer (B^i, p^i) at $t = 0$ if the following intention-to-switch condition holds:

$$-s - Tp^i + B^i \geq -Tv \quad \Longleftrightarrow \quad s \leq T(v - p^i) + B^i, \tag{14}$$

in which case, her anticipated total payoff when selecting the rival's offer first is $-s-Tp^i+B^*+B^i = -Tp^i + B^i$.

Consider first the case in which (14) does not hold. Then, the consumer will not switch from firm j to firm i at any switching opportunity since $s > T(v-p^i)+B^i$ implies that $s > \beta(T(v-p^i)+B^i)$. Because $p^i < v = p^*$, the consumer also never switches from firm i to firm j. As violating (14) is equivalent to $-Tp^i+B^i < -Tv+s$, the consumer strictly prefers to take up firm j's offer at $t = -1$ (and will never switch away from firm j). Hence, such a deviation by firm i is not profitable.

Consider second the case in which (14) holds. The consumer's anticipated total payoff when selecting firm i's offer at $t = -1$ is $-Tp^i + B^i$, because the consumer expects not to switch to firm j given $p^* = v > p^i$ and $B^* = s$. The consumer's anticipated total payoff when selecting firm j's offer at $t = -1$ and then switch to firm i at $t = 0$ is also $-s - Tp^i + B^* + B^i = -Tp^i + B^i$. As the consumer is indifferent at $t = -1$, by the tie-breaking rule she plans to collect both bonuses and randomly (with equal probability) chooses an offer at $t = -1$. Note that (14) implies that $Tv - s \geq Tp^i - B^i$, and hence conditionally on a consumer selecting firm i at $t = -1$ and not switching, firm i earns less from the consumer following the deviation. For a profitable deviation to exist, thus, the deviation must induce the consumer to switch from firm j to firm i and the firm must earn profits from the consumer's switching decision. Given that the consumer has selected firm j's offer $(B^*, p^*) = (s, v)$ at $t = -1$, the consumer does not procrastinate switching to firm i at switching opportunity $\kappa < K$ only if

$$-s + \beta\left(-\frac{T}{m}p^i + B^i\right) > \beta\left(-s - \frac{T}{m}v + B^i\right) \quad \Longleftrightarrow \quad p_i + \frac{1-\beta}{\beta}\frac{m}{T}s < v. \tag{15}$$

Condition (4), however, implies that (15) is violated for all $p^i \in [0, v]$, and thus the consumer does not switch prior to the last switching opportunity K. The consumer switches at switching opportunity K only if

$$-s + \beta \times \underbrace{\left[-\left(T - K\frac{T}{m}\right)p^i + B^i\right]}_{\equiv -\pi'} \geq -\beta\left(T - K\frac{T}{m}\right)v,$$

where π' are the profits the firm earns from the consumer switching at switching opportunity K. Rewriting shows that

$$\pi' = \left(T - K\frac{T}{m}\right)v - \frac{s}{\beta} \leq \frac{T}{m}v - \frac{s}{\beta} < 0,$$

where the strict inequality follows from Condition (4). Thus, either the consumer does not switch following the deviation or firm i makes a loss from the consumer who switches to firm i. Hence, the deviation is unprofitable.

Given Condition (4), we conclude that our candidate equilibrium is indeed an equilibrium.

(ii). We show that the above equilibrium is unique in pure strategies, given $Tv > s$, $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$, and the tie-breaking rule in which if the consumer is indifferent whether to plan on collecting a bonus at $t = -1$, she does plan to do so, and if the consumer is indifferent as to which initial offer to take at $t = -1$, she selects each firm with probability 1/2. We establish uniqueness in eight steps.

Step (I). At any switching opportunity except for the last one (i.e., at opportunities $\kappa = 0, 1, \cdots, K-1$), the consumer does not switch. Note that the consumer switches from firm j to firm i only if she does not have an incentive to procrastinate, that is,

$$-s + \beta\left(-\frac{T}{m}p^i + B^i\right) \geq \beta\left(-s - \frac{T}{m}p^j + B^i\right) \quad \Longleftrightarrow \quad p_i + \frac{1-\beta}{\beta}\frac{m}{T}s \leq p^j.$$

Because of the assumption $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$, however, this condition is not satisfied for any $p^i, p^j \in [0, v]$.

Step (II). Whenever the consumer switches from firm j to firm i at the last switching opportunity $\kappa = K$, firm i earns negative profits. The consumer switches from firm j to firm i at the last switching opportunity only if

$$
\begin{aligned}
& -s + \beta \left[-\left(T - K\frac{T}{m} \right) p^i + B^i \right] \geq -\beta \left(T - K\frac{T}{m} \right) p^j \\
\iff \quad & \left(T - K\frac{T}{m} \right) p^i - B^i \leq \left(T - K\frac{T}{m} \right) p^j - \frac{s}{\beta}. \qquad (16)
\end{aligned}
$$

Notice that firm i's profits from the consumer switching at opportunity K is $\left(T - K\frac{T}{m}\right) p^i - B^i$; and that

$$\left(T - K\frac{T}{m} \right) p^j - \frac{s}{\beta} \leq \frac{T}{m} v - \frac{s}{\beta} < 0,$$

where the strict inequality follows from the assumption that $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$. Thus, if a consumer switches at switching opportunity K, the firm attracting her makes a loss from this consumer.

We thus conclude that independently of whether firms choose equilibrium or non-equilibrium offers, the consumer either does not switch or the firm that attracts a switching consumer makes a loss from it. From now on, we specify properties of pure-strategy equilibrium offers.

Step (III). No firms sets $B^i > s$. Suppose otherwise. Let firm i sets $B^i > s$. Consider first the case in which firm $j \neq i$ sets $B^j \geq s$. In this case, the consumer at $t = -1$ plans to collect both firms' sign-up bonuses. Suppose firm i deviates to an offer $B^{i'}$ and $p^{i'} = p^i$ for which $s < B^{i'} < B^i$. This deviation does not change the consumer's decision at $t = -1$ because of the tie-breaking rule, and the deviation also makes the consumer (weakly) less likely to switch from firm j to firm i at $t \geq 0$. Hence, it is a profitable deviation — a contradiction. Consider second the case in which firm $j \neq i$ sets $B^j < s$. In this case, the consumer at $t = -1$ plans to collect firm i's offer by the tie-breaking rule. Then, using the same deviation as above, firm i can decrease its sign-up bonus and earn larger profits — a contradiction.

Step (IV). The consumer does not switch at the last switching opportunity following equilibrium offers. To see this, note that Condition (16) never holds when $B^i \leq s$ and $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$.

Step (V). Both firms earn positive profits in any equilibrium. Suppose otherwise. Let firm i earn zero profits. Consider a deviation by firm i in which the firm offers $B^{i'} = s + \epsilon$ and $p^{i'} = v$ for a sufficiently small $\epsilon > 0$. At $t = -1$, the consumer is either indifferent between which offer to choose or strictly prefers to take up firm i's offer (with potentially planning to switch to firm j). Also, if the consumer takes up firm i's offer at $t = -1$, then she never switches as shown in

Steps (I) and (IV) above. Hence, by the tie-breaking rule, the consumer takes up firm i's offer with positive probability at $t = -1$, and given that the consumer selects firm i at $t = -1$, firm i earns at least $p^{i'} - B^{i'} = Tv - s - \epsilon$. For a sufficiently small $\epsilon > 0$, firm i's profits from this deviation is positive — a contradiction.

Combined with the result that the consumer does not switch at $t \geq 0$ in any equilibrium, we conclude that the consumer must be indifferent between both offers at $t = -1$, and thus chooses each firm's offer with probability 1/2 at $t = -1$ in any pure-strategy equilibrium.

Step (VI). At least one firm i offers $B^i = s$. Suppose otherwise: both firms offer $B^i < s$. Then, the consumer does not plan to switch to another firm at $t \geq 0$ when initially selecting a firm for whom $p^i = \min\{p^1, p^2\}$. Given that we established that the consumer is indifferent between two firms at $t = -1$, by offering a slightly higher B^i, firm i can attract the consumer with probability 1 instead of 1/2 at $t = -1$, and hence can increase its profits — a contradiction.

Step (VII). Whenever firm i sets $B^i = s$, it also sets $p^i = v$. Suppose otherwise: $B^i = s$ and $p^i < v$. By deviating to $p^{i'} = v$ and $B^{i'} = B^i = s$, at $t = -1$, the consumer is either indifferent or strictly prefers to take up firm i's offer (with potentially planning to switch to firm $j \neq i$). Hence, firm i can still attract the consumer with probability at least 1/2 at $t = -1$, and given that firm i attracts her at $t = -1$, it earns larger profits — a contradiction.

Step (VIII). If firm i sets $B^i = s$ and $p^i = v$, firm $j \neq i$ also sets $B^j = s$ and $p^j = v$. First, firm j must set $p^j = v$ to make positive profits; if $p^j < v$, by the tie-breaking rule, the consumer strictly prefers to choose firm i at $t = -1$ (with planning to switch to firm j at $t = 0$), and since she actually never switches, firm j earns zero profits. Second, if firm j sets $B^j < s$ and $p^j = v$, then the consumer strictly prefers to choose firm i at $t = -1$ (with planning to never switch away from firm i), so again firm j earns zero profits — a contradiction.

We thus conclude that the equilibrium derived in (i) is the unique pure-strategy equilibrium. □

Proof of Proposition 6. Consider the following candidate equilibrium in which all three firms offer $(B^*, p^*) = (s, v)$. If the consumer is indifferent whether to plan on collecting a bonus at $t = -1$ or $t = T'$, she does plan to do so. If she is indifferent as to which of the two initial offers to take at $t = -1$ or which of the two offers she sees at $t = T'$, she selects each firm with probability 1/2. At the switching opportunity $t = 0$ and the following switching opportunities up to opportunity K, whenever the consumer is indifferent, she does not switch. Similarly, at the switching opportunity

T and any opportunity thereafter, whenever the consumer is indifferent, she does not switch.

As a preliminary observation, note that because the consumer cannot collect the bonus twice prior to T' and she always sees the same switching offer prior to T', she both plans to and actually switches at most once following any history prior to T', independently of whether firms choose equilibrium or non-equilibrium offers. By the same reasoning, she both plans to and actually switches at most once following any history where she choose to act at time $t > T'$. We thus focus on the first time the consumer switches for all histories in which she acts at time $t \in [0, T')$, and then again on the first time she switches for histories in which she acts at time $t \in (T', 2T]$. Also, since a naive consumer solves an optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we use the convention below that in such a case she believes her future self will switch immediately when indifferent.

Below, we will rename firms after they have been randomly assigned as to whether they make offers first at $t = -1$ or T'. We use the convention that the consumer sees the offers of firms 1 and 2 at $t = -1$, and that of firm 3 (as well as the initial firm the consumer did not select at $t = -1$) at T'. Note firms chooses their offers simultaneously after the assignment. We will show that given the firms' assignment, there is a unique pure-strategy equilibrium.

(i). We begin by showing show that the above strategies constitute an equilibrium before turning to uniqueness under (ii) below. Since $p^*T - B^* = Tv - s > 0$, each firm prefers the equilibrium offer to any offer which results in the firm having no sales.

We now establish that a deviation to $B^i > s$ and $p^i = v$ is unprofitable. Consider firm 3 first. Because $p^1 = p^2 = v$ and $B^1 = B^2 = s$, when setting $p^3 = v$ and $B^3 > s$, the consumer is indifferent as to whether she collects the bonus only from firm 3 or both firms whose offer she sees in period T', and hence she expects to collect both bonuses by our equilibrium-selection assumption. She is, thus, indifferent whether to select firm 3 or its rival in period T', and hence selects either offer with equal probability by the tie-breaking rule. Because $-s + \beta B^i < \beta(-s + B^i)$ for all $B^i > 0$, following time T' the consumer does not switch until the last opportunity. At the last opportunity, the consumer switches to firm i only if $-s + \beta B^i \geq 0$ or equivalently $B^i \geq \frac{s}{\beta}$, so firm i's profits from it are at most $(T - K\frac{T}{m})p^i - B^i \leq \frac{T}{m}v - \frac{s}{\beta}$, which is negative by the assumption $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$. Hence, a deviation $B^3 > s$ and $p^3 = v$ merely decreases firm 3's total profits. We next show that a deviation to $B^i > s$ and $p^i = v$ is unprofitable for firms $i = 1, 2$. Without loss of generality, consider such a deviation by firm 1. Conditionally on the consumer seeing firm 1's offer at T', this deviation is unprofitable in the subgame starting at T' by the exact same argument as the one for firm 3. No matter how the consumer plans to switch prior to time T', she (correctly) believes that she will switch in period T'. Hence, her continuation payoff derived from decisions at or after T' are independent of whether or not the consumer switches at the first K switching opportunity. Hence, if a consumer does not select the deviant firm's contract at $t = -1$, she will not switch prior to opportunity K, and if she switches at opportunity K the firm makes negative profits from the contract up to time T. Furthermore, at $t = -1$, the consumer strictly prefers to take the non-deviant firm's offer so that she is being offered the higher deviant firm's bonus $B^i > s$ at T' again. Thus, such a deviation generates non-positive profits. We hence conclude that any deviation in which $B^i > s$ and $p^i = v$ is unprofitable for all firms.

When deviating to a contract with $p^i = v$ and $B^i < s$, the consumer both at T' and at $t = -1$ strictly prefers to take firm i's rival's offer and she does not want to switch at any switching opportunity in which her switching costs are s. We conclude that a deviation in which $B^i < s$ and $p^i = v$ is unprofitable.

We next consider deviations by firm i in which $p^i < v$, supposing the rivals offer the candidate equilibrium offer $(B^j, p^j) = (s, v)$. Again, we begin by considering a deviation of firm 3 to an offer in which $p^3 < v$. If the consumer selects the offer of firm $j \neq 3$ at $t = T'$, the consumer believes that she will switch to offer (B^3, p^3) at $t = T$ only if the following intention-to-switch condition holds:

$$-s - Tp^3 + B^3 \geq -Tv \quad \Longleftrightarrow \quad s \leq T(v - p^3) + B^3, \tag{17}$$

in which case, her anticipated total payoff when selecting the rival's offer first is $-s - Tp^3 + B^j + B^3 = -Tp^3 + B^3$.

Consider first the case in which (17) does not hold. Then, the consumer will not switch from firm j to firm 3 at any switching opportunity following T'. Because $p^3 < v = p^j$, the consumer also never switches from firm 3 to firm j. As violating (17) is equivalent to $-Tp^3 + B^3 < -Tv + s$, the consumer strictly prefers to take up firm j's offer at $t = -1$ (and will never switch away from firm j). Hence, such deviation by firm 3 is unprofitable.

Consider second the case in which (17) holds. The consumer's anticipated total payoff when selecting firm 3's offer at $t = T'$ is $-Tp^3 + B^3$, because the consumer expects not to switch to firm j given $p^j = v > p^3$ and $B^j = s$. The consumer's anticipated total payoff when selecting firm j's offer at $t = T'$ and then switch to firm 3 at $t = T$ is also $-s - Tp^3 + B^j + B^3 = -Tp^3 + B^3$. As the consumer is indifferent at $t = T'$, by the tie-breaking rule she plans to collect both bonuses and randomly chooses an offer at $t = T'$. For a profitable deviation to exist, it must induce the consumer to switch from firm j to firm 3. Given that the consumer has selected $(B^j, p^j) = (s, v)$ at $t = T'$, (17) implies that if the consumer does not have the intention to switch at time T, then she does not have the intention to switch at any time thereafter, so without loss of generality we focus on whether the consumer switches from firm j to firm 3 at $t = T$.

At $t = T$, the consumer switches from firm j to firm 3 only if the consumer does not prefer procrastinate switching, i.e., only if

$$-s + \beta\left(-\frac{T}{m}p^3 + B^3\right) \geq \beta\left(-s - \frac{T}{m}v + B^3\right) \quad \Longleftrightarrow \quad p^3 + \frac{1-\beta}{\beta}\frac{m}{T}s \leq v. \tag{18}$$

However, (18) is not satisfied for any $p^i \in [0, v]$ if Condition (4) holds. Hence the consumer never switches at $t = T$ from firm j to firm 3, and a deviation to any contract in which $p^3 < v$ is unprofitable for firm 3.

We are left to consider deviations by firm $i \in \{1, 2\}$ to an offer in which $p^i < v$. We will first consider deviations for which $-Tp^i + B^i < -Tv + s$. In this case, the if firm i's contract is available at T', from period T on the consumer gets an expected continuation benefit of $-Tp^i + B^i$ when selecting firm i's offer because thereafter the consumer does not want to switch to firm 3 which offers a price of $p^3 = v$ and a bonus for switching of s. When selecting firm 3 and then not switching, on the other hand, from period T on the consumer gets expected continuation benefit of $-Tv + s$, which implies that she selects firm 3's offer at T'. Furthermore, in this case she has no intention to switch at time T because $-s + B^i - Tp^i < -Tv$, and switching at a later opportunity is even less beneficial. Hence, here expected continuation benefit from period T on is $-Tv + s$ when selecting the firm $j \neq i$ at $t = -1$. Similarly, when selecting firm i the consumers continuation value from period T on is $-Tv + s$ because she strictly prefers switching at T' to not doing so, and with both offers being equal to (s, v) at T', her continuation value from period T on is $-Tv + s$. Therefore, in this case her choice between firm 1 and firm 2 as well as her switching behavior up to opportunity K is identical to that in the game of Proposition 5, and hence it follows from the proof of Proposition 5 that a deviation to a contract for which $-Tp^i + B^i < -Tv + s$ is unprofitable.

Next consider deviations for which $-Tp^i + B^i \geq -Tv + s$ and $p^i < v$ by a firm $i \in \{1, 2\}$. Let $j \neq i$, $j \in \{1, 2\}$ denote the rival firm i faces at $t = -1$. Since $p^i < v$, if the consumer selects firm i's contract at T', she will not switch away from it thereafter. Hence, her expected continuation value from T on is $-Tp^i + B^i$ when doing so. If she selects firm 3's contract and switches at time T, on the other hand, she gets an expected payoff of $-s + B^3 - Tp^i + B^i = -Tp^i + B^i$ from T on. Hence, the consumer weakly prefers to select firm 3 in period T' and by the tie-breaking rule, she does so with probability of at least 1/2. Furthermore, the profits conditional on the consumer selecting i's offer at T' or thereafter are weakly lower, and thus such a deviation can only increase profits from period T on if it induces the consumer to switch from firm 3 to firm i at time T (or thereafter). The consumer is willing to switch only if she does not prefer procrastinating, i.e. only if

$$-s + \beta\left(-\frac{T}{m}p^i + B^i\right) \geq \beta\left(-s - \frac{T}{m}v + B^i\right) \quad \Longleftrightarrow \quad p^3 + \frac{1-\beta}{\beta}\frac{m}{T}s \leq v,$$

which, however, does not hold by Condition (4). We conclude that the deviation by firm i does not raise profits from period T onwards.

Now consider the consumer's choice at period $t = -1$. If the consumer selects the deviant firm i's offer, she will not switch prior to period T' because $p^i < v = p^j$. At T', where is is offered the opportunity to switch to firm j's or firm 3's offer for free, she switches if and only if $s - Tv \geq -Tp^i$. Hence, her expected payoff of selecting firm i's offer is $-Tp^i + B^i + \max\{s - Tv, -Tp^i\}$. If she selects firm j's offer, switches at $t = 0$, and switches to firm i's offer at $t = T'$ (or firm 3's offer and then switches at $t = T$ to firm i), the consumer gets a payoff of $B^j - s + B^i - Tp^i + B^i - Tp^i = 2(B^i - Tp^i)$, which is weakly greater. Therefore. the naive consumer's expected payoff from selecting contract j is weakly higher, and by the tie-breaking assumption, she selects firm j's offer with probability of at least 1/2. Since the deviation does not increase profits when the consumer sees firm i's offer at time T' or thereafter and the deviation cannot increase the probability that she sees firm i's offer at T', a necessary condition for the deviation to increase profits is that the consumer switches form firm j to firm i prior to opportunity K when selecting firm j's contract first. Given that the consumer expects to switch at T' in this case (to firm 3 or firm i) independently of whether she switched beforehand, her incentives to switch at opportunities prior to K are the same in the game of Proposition 5, and it follows from the proof that the consumer does not switch, and hence, the deviation is unprofitable for firm i. We conclude that there is no profitable deviation for which

$p^i < v$.

Hence, since there exists no profitable deviation in which $p^i = v$ and no profitable deviation in which $p^i < v$ for any of the three firms, we conclude that all three firms offering (s, v) indeed constitutes an equilibrium.

(ii). We now show that the above equilibrium is unique in pure strategies, given $Tv > s$, $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$, and the tie-breaking rules specified above. That is, if the consumer is indifferent whether to plan on collecting a bonus at $t = -1$ or $t = T'$, she does plan to do so; if she is indifferent as to which of the two initial offers to take at $t = -1$ or which of the two offers she sees at $t = T'$, she selects each firm with probability $1/2$; at the switching opportunity $t = 0$ and the following switching opportunities up to opportunity K, whenever the consumer is indifferent, she does not switch; similarly, at the switching opportunity T and any opportunity thereafter, whenever the consumer is indifferent, she does not switch.

Below, we refer to times $t = -1, T'$ (where the consumer does not incur s) as *contract selection opportunities* and other times where she incurs s upon switching as switching opportunities. Note that switching opportunities $\kappa = 0, 1, \cdots, K$ occur before T', switching opportunity $K + 1$ occurs at time T, and the final switching opportunity is $2K + 1$. We establish uniqueness in nine steps.

Step (I). At any switching opportunity after T' except for the last one (i.e., at opportunities $\kappa = K+1, K+2, \cdots, 2K$), the consumer does not switch in any pure-strategy equilibrium. Note that the consumer switches from firm j to firm i only if she does not have an incentive to procrastinate, that is,

$$-s + \beta\left(-\frac{T}{m}p^i + B^i\right) \geq \beta\left(-s - \frac{T}{m}p^j + B^i\right) \quad \Longleftrightarrow \quad p^i + \frac{1-\beta}{\beta}\frac{m}{T}s \leq p^j.$$

Because of the assumption $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$, however, this condition is not satisfied for any $p^i, p^j \in [0, v]$. Similarly, the consumer does not switch at any switching opportunity prior to T' except possibly the last one (i.e. at opportunities $\kappa = 0, 1, \cdots, K - 1$) because she prefers to procrastinate doing so.

Step (IIa). Whenever the consumer switches from firm j to firm i at the last switching opportunity $\kappa = 2K + 1$, firm i prefers the consumer not to switch. The consumer switches from firm j

to firm i at the last switching opportunity only if

$$\begin{aligned}
& -s+\beta\left[-\left(T-K\frac{T}{m}\right)p^i+B^i\right] \geq -\beta\left(T-K\frac{T}{m}\right)p^j \\
\iff \quad & \left(T-K\frac{T}{m}\right)p^i - B^i \leq \left(T-K\frac{T}{m}\right)p^j - \frac{s}{\beta}. \qquad (19)
\end{aligned}$$

Notice that firm i's profits from the consumer switching at opportunity K is $\left(T-K\frac{T}{m}\right)p^i - B^i$; and that

$$\left(T-K\frac{T}{m}\right)p^j - \frac{s}{\beta} \leq \frac{T}{m}v - \frac{s}{\beta} < 0,$$

where the strict inequality follows from the assumption that $\frac{1-\beta}{\beta}\cdot\frac{m}{T}\cdot s > v$. Thus, if a consumer switches at switching opportunity $2K+1$, the firm attracting her would prefer the consumer not to switch.

Step (IIb). Whenever the consumer switches from firm $j \in \{1,2\}$ to firm $i \in \{1,2\}$ at the switching opportunity $\kappa = K$, firm i would prefer that the consumer does not switch at K. It follows from the exact same calculation as in Step (IIa) that firm i makes a loss from attracting the consumer up to time T. Thus, firm i can benefit from this switching only if it affects contract selection at period T' or switching behavior thereafter. By the tie-breaking assumption, however, it does not affect the choice between firm i and firm 3 at T'. Hence, whenever the consumer switches at switching opportunity $\kappa = K$, the firm would prefer that the consumer does not switch and wait for T'.

We thus conclude that independently of whether firms choose equilibrium or non-equilibrium offers, the consumer either does not switch at a switching opportunity or the firm that attracts a switching consumer at a switching opportunity would prefer the consumer not to switch.

Step (III). No firms sets $B^i > s$ in equilibrium. Suppose otherwise: firm i sets $B^i > s$. Consider a deviation by firm i to an offer $B^{i'} = B^i - \epsilon, p^{i'} = p^i$ for sufficiently small ϵ so that $s < B^{i'} < B^i$. We will first argue that this benefits firm i in case the consumer sees its offer at contract selection period T'. Consider the subcase in which firm $j \neq i$ which offers a contract at T' sets $B^j \geq s$. In this case, the consumer at $t = T'$ plans to collect both firms' sign-up bonuses both prior and after the deviation: under the tie-breaking rule the consumer must plan to collect one bonus at t and the other at the next switching opportunity, and in this case her preferences as to which offer to choose first only depend on the prices and not on the bonuses, and hence, her contract selection is unaffected by the deviation. Additionally, the deviation makes the consumer (weakly) less likely to switch from firm j to firm i at one of the following K switching opportunities, which is beneficial to firm i by Step (IIa). We are left to consider the subcase in which firm $j \neq i$ that offers a contract at T' sets $B^j < s$. In this case, the consumer at $t = T'$ selects firm i's offer by the tie-breaking rule, and her subsequent switching behavior is independent of B^i. Thus, the deviation is also profitable in this case.

Similarly, the deviation is also beneficial in case the consumer sees firm i's offer at $t = -1$. By the same reasoning, the deviation does not affect the offer the consumer selects at the contract selection opportunity $t = -1$ both when the rival $j \in \{1, 2\}$, $j \neq i$ offers an offer with $B^j \geq s$ or one with $B^j < s$. And in case the consumer selects the offer of the rival firm $j \neq i$ at $t = -1$, it makes (weakly) less likely that the consumer switches from j to firm i prior to T', which is profit-increasing by Step (IIb), and when calculating continuation profits starting at T' increases firm i's profits. If the consumer selects firm i's offer, the deviation is profitable as the switching behavior is unaffected and firm i pays a lower bonus. We conclude that all three firms set bonuses $B^i \leq s$.

Step (IV). The consumer does not switch at the last switching opportunity or the last switching opportunity prior to T' in any equilibrium. To see this, note that Condition (19) never holds when $B^i \leq s$ and $\frac{1-\beta}{\beta} \cdot \frac{m}{T} \cdot s > v$.

Step (V). All firms earn positive profits in any equilibrium. Suppose otherwise: there exists a firm i that earns zero profits. Consider a deviation by firm i in which it offers $B^{i'} = s + \epsilon$ and $p^{i'} = v$ for a sufficiently small $\epsilon > 0$. If the consumer sees and selects this deviation offer at $t = -1$, firm i earns positive profits. So suppose otherwise that the consumer sees firm i's offer at the contract selection opportunity T'. At T', the consumer is either indifferent between choosing either offer or strictly prefers to take up firm i's offer (with planning to switch to firm j). Furthermore, if the consumer selects firm i's offer at T', then she never switches as shown above. Hence, by the tie-breaking rule, the consumer takes up firm i's offer with positive probability at the contract selection opportunity T', and given that the consumer selects firm i at T', firm i earns at least $p^{i'} - B^{i'} = Tv - s - \epsilon$. For a sufficiently small $\epsilon > 0$, firm i's profits from this deviation is positive — a contradiction.

Step (VI). With positive probability, the consumer sees a pair of offers at $t = -1$ or $t = T'$ between which the consumer is indifferent. By Steps (I) and (IV), the consumer does not switch at a switching opportunity (in contrast to a contract selection opportunity) in equilibrium. If the consumer strictly prefers firm 3 to both initial firms, then the consumer must be indifferent between the initial firms for both of these to earn positive profits. And if the consumer prefers one initial firm, say firm 1, to its rival at $t = -1$, then the consumer must be indifferent between that rival and firm 3 at T'. Hence, with positive probability, at one of the two contract selection opportunities the consumer must be indifferent between the two offers she sees, and in this case she selects either one with probability 1/2.

Step (VII). At least one firm offers $B^i = s$ in equilibrium. Let i' and j' be a pair of firms between whose offers the consumer sees at either $t = -1$ or $t = T'$ with positive probability and between which the consumer is indifferent, which exists by Step (VI). We now establish that at least one firm $i \in \{i', j'\}$ offers $B^i = s$ in equilibrium. Suppose otherwise, that is, both i' and j' offer $B^i < s$. Note that since both i' and j' offer $B^i < s$, the consumer does not plan to switch before the next contract selection opportunity at $t = -1$ or before $2T$ in case she is indifferent at T': for if she would plan to switch, she must switch to a lower price and she would plan to do so at the next switching opportunity. But then she is better off selecting the contract she plans to switch to immediately and saving the difference between switching costs and the bonus. In case the consumer is indifferent at T', by offering a slightly higher B^3, firm 3 can profitably attract the consumer with probability 1 instead of 1/2 conditional on the consumer being indifferent at T' while not affecting the consumers choice when the consumer has a strict preference; hence firm 3 can increase its profits — a contradiction. Thus, the consumer must be indifferent between firm 1 and firm 2 at $t = -1$. Furthermore, the consumer must (weakly) prefer to select firm 3's offer to the offer of firm 1 or firm 2 at T'; otherwise, firm 3 earns zero profits. Let firm $i \in \{1, 2\}$ be a firm whose offer the consumer selects with probability no greater than 1/2 conditional on seeing it at T'. This firm i get selected with probability 1/2 at $t = -1$ and then keeps the consumer until time T (earning $Tp^i - B^i > 0$ conditional on the sale at $t = -1$), and with probability 1/2 makes another offer at T'. In the latter case, with probability no greater than 1/2 firm i gets selected at T' (earning again $B^i - Tp^i > 0$ conditionally on being selected). Hence, the expected profits in this case are no greater than $\frac{3}{4}(Tp^i - B^i)$. By offering $B^i + \epsilon$ instead, firm $i \in \{1, 2\}$ would attract the consumer at $t = -1$ with probability one and would earn $Tp^i - B^i - \epsilon$, which is a profitable deviation for a sufficiently small $\epsilon > 0$.

Step (VIII). Whenever firm i sets $B^i = s$, it also sets $p^i = v$. Suppose otherwise: $B^i = s$ and $p^i < v$. Consider a contract selection opportunity $t = -1, T'$ at which the consumer sees firm i's offer as well as a competing offer by firm $j \neq i$. Note that since $B^i = s$, by the tie-breaking assumption, the consumer plans to collect firm i's bonus; and she selects firm i with probability 1 in case $p^i > p^j$ for the competing offer, with probability zero if $p^i < p^j$, and with probability 1/2 in case $p^i = p^j$. By deviating to $p^{i'} = v$, the firm thus weakly increases the probability it is selected if the contract selection opportunity is $t = T'$ as well as strictly increases the profits conditionally on being selected. And because a firm weakly prefer to be selected at $t = -1$ and earning $Tp^i - B^i$ for sure to reaching T' and earning at most $Tp^i - B^i$, the deviation also increases profits if the consumer sees firm i's offer at contract selection opportunity $t = -1$.

Step (IX). All firms offer (s, v). By Steps (VI) to (VIII), at least one firm must make an offer $(B^i, p^i) = (s, v)$. Suppose first that firm 3 makes an offer (s, v). We now argue that its rival j whose offer the consumer sees T' is chosen with probability zero at T' whenever $(B^j, p^j) \neq (s, v)$. It is because by Step (III) $B^j \leq s$ and by Step (VIII) $B^i < s$; thus, either (a) $B^j < s$ and $p^j = v$ in which case the consumer selects firm 3's offer at T' planning not to switch or (b) $B^j < s$ and $p^j < v$ in which case the consumer selects firm 3 at T' planning to switch to firm j in case $-s + B^j - Tp^j \geq -Tv$ and not switch otherwise. The fact that firm $i \in \{1, 2\}$ who makes an offer $(B^i, p^i) \neq (s, v)$ is chosen at T' with probability zero implies that not both firm 1 and firm 2 can do so; if both firms did so, they would earn zero profits from period T on. To earn positive profits, which firms do by Step (V), hence, the consumer must be indifferent between choosing the offer of firm 1 and that of firm 2 at $t = -1$. By Step (VII), at least one of these two firms must set $B^i = s$, and by Step (VIII), offers $(B^i, p^i) = (s, v)$. The consumer, who plans to select an offer at T' by the tie-breaking assumption, will then select the rival firm $j \in \{1, 2\}$ with probability zero at $t = -1$ in case $(B^j, p^j) \neq (s, v)$ by the same argument as above at time T'. Hence, if firm 3 makes an offer (s, v), all firms do so.

Now suppose that firm 3 makes an offer $(B^3, p^3) \neq (s, v)$. Then one of the initial firms must offer (s, v), and without loss of generality suppose firm 1 does so. At contract selection opportunity T', whenever the consumer sees the offers of firm 1 and 3, she selects firm 1 by the same argument with the last paragraph. For firm 3 to earn positive profits, which it must do by Step (V), the consumer must therefore be indifferent between the offers of firm 2 and firm 3 at T'. But then by Step (VII) either firm 2 or firm 3 must set a bonus $B^i = s$, and by Step (VIII) it offers offer (s, v). In case firm 3 does so, we already established that all firms offer (s, v), so suppose firm 2 does. In that case, firm 3 faces a rival at contract selection opportunity T' that offers (s, v) with probability one, and hence firm 3 is selected with probability zero at T', contradicting that it must earn positive profits. Therefore, we conclude that all firms make offers (s, v). $\square$

Proof of Proposition 7. Consider the following candidate equilibrium in which both firms offer $(B^*, p^*) = (s, v + \frac{1-\beta}{\beta}\frac{m}{T}s)$. If the consumer is indifferent whether to plan on collecting a bonus at $t = -1$, she does plan to do so. If she is indifferent as to which initial offer to take at $t = -1$, she selects each firm with probability 1/2. Between $t = 0$ and $t = T$, whenever the consumer is indifferent, she does not take any action.

Since a naive consumer solves an optimization problem, her decision on whether she wants to switch or not at a given switching opportunity is independent of how she thinks she will act in the future whenever she is indifferent; we use the convention below that in such a case she believes her future self will switch and/or cancel a contract at the earliest opportunity whenever indifferent.

To proceed and establish that the above strategies constitute an equilibrium, we proceed in eight steps. As a preliminary observation, Step (I) shows that because the consumer cannot collect the bonus twice and she sees the same switching offer, she both plans to and actually takes up a different contract at most once independently of whether firms choose equilibrium or non-equilibrium offers. Steps (II) to (VI) derive the firms' profits from the candidate equilibrium contract offers. Steps (VII) and (VIII) establish that there is no profitable deviation.

Step (I). Given any pair of contracts, no self plans to hold the same contract twice. Note this implies trivially that the consumer does not hold the same contract twice. We first establish that if a self $\kappa \in \{0, \cdots, K\}$ holding the initial contract j cancels it without switching then neither self κ nor any future self plans to hold (or holds) a contract twice. Self $\kappa \in \{0, \cdots, K\}$ will only cancel the initial contract j without switching if $-s \geq \beta\tau(v - p^j) - \beta\mathbb{I}_{T>\kappa T_w+\tau}s$, where τ stands for the time interval until either T is reached or the consumer plans to take up a contract again. Note that it holds only if $p^j > v$. In this case, no self $\kappa' \geq \kappa$ plans to take up contract j again, because she would be strictly better off by not holding any contract until the next time she plans to incur the switching cost. If self κ takes up contract $i \neq j$, then we already established that she will not switch back to or otherwise take up contract j again. For self κ to plan to hold contract i twice, self

κ hence must cancel it and then take it up again at some $\kappa' > \kappa$. If τ is the time interval between κ and κ', this requires $-s \geq \beta\tau(v-p^i) - \beta s$ for the consumer to cancel, which implies that neither self κ nor any future self plans to take up either contract again.

We now consider self -1's problem. Let τ^i be the overall length of time self -1 plans to hold contract $i = 1, 2$, so that $T - (\tau^1 + \tau^2)$ is the time that the consumer goes with consuming the service. We consider three cases: (a) self -1 plans to take up both contracts; (b) self -1 plans to only take up contract i; (c) self -1 plans to take up no contract.

Consider case (a). Self -1's payoff induced by a plan in which the consumer takes up both contracts is

$$\beta\left[\tau^i(v-p^i) + \tau^j(v-p^j) + B^i + B^j - xs\right],$$

where x is the number of times self -1 plans to cancel, switch or take up a contract at any switching opportunity $\kappa \in \{0, \cdots, K\}$.

Observe next that in case $p^i = \min\{p^1, p^2\} \leq v$, in any optimal plan satisfying our tie-breaking assumption in which self -1 plans to take up both contracts. Specifically, self -1 takes up a contract j in $t = -1$ and plans to switch to a contract i according to which she pays p^i in period 0; this ensures that $x = 1$ and that she receives $v - p^i$ for the entire period T (and if $p^1 = p^2$ she plans to switch at the earliest opportunity due to our tie-breaking rule). Hence, in this case she plans to take both contracts at $t = -1$, she does not plan to hold a given contract twice.

We now rule out that some self $\tilde{\kappa} \in \{0, \ldots, K\}$ plans to hold a contract twice. We already establish this for the case in which self κ holding the initial contract cancels it without switching. Suppose, thus, self κ holding the initial contract j plans to switch to firm i. Consider first the case in which self κ holding the initial contract j plans to switch firm i and then back to firm j at κ' without canceling inbetween, and then plans to hold contract j for a time interval of $\tau \geq 0$ before acting again (or T is reached). Obviously, $\tau > 0$ as otherwise the consumer could save on the switching cost to firm j and either just keep holding contract i or cancel immediately without switching first. Hence, to plan to switch back at κ', it must be that $\tau(v-p^j) > \tau(v-p^i)$ as otherwise she could save on the switching cost, and $\tau(v-p^j) \geq 0$ as otherwise she would prefer canceling. But this implies that $p^j < p^i$ and that $\min\{p^1, p^2\} \leq v$, which contradicts the fact established above that $p^j \geq p^i$. So self κ cannot plan to switch to firm i and then back without canceling inbetween. Furthermore, following a switch to contract i, since $p^i \leq p^j$, no future self will actually switch back to firm j. Consider second the case in which self κ holding the initial contract j plans to switch to firm i and then cancel contract i before taking up contract j again. To prefer to plan to cancel contract i to taking up contract j, it must be that $(v-p^j)<0$, which however contradicts the fact that self κ plans to take up the contract at a later date. Furthermore, if some self $\kappa'>\kappa$ cancels contract i, we have $(v-p^j)<0$ and so no self $\kappa''\geq\kappa$ will take up contract j. We conclude that no $\tilde{\kappa}\in\{0,\ldots,K\}$ plans to hold a contract twice in case (a).

Consider case (b). Self -1's payoff induced by a plan in which the consumer takes up a single contract i is

$$\beta\left[\tau^i(v-p^i)+B^i-xs\right],$$

where τ is the length of time she plans to hold contract i and x is the number of times self -1 plans to cancel or take up contract i at switching opportunities $\kappa\in\{0,\ldots,K\}$. Clearly, if $p^i\leq v$, then $x=0$. If $p^i>v$, the payoff of taking up contract i at $t=-1$ and canceling it at $t=0$, which is $\beta(B^i-s)$, dominates that of taking the contract at $\kappa\geq 0$ and either canceling it immediately (i.e., $\beta(B^i-2s)$) or taking up the contract at $\kappa\geq 0$ and holding it for a non-zero amount of time. In either case, hence, self -1 takes up the contract i at $t=-1$, and we already established that if she cancels contract i she does not plan to take it up again. We conclude that self -1 does not plan to hold a contract twice.

Because self -1 does not plan to hold a contract twice, planing to cancel the contract at $\kappa\in\{0,\ldots,K\}$ yields payoff

$$\beta\left[(T-\kappa T_w)(v-p^i)+B^i-s\right],$$

while holding it to the end yields payoff $T(v-p^i)+B^i$. As self -1's canceling payoffs are decreasing in κ for $p^i>v$, she either plans to cancel immediately or not at all. Suppose the consumer plans not to cancel the contract at all. For the sake of contradiction, suppose furthermore that self κ holding the initial contract i wants to switch to contract j or cancel contract i at κ. Let $-s+\beta V$ denote self κ's payoff from its optimal plan, and note that it must be greater than self κ's payoff of not canceling, i.e., $-s+\beta V>\beta(T-\kappa T_w)(v-p^i)$. But if self -1 would plan to follow the same plan, then it would get a payoff of

$$\beta\left[\kappa T_w(v-p^i)+B^i-s+V\right]>\beta\left[T(v-p^i)+B^i\right],$$

contradicting that not canceling is optimal for self -1. We conclude that no self κ holding the initial contract switches to firm j or cancels the initial contract immediately. And here benefit from switching or canceling at a future date $\kappa' > \kappa$ are the same as that of self -1, so self κ must plan to follow the same non-canceling plan as self -1.

Now suppose that self -1 plans to cancel immediately at switching opportunity $\kappa = 0$, which requires $p^i > v$. We will argue that no self $\kappa \in \{0, \cdots, K\}$ holding the initial plans to switch to contract j. Because self -1 does not plan to hold contract j, it must be that $B^j < s$ since otherwise she would be at least as well of planing to take up contract j at $t = -1$, and then immediately switching to contract i at $\kappa = 0$; and when weakly better off taking both contracts self -1 must plan to do so by our tie-breaking assumption. Furthermore, because self -1 prefers to cancel contract i rather than to switch to contract j, it must be that $\beta\left(-s + B^j + T(v - p^j)\right) < 0$. Now for the sake of contradiction, suppose some self $\kappa \in \{0, \ldots, K\}$ holding the initial contract prefers to switch to contract j at $\kappa' \geq \kappa$. Let τ be the amount of time she plans to hold contract j, and note that since $B_j < s$, for planing to switch to be optimal it must be that $\tau > 0$. For switching to dominate planing to canceling the contract at κ' a necessary condition is that $\beta\left(-s + B^j + \tau(v - p^j)\right) \geq 0$, which requires that $v > p^j$. But then

$$0 > \beta\left(-s + B^j + T(v - p^j)\right) \geq \beta\left(-s + B^j + \tau(v - p^j)\right) \geq 0,$$

a contradiction. We conclude that no self κ plans to or does switch to contract j, and hence either plans to hold contract i to the end or cancel it. Because if any self plans to cancel contract i (immediately or with delay) then she does not take it up again, she must take up contract j twice to hold a contract twice. But if self κ gets a non-negative payoff from planing to taking up contract j for the first time at some $\kappa' \geq \kappa$, self -1 would get a non-negative incremental payoff of taking contract j up at κ' after canceling the contract at $\kappa = 0$, contradicting our tie-breaking rule that self -1 must take up both contract when indifferent. We conclude if self -1 plans to hold only one contract, no self plans to or does take up a contract twice.

Consider case (c). Because self -1 does not plan to take up a contract, no self who does not hold a contract plans to do so either immediately or in the future. If self κ expects a non-negative payoff from a plan that involves her first taking up a contract at $\kappa' \geq \kappa$, self -1 can also plan to do so. But then, self -1 must plan to do so by our tie-breaking rule, a contradiction.

We conclude that for any equilibrium or non-equilibrium pair of offers, the consumer never plans to or does hold a contract twice. *We thus from now on focus on the first time the consumer switches or cancels a contract and, take for granted that she never takes up the contract for a second* *time.*

Step (II). Given the candidate equilibrium offers, self $t = -1$ plans to take up both contract offers. Suppose otherwise. Then she plans to either (a) take no contract or (b) to take one contract. Consider case (a). Self -1 could instead plan to select firm i's contract and then cancel it at $t = 0$, yielding a payoff of $\beta(B^i - s) = 0$; hence, since she at least weakly prefers to take a contract, by the tie-breaking assumption she must do so, a contradiction. Next, consider case (b). Let self -1 plan to take up firm i's contract at some opportunity. Suppose self -1 plans to take up the contract of firm i at switching opportunity $t = 0$ or later; note first that she cannot take the contract at $t = 0$ because in that case she would be better off taking firm i's contract at $t = -1$ and saving the cost s of taking up the contract. But then by essentially the same argument as in case (a), she at least weakly prefers to take up the contract of firm $j \neq i$ at $t = -1$ and cancel it at $t = 0$, and by the tie-breaking assumption she must do so, a contradiction. Hence, self -1 must take up firm i's contract at $t = -1$. Let the anticipated continuation value from self -1's optimal plan starting at $t = 0$ be V, so the self -1's anticipated payoff is βV. If self -1, however, selects firm j's contract first and then switches to firm i's contract at $t = 0$ and thereafter follows the same plan as before her anticipated payoff is at least $\beta B^j - \beta s + \beta V = \beta V$; thus, by the tie-breaking assumption, she must plan to take both contracts.

Step (III). Given the candidate equilibrium contract offers, if the consumer does not have a contract at some switching opportunity κ, then she will neither take up a contract at κ nor plan to take up an equilibrium contract at any future switching opportunity. Suppose, toward a contradiction, that self κ takes up a contract. Let self κ plan to take up the contract immediately and pay the price p^i for time interval of length $\hat{T} \in [0, T]$ before either switching to firm j or canceling the contract. Then, the incremental payoff from holding the contract of firm i instead of no contract is at most $-s + \beta B^i + \beta\hat{T}(v - p^i) < 0$, so self κ gets a higher anticipated payoff when not planing to take up firm i's contract. We next show that, if self $\kappa' < \kappa$ does not have a contract, she does not plan to take up a contract in future. To see why, note that (a) if self κ' plans to take up the contract at κ and immediately cancel it, then her incremental anticipated payoff from doing so is $\beta(-s + B^i - s) = -\beta s < 0$; (b) if self κ' plans to take up the contract at κ and plans to pay p^i for time interval of length $\hat{T} \in (0, T]$, then her incremental anticipated payoff from doing so is no greater than $\beta(-s + B^i + \hat{T}(v - p^i)) < 0$; (c) if she plans to take up the contract and immediately switch to firm j, then by analogous arguments from (a) and (b), her anticipated incremental payoff from switching to firm j's contract itself is negative, and since the incremental anticipated payoff of taking up and immediately switching away from firm i is zero, her incremental anticipated payoff from taking up both contracts is negative. Hence, if the consumer does not hold a contract at switching opportunity κ, she does not plan to acquire it.

Step (IV). The consumer takes up each equilibrium contract offer at $t = -1$ with probability $1/2$. Since the consumer at $t = -1$ plans to take up both contract offers in the candidate equilibrium, and if she does not hold a contract at $\kappa = 0$ then earlier selves do not plan to acquire one at $t = 0$ or thereafter, she must select one contract offer at $t = -1$, and by the tie-breaking rule she chooses either firm with probability $1/2$ in the candidate equilibrium.

Step (V). Given the candidate equilibrium contract offers, self κ neither cancels nor switches away from the contract she chose initially at $t = -1$. Suppose otherwise. Then the consumer either (a) cancels firm i's contract without switching; (b) switches to firm j's contract and does not cancel it immediately; or (c) switches to firm j's contract and cancels it immediately. In case (a), for any $\kappa \leq K$, we already established that once the consumer does not hold a contract, she does not plan to acquire one in the future, so her continuation value after canceling is zero. For any $\kappa < K$, if self κ plans to delay canceling to $\kappa+1$, then her change in anticipated payoff is $-\beta T_w(p^i - v) - \beta s = -s$, so she weakly prefers delaying to canceling immediately, and does so by the tie-breaking assumption. If $\kappa = K$, self K's anticipated payoff when not canceling is $-\beta(T - K\frac{T}{m})(p^i - v) \geq -\beta\frac{T}{m}(p^i - v) > -s$ and hence self K does not cancel, a contradiction. In case (b), for $\kappa < K$ self κ's anticipated payoff (net of any predetermined βB^i) is $-s + \beta B^j - \beta T_w(p^j - v) + \beta V = -(1-\beta)s - \beta T_w(p^* - v) + \beta V$, where V is the anticipated continuation value from following self κ's optimal "contract cancellation" plan from $\kappa+1$ onwards. By not switching to firm j now and following the same cancellation plan in the future, self κ's anticipated payoff increases to $-\beta T_w(p^i - v) + \beta V = -\beta T_w(p^* - v) + \beta V$, and hence, she prefers not to switch. Similarly, since $-\beta(T - K\frac{T}{m})(p^* - v) > -s + \beta B^j - \beta(T - K\frac{T}{m})(p^* - v)$, the consumer prefers not to switch at switching opportunity K, a contradiction. Finally, in case (c) self κ is better off just canceling rather than switching and canceling because $-s + \beta B^* < 0$. We conclude that the consumer neither cancels nor switches following the candidate equilibrium contract offers.

Step (VI). Each firm earns $\frac{1}{2}(Tp^* - B^*)$ in the candidate equilibrium. This follows immediately from Step (IV) and (V). Because Condition (5) implies $0 < Tv < \frac{1}{2}[T(v+\frac{1-\beta}{\beta}\frac{m}{T}s)-s] = \frac{1}{2}(Tp^*-B^*)$, each firm prefers the equilibrium offer to any offer which results in the firm having no sales.

We next turn to the implications of firm i deviating. We begin by bounding the profits a firm earns when attracting the consumer at the last switching opportunity, which we then use to show that there is no profitable deviation.

Step (VII). A deviating firm i that attracts the consumer at the last switching opportunity K earns less than $\frac{T}{m}v - s$ from doing so. At the last switching opportunity, if the consumer does not have a contract, then she takes up firm i's offer if and only if one of the following two conditions holds: either self K's payoff when she immediately cancels i's contract is positive, i.e., $-2s + \beta B^i > 0$, or self K's payoff when she takes up and does not cancel i's contract is positive, i.e., $-s + \beta\left[(T - K\frac{T}{m})(v - p^i) + B^i\right] > 0$. In the former case, firm i's profits from attracting the consumer at the last switching opportunity are $-B^i < -\frac{2s}{\beta} < 0$. In the latter case, firm i's profits from attracting the consumer at the last switching opportunity are at most

$$\left(T - K\frac{T}{m}\right)p^i - B^i < \left(T - K\frac{T}{m}\right)v - \frac{s}{\beta} \leq \frac{T}{m}v - \frac{s}{\beta}. \tag{20}$$

Because Condition (5) implies that $\frac{m}{\beta T}s > v$, a firm makes a loss from attracting a consumer who does not have a contract at the last switching opportunity.

At the last switching opportunity, if firm i attracts the consumer from firm j, then she either (a) cancels firm i's contract immediately or (b) does not cancel firm i's contract. Consider case (a). As self K needs to receive $\beta B^i \geq s$ to prefer switching and canceling to just canceling firm j's contract, firm i makes a loss from attracting the consumer. Next consider Case (b). In this case self K needs to prefer switching to continuing to use firm j's contract, i.e.,

$$-s + \beta\left[\left(T - K\frac{T}{m}\right)(v - p^i) + B^i\right] \geq \beta\left(T - K\frac{T}{m}\right)(v - p^j). \tag{21}$$

By (21) and the fact that if firm j offers the equilibrium contract $p^j = p^*$, the deviant firm i's profits conditional on the consumer switching at K are at most $(T - K\frac{T}{m})p^i - B^i \leq (T - K\frac{T}{m})p^* - \frac{s}{\beta}$. Note also that $(T - K\frac{T}{m})(v + \frac{1-\beta}{\beta}\frac{m}{T}s) - \frac{s}{\beta} \leq \frac{T}{m}(v + \frac{1-\beta}{\beta}\frac{m}{T}s) - \frac{s}{\beta} = \frac{T}{m}v - s$, which completes the argument for Step (VI).

Step (VIII). There is no profitable deviation. We now partition the set of deviant contract offers (B^i, p^i) by firm i into those for which: *(A)* $p^i > p^*$; *(B)* $p^i \in (v, p^*)$; *(C)* $p^i = p^*$ and $B^i \neq s$; and *(D)* $p^i \leq p^*$ and the rule out a profitable deviation case by case.

(A). Consider a deviation by firm i to an offer (B^i, p^i) for which $p^i > v + \frac{1-\beta}{\beta}\frac{m}{T}s$. Recall that if the consumer has canceled firm i's contract at switching opportunity κ, by Steps (I) and (III), she neither takes up nor plans to take up firm j's candidate equilibrium contract at any future switching opportunity $\kappa' > \kappa$. Given that the consumer takes up firm i's contract, self κ strictly prefers canceling it immediately at κ to canceling it at any $\kappa' > \kappa$ because $-s > -\beta\hat{T}(v - p^i) - \beta s$ for any time interval $\hat{T} \geq \frac{T}{m}$. Similarly, given that the consumer takes up firm i's contract, self κ strictly prefers to cancel it immediately rather than to plan to switch to firm j at any switching opportunity $\kappa' > \kappa$; to see why, let $\hat{T} \geq \frac{T}{m}$ be the amount of time until she switches to firm j and $\tilde{T} \geq 0$ be the amount of time she pays p^j. Suppose first she plans to switch to firm j at $\kappa' > \kappa$ and cancels it at $\kappa'' \in \{\kappa', \ldots, K\}$. Then, self κ's anticipated payoff (net of βB^i) is $\beta\hat{T}(v - p^i) - \beta s + \beta B^* + \beta\tilde{T}(v - p^*) - \beta s \leq \beta\hat{T}(v - p^i) - \beta s < -s$, so she strictly prefers canceling firm i's contract immediately at κ. Suppose second she plans to switch to firm j at $\kappa' > \kappa$ and plans not to cancel firm j's contract at any future switching opportunity. Then, self κ's anticipated payoff (net of βB^i) is $\beta\hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T} - \kappa\frac{T}{m})(v - p^*) < \beta T(v - p^*) = -(1-\beta)ms < -s$, which holds by the assumption that $(1-\beta)m > 1$, so she prefers to cancel immediately. We conclude that, at any switching opportunity κ, the consumer either cancels firm i's contract immediately or plans to hold it until T.

We now argue that firm i's profits from attracting the consumer at a switching opportunity κ are bounded from above by Tv. If the consumer cancels the contract immediately at a switching opportunity κ, attracting her at κ is unprofitable. Thus, consider the case in which self κ plans to hold it until T. If self κ plans to hold the contract of firm i until T, her payoff is $\beta(T - \kappa\frac{T}{m})(v - p^i) + \beta B^i - s$. When planing to hold firm i's contract, self κ is willing to take up this contract at κ only if $\beta B^i + \beta(T - \kappa\frac{T}{m})(v - p^i) \geq 0$. Thus, the firm's profits from attracting the consumer at κ are $(T - \kappa\frac{T}{m})p^i - B^i \leq (T - \kappa\frac{T}{m})v \leq Tv$.

We now argue that firm i's profits from attracting the consumer at $t = -1$ are also bounded from above by Tv. Recall that self -1 either plans to hold firm i's contract until T or plans to cancels firm i's contract at any switching opportunity $\kappa = 0$. Because taking up firm i's contract at $t = -1$ and planing to cancel it at $\kappa = 0$ requires $B^i \geq s$, firm i makes a loss from attracting the consumer at $t = -1$ in this case. Suppose next that the consumer plans to hold firm i's contract until T. Self -1's anticipated payoff at $t = -1$ from doing so must be non-negative; i.e. $\beta B^i + \beta T(v - p^i) \geq 0$.

In this case, the upper bound of firm i's total profits (when the consumer takes up firm i's contract at $t = -1$ with probability 1 and does not cancel thereafter) are $Tp^i - B^i \leq Tv$.

Hence, both in case firm i attracts the consumer at $t = -1$ or thereafter, her profits from attracting the consumer are bound by Tv. Furthermore, a consumer takes up firm i's contract at most once, so the deviant firm i's profits are bounded from above by Tv. Because Condition (5) implies $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta}\frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$, a deviation by firm i to an offer (B^i, p^i) for which $p^i > p^*$ is unprofitable.

(B). Consider a deviation by firm i to an offer (B^i, p^i) for which $p^i \in (v, v + \frac{1-\beta}{\beta}\frac{m}{T}s)$. We first show that the consumer chooses firm i's offer with at most probability 1/2 at $t = -1$. Suppose otherwise, i.e. self -1 chooses firm i's offer at $t = -1$ with probability 1. There are four cases: (a) self -1 plans to cancel firm i's offer at $\kappa \geq 0$, (b) self -1 plans to switch to firm j's offer at $\kappa = 0$, (c) self -1 plans to switch to firm j's offer at $\kappa \geq 1$, and (d) self -1 plans to hold firm i's contract until T. Consider case (a). Because a consumer does not plan to take up a contract more than once by Step (I), after canceling firm i's offer, self -1 does not plan to take up firm j's equilibrium offer by Step (III) above. But then, self -1 attains the same anticipated payoff when planning to take up firm j's contract at $t = -1$ and switching to firm i at $t = 0$ (and thereafter following the same "cancellation plan" as before). By the tie-breaking assumption, self -1 must plan to take both contracts, a contradiction. Consider case (b). If self $t = -1$ plans to cancel firm j's contract at $t = 0$, then her anticipated payoff is $\beta(B^i + B^j - s)$ and she attains the same anticipated payoff when taking up firm j's contract first and then switching to and canceling firm i's contract at $t = 0$, so our tie-breaking rule contradicts that she takes up firm i's contract with probability one. If self $t = -1$ plans to pay firm j's price $p^j = p^*$ up to time $\hat{T}$, self -1's anticipated payoff is $\beta(B^i + B^j - s + \hat{T}(v - p^*) + \mathbb{I}_{\hat{T}<T}s)$; but by taking up firm j's contract first and then planing to pay the lower price p^i up to time $\hat{T}$, self -1's anticipated payoff strictly increase to $\beta(B^i + B^j - s + \hat{T}(v - p^i) + \mathbb{I}_{\hat{T}<T}s)$, a contradiction. Consider case (c). Let $\hat{T} \geq \frac{T}{m}$ be the time until she switches to firm j and $\tilde{T} \geq 0$ be the amount of time she pays p^j. Suppose first self $t = -1$ plans to switch to firm j at $\kappa' \geq 1$ and cancels its contract at $\kappa'' \in \{1, \ldots, K\}$. Then, her anticipated payoff from following the plan is $\beta B^i + \beta\hat{T}(v - p^i) - \beta s + \beta B^* + \beta\tilde{T}(v - p^*) - \beta s \leq \beta B^i + \beta\hat{T}(v - p^i) - \beta s - \beta s + \beta B^* < \beta(B^i + B^* - 2s)$, so self -1 strictly prefers to plan to switch to firm j's contract at $t = 0$ and then cancel it immediately. Suppose second she plans to switch to

firm j at $\kappa' > \kappa$ and not to cancel its contract. Then, self -1's anticipated payoff from following the plan is $\beta B^i + \beta\hat{T}(v-p^i) - \beta s + \beta B^* + \beta(T-\hat{T})(v-p^*) < -\beta s + \beta B^* + \beta B^i + \beta T(v-p^i)$, so self -1 strictly prefers to plan to take up firm j's offer at $t=-1$, switch to firm i's offer at $t=0$, and hold it until the end, a contradiction. Consider case (d). Then, for the consumer to be willing to take up firm i's contract at $t=-1$, $\beta T(v-p^i) + \beta B^i \geq 0$ must hold. In this case, firm i's total profits are at most $Tp^i - B^i \leq Tv$. Because Condition (5) implies $Tv < \frac{1}{2}[T(v + \frac{1-\beta}{\beta}\frac{m}{T}s) - s] = \frac{1}{2}(Tp^* - B^*)$, any deviation that induces the consumer to take up firm i's contract with probability one is not a profitable deviation. We conclude that self $t=-1$ chooses firm i's offer with at most probability $1/2$ for any possibly profitable deviation.

We next show that firm i's deviating offer does not induce the consumer to switch from firm j to firm i except possibly at the last switching opportunity K. We first argue that self κ prefers to procrastinate switching from firm j to firm i at any switching opportunity $\kappa < K$. Note first that if the consumer plans to cancel firm i's contract immediately after switching to it, then she prefers to procrastinate switching rather than to do so since

$$\beta\left(-2s + \frac{T}{m}(v-p^*) + B^i\right) > -2s + \beta B^i \quad \Longleftrightarrow \quad (1-\beta)s > 0.$$

Thus, consider the case in which self κ plans to hold firm i's contract at least until the next switching opportunity. Again, she prefers to procrastinate switching to switching immediately because

$$\beta\left(-s + \frac{T}{m}(v-p^*) + B^i\right) > -s + \beta\left(\frac{T}{m}(v-p^i) + B^i\right) \quad \Longleftrightarrow \quad p^i > v. \tag{22}$$

Hence, the consumer does not switch from firm j to firm i at any switching opportunity $\kappa < K$.

We now show that if firm i's deviation induces the consumer to switch from firm j to firm i at the last switching opportunity K, firm i's deviation is unprofitable. If the consumer switches at K, (21) must hold, which requires that $(T - K\frac{T}{m})p^i - B^i \leq (T - K\frac{T}{m})p^* - \frac{s}{\beta}$ and hence also that $Tp^i - B^i \leq Tp^* - \frac{s}{\beta}$. By Step (VII), firm i earns at most $\frac{T}{m}v - s$ conditional on the consumer switching to i's contract at K. Because self $t=-1$ chooses firm i with probability at most $1/2$, firm i's total profits from this deviation are at most

$$\frac{1}{2}(Tp^i - B^i) + \frac{1}{2}(\frac{T}{m}v - s) \leq \frac{1}{2}(Tp^* - \frac{s}{\beta}) + \frac{1}{2}(\frac{T}{m}v - B^*) = \frac{1}{2}(Tp^* - B^*) + \frac{1}{2}(\frac{T}{m}v - \frac{s}{\beta}). \tag{23}$$

Because Condition (5) implies $\frac{m}{\beta T}s > v$, firm i's deviation is unprofitable. Hence, we established that for any potentially profitable deviation, after taking up firm j's contract the consumer does not switch to firm i. This implies that in any possibly profitable deviation, firm i must attract the consumer at $t=-1$ with probability $1/2$.

Suppose such a profitable deviation in which firm i attracts the consumer at $t=-1$ with probability $1/2$ exists. Note that if $B^i \geq s$, since the consumer takes up firm i's contract with probability $1/2$ and does not switch away from firm j's contract, firm i's deviation profits are at most $\frac{1}{2}(Tp^i-B^i) < \frac{1}{2}(Tp^*-B^*)$, a contradiction. Hence $B^i < s$. If self -1 takes up firm i's contract at $t=-1$ she plans not to cancel it at $\kappa=0$, because otherwise the incremental anticipated payoff from taking firm i's contract would be $\beta(B^i-s)<0$. Similarly, because $p^i > v$, self -1 does not plan to cancel the contract at any switching opportunity κ, because otherwise the incremental anticipated payoff from taking firm i's contract would be negative. In addition, because $B^i < s$, the consumer who takes up firm i's contract at $t=-1$ does not plan to switch to firm j's contract at $t=0$; otherwise, self -1 strictly prefers the plan to take up firm j's contract at $t=-1$ and save on the switching costs. Thus, self -1 plans to pay p^i for a positive amount of time. Let $\hat{T} \geq \frac{T}{m}$ be the time until she plans to switch to firm j and $\tilde{T} \geq 0$ be the time she pays p^j. Suppose first self $t=-1$ plans to switch to firm j at $\kappa' \geq 1$ and cancels it at $\kappa'' \in \{1,\ldots,K\}$. Then, her anticipated payoff from following the plan is $\beta B^i+\beta\hat{T}(v-p^i)-\beta s+\beta B^*+\beta\tilde{T}(v-p^*)-\beta s \leq \beta\hat{T}(v-p^i)-\beta(s-B^i)<0$, so she strictly prefers the plan in which she takes up no contact. Suppose second she plans to switch to firm j at $\kappa' > \kappa$ and plans not to cancel firm j's contract thereafter. Then her anticipated payoff from following this plan is $\beta B^i+\beta\hat{T}(v-p^i)-\beta s+\beta B^*+\beta(T-\hat{T})(v-p^*) < \beta B^i+\beta T(v-p^i)$, so it is lower than the anticipated payoff when self -1 plans to take up firm i's offer at $t=0$ and refrain from canceling it at every switching opportunity. We thus established that if the consumer takes up firm i's contract at $t=-1$, she plans to pay p^i until T. For the consumer to take up firm i's contract at $t=-1$, thus, $\beta T(v-p^i)+\beta B^i \geq 0$ must hold. This implies that firm i's total profits are at most $Tp^i-B^i \leq Tv$. Because Condition (5) implies $Tv < \frac{1}{2}[T(v+\frac{1-\beta}{\beta}\frac{m}{T}s)-s] = \frac{1}{2}(Tp^*-B^*)$, this contradicts that the deviation is profitable. We conclude that any deviation to an offer for which $p^i \in (v, v+\frac{1-\beta}{\beta}\frac{m}{T}s)$ is unprofitable.

(C). Consider a deviation by firm i to an offer (B^i,p^i) for which $B^i \neq s$ and $p^i = v+\frac{1-\beta}{\beta}\frac{m}{T}s$. Suppose first $B^i > s$. Suppose self -1 would only take one firm's contract, say the contract of firm $n \in \{i,j\}$ and denote the continuation value of this plan starting at $t=0$ by V, so her anticipated payoff is βV. By taking firm $n' \neq n$'s contract at $t=-1$, switching to firm n at $\kappa=0$ and then

following the same continuation plan as before from $\kappa = 0$ onwards, self -1 anticipated payoff would become $\beta(B^{n'} - s + V) \geq \beta V$; by our tie-breaking rule, hence, self -1 must plan to take both contracts. Since the consumer plans to collect both bonuses, and because $p^i = p^j$, self $t = -1$ is indifferent between taking up either contract i or j first and thus must select each with equal probability. As before, for any $\kappa < K$, if self κ who holds contract n plans to delay canceling to $\kappa + 1$, then her change in anticipated payoff is $-\beta T_w(p^n - v) - \beta s = -s$, so she weakly prefers delaying to canceling immediately, and does so by our tie-breaking assumption. If the consumer switches at K, (21) must hold, which requires that $(T - K\frac{T}{m})p^i - B^i \leq (T - K\frac{T}{m})p^* - \frac{s}{\beta}$ and hence also that $Tp^i - B^i \leq Tp^* - \frac{s}{\beta}$. By Step (VII), firm i earns at most $\frac{T}{m}v - s$ conditional on the consumer switching to i's contract at K. Because self $t = -1$ chooses firm i with probability at most $1/2$, by the exact same calculation as in (23), firm i's deviation is unprofitable in case it induces the consumer to switch at K.

Suppose second $B^i < s$. Because the level of firm i's bonus is less than the switching cost and $p^i = p^j > v$, self $t = -1$'s prefers to take up firm j's contract and cancels it at $t = 0$ yielding an anticipated payoff of $\beta(B^j - s) = 0$ to all other plans in which she signs a contract, and because the consumer weakly prefers this plan to never taking up any contract, she selects firm j's contract. Furthermore, at $p^i = p^* > v$ and $B^i < B^*$, no self κ takes up or switches to firm i's contract at any switching opportunity. Hence, firm i has no sales, a contradiction. We conclude that any deviation by firm i to an offer (B^i, p^i) for which $B^i \neq s$ and $p^i = p^*$ is unprofitable.

(D). Consider a deviation by firm i to an offer (B^i, p^i) for which $p^i \leq v$. We first show that, for firm i's deviation to be profitable, the consumer must choose firm i's offer with probability $1/2$ at $t = -1$. Suppose otherwise; then by our tie-breaking assumption she chooses firm i's offer at $t = -1$ with either (a) probability 1 or (b) probability 0. Consider first case (a). By the arguments in Step (III), a consumer who canceled contract i at or prior to any switching opportunity κ does neither take up or plan to take up contract j at any switching opportunity $\kappa' \geq \kappa$. Because $p^i \leq v$, self $t = -1$'s anticipated continuation payoff at any switching opportunity κ of simply keeping the contract until T is non-negative, while planing to cancel yields an anticipated continuation payoff of $-s$, planing to switch to firm j and cancel contract j immediately yields an anticipated continuation payoff of $-2s + B^j = -s$, and planing to switch without canceling immediately yields a strictly negative anticipated continuation payoff. Hence, self $t = -1$ anticipated payoff of selecting firm i's

contract is $\beta(B^i+T(v-p^i))$. Self -1's anticipated payoff when selecting firm j's contract at $t=-1$ and planing to switch to firm i at $t=0$ is $\beta(B^j-s+B^i+T(v-p^i))=\beta(B^i+T(v-p^i))$, and thus by our tie-breaking rule self -1 cannot select contract i with probability greater than 1/2 at $t=-1$, a contradiction. Because $p^i\leq v$, an upper bound to an self $\kappa\geq 0$ of taking up the contract of firm i is $\beta(T(v-p^i)+B^i)$, and hence a necessary condition for self κ to take up contract i is that

$$T(v-p^i)+B^i\geq 0. \tag{24}$$

Hence, firm' i's deviation profits are at most $(T-\kappa\frac{T}{m})p^i-B^i\leq(T-\kappa\frac{T}{m})v\leq Tv$. Because Condition (5) implies $Tv<\frac{1}{2}[T(v+\frac{1-\beta}{\beta}\frac{m}{T}s)-s]=\frac{1}{2}(Tp^*-B^*)$, deviating and attracting the consumer at switching opportunities only is not a profitable deviation. We conclude that the consumer must choose firm i's offer with probability 1/2 (and hence chooses firm j's offer with probability 1/2 by the tie-breakig rule) at $t=-1$.

If the consumer selects firm j's offer at $t=-1$, she plans to switch to offer (B^i,p^i) at $t=0$ and then not cancel it rather than to merely cancel firm j's offer only if (24) holds. Consider first the case in which (24) does not hold. Then, any self $\kappa\geq 0$ will refrain from taking up firm i's offer. Also, self -1's anticipated payoff from taking up firm i's offer at $t=-1$ is at most $T(v-p^i)+B^i<0$, so she strictly prefers to not taking up firm i's offer at $t=-1$. Hence, firm i has no sales, a contradiction.

Consider second the case in which (24) holds. Because $p^*>v\geq p^i$ and $B^*=s$, the consumer plans to not switch from firm i to firm j, so the consumer's anticipated total payoff when selecting firm i's offer at $t=-1$ is $T(v-p^i)+B^i$. Self -1's anticipated total payoff when selecting firm j's offer at $t=-1$ and then switching to firm i at $t=0$ is also $-s+T(v-p^i)+B^*+B^i=T(v-p^i)+B^i$. As the consumer is indifferent at $t=-1$, by the tie-breaking rule self $t=-1$ plans to collect both bonuses. We next argue that the consumer's uniquely optimal plan is to take up firm j's offer at $t=-1$ and then switch to firm i at $t=0$ and then not incur the cost of canceling firm i's contract. To see why, suppose first that the consumer takes up firm j's offer at $t=-1$. As self $t=-1$ plans to collect both bonuses, self $t=-1$ plans to switch to firm i at some opportunity. Also, because $p^i\leq v$, self $t=-1$ does not plan to cancel firm i's contract. Hence, self $t=-1$'s anticipated payoff is $\beta B^*+\beta\hat{T}(v-p^*)-\beta s+\beta B^i+\beta(T-\hat{T})(v-p^i)$, where $\hat{T}\geq 0$ is the amount of time until she switches to firm i. Because $p^*>v\geq p^i$, the anticipated payoff is maximized at

$\hat{T} = 0$, which is $\beta T(v - p^i) + \beta B^i$. Suppose second that the consumer takes up firm i's offer at $t = -1$. Let $\hat{T} \geq 0$ be the amount of time until she switches to firm i and $\tilde{T} \geq 0$ be the amount of time she pays p^*. As self $t = -1$ plans to collect both bonuses, self $t = -1$'s anticipated payoff is $\beta B^i + \beta\hat{T}(v - p^i) - \beta s + \beta B^* + \beta\tilde{T}(v - p^*) - \beta s \leq \beta T(v - p^i) + \beta B^i - \beta s < \beta T(v - p^i) + \beta B^i$ if self $t = -1$ plans to cancel firm j's contract at some opportunity, and it is $\beta B^i + \beta\hat{T}(v - p^i) - \beta s + \beta B^* + \beta(T - \hat{T})(v - p^*) \leq \beta(T - K\frac{m}{T})(v - p^i) + K\frac{m}{T}(v - p^*) + \beta B^i < \beta T(v - p^i) + \beta B^i$ if self $t = -1$ plans to hold firm j's contract until the end. Thus, in either case, the anticipated payoff is lower than the case in which self $t = -1$ plans to take firm j's offer, switch to firm i's offer at $t = 0$, and hold it until the end. These results imply that, for the deviation offer to be profitable, firm i must attract the consumer at some switching opportunity. But then (24) must hold, implying that firm i's total profits are at most $Tp^i - B^i \leq Tv$. Because $Tv < \frac{1}{2}(Tp^* - B^*)$ by Condition (5), firm i's deviation is unprofitable.

We conclude that there is no profitable deviation for firm i, so the candidate equilibrium specified above is indeed an equilibrium. $\square$

Analysis when B^n is exogenously fixed. Suppose that $B^n = \bar{B}$, where $s < \bar{B} < \min\{2s - Tv/m, s/\beta\}$. We check that firms have no profitable deviation from offering $p^* = v + \left(\frac{1-\beta}{\beta} \cdot s + \bar{B} - s\right) \cdot \frac{m}{T}$. Note that the assumption that m is an integer implies $T - K\frac{T}{m} = \frac{T}{m}$, i.e., the length of the last period is the same as other periods.

The proof of Proposition 7 holds in the same manner up to the end of Step (II). With using $T - K\frac{T}{m} = \frac{T}{m}$ and $\bar{B} < \min\{2s, s/\beta\}$, the proof of Step (III) is also analogous to the proof of Proposition 7. The proof of Step (IV) holds as it is. So we start from Step (V) in the proof of Proposition 7.

Step (V). Given the candidate equilibrium contract offers, self κ neither cancels nor switches away from the contract she chose initially at $t = -1$. Suppose otherwise. Then the consumer either (a) cancels firm i's contract without switching; (b) switches to firm j's contract and does not cancel it immediately; or (c) switches to firm j's contract and cancels it immediately. In case (a), if $\kappa = K$, self K's anticipated payoff when not canceling is $-\beta\frac{T}{m}(p^i - v) = -s + \beta(2s - \bar{B}) > -s$ and hence self K does not cancel. For any $\kappa \leq K$, we already established that once the consumer does not hold a contract, she does not plan to acquire one in the future, so her continuation value after canceling is zero. Thus, for any $\kappa < K$, if self κ cancels firm i's contract, then her anticipated payoff is $-s$. If self κ plans to switch to firm j's contract next time and immediately cancel it, her anticipated payoff is $-\beta T_w(p^* - v) - \beta \cdot 2s + \beta\bar{B} = -s$. So self κ weakly prefers to switch from firm i's contract next time rather than to cancel firm i's contract immediately, and does so by the tie-breaking assumption. The proofs of case (b) and case (c) are analogous to the proof of Proposition 7, and hence is omitted.

Step (VI)'. Each firm earns $\frac{1}{2}(Tp^* - \bar{B})$ in the candidate equilibrium. Steps (IV) and (V) imply that each firm earns $\frac{1}{2}(Tp^* - \bar{B})$ in the candidate equilibrium. Because Condition (5) implies $0 < Tv < \frac{1}{2}(Tp^* - \bar{B})$, each firm prefers the equilibrium offer to any offer which results in the firm having no sales.

We next turn to the implications of firm i deviating. We begin by bounding the profits a firm earns when attracting the consumer at the last switching opportunity, which we then use to show that there is no profitable deviation.

Step (VII)'. Whenever the consumer switches from firm $j \in \{1,2\}$ to firm $i \in \{1,2\}$ at the switching opportunity $\kappa = K$, firm j makes a loss from it up to time T. We first show that, if the consumer does not have a contract at the last switching opportunity K, then self K does not take up firm i's contract. First, self K's payoff when she immediately cancels i's contract is negative, i.e., $-2s + \beta\bar{B} < 0$, so she does not do so. Second, self K's payoff when she takes up and does not cancel i's contract is positive only if $-s + \beta\left[\frac{T}{m}(v - p^i) + \bar{B}\right] > 0$. In this case, firm i's profits from attracting the consumer at the last switching opportunity are at most

$$\frac{T}{m}p^i - \bar{B} < \frac{T}{m}v - \frac{s}{\beta}.$$

Because Condition (5) implies that $\frac{m}{\beta T}s > v$, a firm makes a loss from attracting a consumer who does not have a contract at the last switching opportunity.

At the last switching opportunity, if firm i attracts the consumer from firm j, then she either (a) cancels firm i's contract immediately or (b) does not cancel firm i's contract. In case (a), because $\bar{B} > s$, firm i makes a loss from attracting the consumer. So consider Case (b). In this case self K needs to prefer switching to continuing to use firm j's contract, i.e.,

$$-s + \beta\left[\frac{T}{m}(v - p^i) + \bar{B}\right] \geq \beta\frac{T}{m}(v - p^j). \tag{25}$$

By (25) and the fact that if firm j offers the equilibrium contract $p^j = p^*$, the deviant firm i's profits conditional on the consumer switching at K are at most $\frac{T}{m}p^i - \bar{B} \leq \frac{T}{m}p^* - \frac{s}{\beta} = \frac{T}{m}[v +$

$\left(\frac{1-\beta}{\beta} \cdot s + \bar{B} - s\right) \cdot \frac{m}{T}] - \frac{s}{\beta} = \frac{T}{m}v + \bar{B} - 2s < 0$, which completes the argument for Step (VII)'.

Step (VIII)'. There is no profitable deviation. We now partition the set of deviant contract offers (B^i, p^i) by firm i into those for which: *(A)'* $p^i > p^*$; *(B)'* $p^i \in (v, p^*)$; and *(C)'* $p^i \leq p^*$. The proofs of case (A)' and case (C)' are essentially the same as the proofs of case (A) and case (D) in Proposition 7, respectively. By using the result of Step (VII)', the proof of case (B)' is analogous to the proof of case (B) in Proposition 7.