This paper makes two contributions to the modeling of addiction. First, we provide new and convincing evidence that smokers are forward-looking in their smoking decisions, using state excise tax increases that have been legislatively enacted but are not yet effective, and monthly data on consumption. Second, we recognize the strong evidence that preferences with respect to smoking are time inconsistent, with individuals both not recognizing the true difficulty of quitting and searching for self-control devices to help them quit. We develop a new model of addictive behavior that takes as its starting point the standard “rational addiction” model, but incorporates time-inconsistent preferences. This model also exhibits forward-looking behavior, but it has strikingly different normative implications; in this case optimal government policy should depend not only on the externalities that smokers impose on others but also on the “internalities” imposed by smokers on themselves. We estimate that the optimal tax per pack of cigarettes should be at least one dollar higher under our formulation than in the rational addiction case.

I. INTRODUCTION

Many of the habits that pervade everyday life can be properly described as addictive. While the degree of addictiveness varies from activity to activity and person to person, habits such as smoking, drinking, eating, and a host of others often meet the two conditions required for addiction: reinforcement, in that the more you partake of the activity, the more you want to partake; and tolerance, in that the more that you partake of the activity, the lower your future utility given the amount of future consumption [Becker and Murphy 1988]. The importance of addiction for a variety of aspects of consumption behavior has led to a long-standing interest in modeling addictive processes. Most of the literature in this area until the mid-1980s modeled addiction as habit formation, capturing the reinforcement aspect of the process through an effect of lagged consumption on the taste for the good.
In a pathbreaking article, Becker and Murphy [1988] explored the detailed dynamic behavior of the consumption of addictive goods, and pointed out that many phenomena previously thought to have been irrational are consistent with optimization according to stable preferences. In the Becker and Murphy model, individuals recognize the addictive nature of choices that they make, but may still make them because the gains from the activity exceed any costs through future addiction. That is, in this rational addiction framework, individuals recognize the full price of addictive consumption goods: both the current monetary price, and the cost in terms of future addiction.

This model of rational addiction has subsequently become the standard approach to modeling consumption of goods such as cigarettes. This standard has been reinforced by a sizable empirical literature, beginning with Becker, Grossman, and Murphy (BGM [1994]), which has tested and generally supported the key empirical contention of the Becker and Murphy model: that consumption of addictive goods today will depend not only on past consumption but on future consumption as well. More specifically, this literature has generally assessed whether higher prices next year lead to lower consumption today, as would be expected with forward-looking addicts. The fairly consistent findings across a variety of papers that this is the case has led to the acceptance of this framework for modeling addiction.

These past tests, however, run into a number of empirical and theoretical problems. On the empirical side, they rely on the assumption that individuals are appropriately forecasting prices far in advance (as much as one year); as we document below, for cigarettes at least, very few price increases are announced this far in advance. Moreover, in many other applications, the fact that the lead of a price variable affects current behavior is taken as the failure of a specification test of the model, not as evidence of forward-looking behavior.

Finally, even if forward-looking behavior can be demonstrated convincingly, there is a more fundamental theoretical problem: forward-looking behavior does not imply time consistency. A key assumption of the rational addiction framework is that individuals are time consistent; their future behavior coincides with their current desires regarding this behavior. But this assumption is at odds with strong evidence from psychological experiments on the nature of choice over time. Moreover, it is at odds with many real world phenomena, such as the inability to
carry out stated desires to quit smoking, and the demand for self-control devices as a means of quitting. Therefore, it is impera-
tive to investigate the implications of incorporating time-incon-
sistent preferences into models of addiction.

The purpose of our paper is to address both these empirical and theoretical issues with the rational addiction literature, in the context of cigarette consumption. We begin by noting the problems with previous tests of rational addiction models. We then suggest an alternative test: examining how consumption changes when a tax change is actually announced, but not yet effective. We do so using monthly data on cigarette consumption, as well as sales, matched to information on the enactment and effective dates of all state level cigarette excise tax increases over the recent past. We find, in fact, that in this framework there is evidence for forward-looking behavior; cigarette consumption does fall when future price increases are announced but not yet effective, clearly ruling out myopic models of addiction. This finding is also robust to the specification tests which prove difficult for previous tests to pass.

We then turn to developing an alternative model that is also consistent with forward-looking consumption decisions. We do so by embedding in the Becker-Murphy framework the hyperbolic discounting preferences pioneered by Laibson [1997]. These preferences provide a sensible parameterization that allows us to maintain the optimizing features of the Becker-Murphy framework, while considering time inconsistency in the decision to smoke. We find that this model also generates the prediction that future prices matter for today’s consumption; indeed, they matter in ways that are sufficiently similar to the Becker-Murphy model that we are unable to empirically distinguish the two with our data. Yet, we show that this model can deliver radically different implications for government policy. In particular, while the rational addiction model implies that the optimal tax on addictive bads should depend only on the externalities that their use imposes on society, the time-inconsistent alternative suggests a much higher tax that depends also on the “internalities” that users impose on themselves. At standard values of a life, these internalities are on the order of $30 per pack of cigarettes, which is 100 times the size of the estimated externalities from smoking. Simulations therefore suggest that the optimal tax can be at least a dollar higher for even modest time inconsistency in this framework.
Our paper proceeds as follows. In Section II we review past attempts to test for forward-looking behavior, and describe our improved empirical strategy for doing so. We implement our new test in Section III. Section IV develops our alternative model of time-inconsistent addiction; Section V solves the model and explores the implications of price changes in this framework. Section VI discusses the implications of the different models for government policy. Section VII concludes.

II. TESTING FOR FORWARD-LOOKING BEHAVIOR

Models of the consumption of addictive goods have a long tradition. Most of the literature until the mid-1980s focused on the habit formation, or reinforcement, aspect of addictive processes. This aspect leads naturally to the prediction that current consumption of addictive goods will be dependent on the path of past consumption, and a number of articles have demonstrated for goods like cigarettes this backwards-looking intertemporal correlation.1

Becker and Murphy [1988] presented a novel analysis that greatly advanced the modeling of addictive processes. The key insight of their model was that just because a good is addictive, there is no reason that its consumption cannot be analyzed in a standard rationally optimizing framework. Their “rational addicts,” in making consumption decisions, recognize that there is a trade-off with current consumption: while utility rises today from the consumption, long-run utility is lower because the individual is building up a stock of the addictive good that has a negative marginal utility. Individuals rationally trade off these factors to consider the appropriate level of consumption of addictive goods.

A key implication of this model is that consumption behavior should exhibit “adjacent complementarity.” Reinforcement arises here through the fact that a larger stock of past consumption raises the marginal utility of current consumption. Thus, the fact that individuals are going to pursue the activity in the future should increase the pursuit today, so as to increase the enjoyment of the activity next period. This insight has led to the central empirical prediction of the rational addiction model: asking whether consumption today is dependent on consumption tomorrow.

The first paper to carry out this test was Becker, Grossman,

1. See Chaloupka and Warner [2000] for a superb review of both the theoretical and empirical literatures in this area.
and Murphy [1994], focusing on cigarette smoking as an addictive behavior. They compile a data set of cigarette consumption and prices across the U. S. states over the 1955 through 1985 period, and match that to information on cigarette prices across the states. They then estimate models that relate current consumption to future consumption. They recognize the important problem that consumption in the future is endogenous, so they propose an instrumental variables strategy that uses future prices as an instrument for future consumption. Thus, in essence, their test amounts to asking whether smoking falls when prices are increased the next year. Doing so, they find significant impacts of future prices (and, in their instrumental variables setup, future consumption) on current cigarette sales, supporting the forward-looking behavior implied by the Becker-Murphy model. This type of test has been carried out by a variety of subsequent studies, on both cigarettes and other substances.2

Unfortunately, these past tests have a number of flaws which render them difficult to interpret. First, conceptually, it is difficult to conceive of individuals being able to forecast well future prices in their state of residence, even if they simply are trying to forecast tax changes. As we document in more detail below, excise tax changes are rarely known one year in advance; only 8 of 160 tax changes over the 1973–1996 period were enacted as far as one year in advance. For individuals to forecast prices this far into the future would require a very sophisticated model of expectations.

Second, the dependent variable is sales of cigarettes, not consumption; in particular, this represents sales from wholesalers to retail distributors of cigarettes.3 If individuals really did anticipate future price changes, then the expected direction of the response is not obvious; to the extent that individuals wish to stockpile cigarettes while they are less expensive, consumption could actually rise in anticipation of price increases.

At an annual frequency, this may not be a major concern, due to cigarette quality deterioration for long periods of storage. But this point interacts with the previous one: if the price change is far in the future, there is unlikely to be stockpiling, but the

3. While the other criticisms levied here apply to all of the studies in this literature, this one only applies to the subset of studies that use aggregate sales, rather than individual consumption, data.
change is also unlikely to be anticipated; if the price change is in the near term, then anticipation is more likely, but so is stockpiling.

Third, there may be endogeneity bias to regressing the quantity of cigarettes consumed on their price. This bias is likely to be small, since the primary determinant in within-state specific price changes is changes in excise taxes; existing evidence suggests that excise tax changes are passed through on a slightly more than one-for-one basis [Federal Trade Commission 1997]. But tax changes explain only about 80 percent of the within-state-year variation in prices, so that there is remaining variation in the price that could lead to endogeneity bias in the price-consumption relationship. The true exogenous variation that should be used to identify this model is taxes.

Fourth, and perhaps most importantly, this test is unable to distinguish true future price effects from other failures of the fixed effects specification. It is plausible that over such a long time period state effects are not truly fixed. If, for any reason, prices or taxes are slowly rising over time in the states where smoking is falling the most, then this will lead to a finding that future prices are correlated with current consumption. Indeed, in many other applications, the fact that the lead of a price variable affects current behavior is taken as the failure of a specification test of the model, not as evidence of forward-looking behavior.4 This relationship between price and lagged consumption is consistent with Showalter [1999], who documents that an oligopolistic tobacco manufacturer facing a relatively inelastic demand for cigarettes will react to declining consumption by raising price. The same behavior may be true of revenue-maximizing state governments faced with declining cigarette demand, leading to the observed correlation even with taxes.

In an earlier version of this paper [Gruber and Köszegi 2000], we replicated the analysis of BGM, extending the sample period through 1997, and showed the fragility of these findings to the problems noted above. Simply replacing prices with taxes in their analysis significantly reduces the significance of the result; indeed, the key reduced-form coefficient (next year’s tax rate) is wrong signed over their sample period. More fundamentally, their result is not at all robust to alternative empirical strategies

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4. For a discussion of identification issues in a panel data context, see Chamberlain [1984].
that control for the potential failure of the fixed effects specification. Introducing either state-specific time trends, or using differences rather than fixed effects, yields a reversal in sign of the coefficient of interest, showing a positive relationship between taxes next year and consumption this year.

We therefore suggest an alternative test of forward-looking behavior that is consistent in spirit with the test employed by BGM, but improves on the problems noted above. In particular, we have collected from state legislative histories since 1973 the date of the legislative enactment of state excise tax increases, and the date that they were actually effective. By examining cigarette consumption in the intervening period, we can test for forward-looking behavior. If individuals are forward-looking, but cannot forecast taxes beyond already announced tax increases, then this provides a more appropriate framework for examining adjacent complementarity.

We summarize the information on these tax changes in Table I, which shows the period of time between the enactment and effective dates of state excise tax increases. Over the full 1973–1996 period, 36 tax changes were enacted and effective in the same month, and 44 in consecutive months. Yet 68 tax changes

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>LENGTH OF TIME BETWEEN ENACTMENT AND EFFECTIVE DATES OF EXCISE TAX INCREASES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Same month</td>
<td>36</td>
</tr>
<tr>
<td>Consecutive months</td>
<td>44</td>
</tr>
<tr>
<td>1 month between</td>
<td>23</td>
</tr>
<tr>
<td>2 months between</td>
<td>27</td>
</tr>
<tr>
<td>3 months between</td>
<td>9</td>
</tr>
<tr>
<td>4 months between</td>
<td>6</td>
</tr>
<tr>
<td>5 months between</td>
<td>2</td>
</tr>
<tr>
<td>7 months between</td>
<td>1</td>
</tr>
<tr>
<td>Multiple changes, &lt; 1 year between</td>
<td>4</td>
</tr>
<tr>
<td>Multiple changes, &gt; 1 year between</td>
<td>8</td>
</tr>
</tbody>
</table>

Source is authors’ tabulations of data on state excise tax enactment and effective dates. Each row shows number of tax changes with the noted length of time between enactment and effective dates. Last two rows refer to tax events where multiple future changes are enacted at the enactment date.
had at least one month between the enactment and effective dates. The longest gap between enactment and effective dates was seven months.

In addition, there were a number of examples of multiple tax increases that were enacted on the same date. The first such change is included in the first eight rows of the table. The last two rows show that there were twelve second or third changes from such multiple change examples, and that eight of them were effective more than one year after being enacted. In the empirical work below, we only use the first effective date for such changes.

We have collected two sources of data on cigarette consumption to test for anticipatory responses. The first is the monthly series of tax-paid cigarette sales that underlies the annual data used by BGM and others. This was constructed using data from state excise tax collections, as archived at the Tobacco Institute and the North Carolina State University. As with the annual data, these represent withdrawals from wholesale distributors, since this is the point at which the excise tax is paid. We have collected these data from January 1982 through December 1996, with the exception of September and October 1982, for which data were not available. As noted above, however, and as will be documented further below, it is problematic to use data on cigarette sales to test for anticipatory price responses of consumption.

At the same time, data on cigarette consumption for this purpose must meet a difficult criterion, as they must provide state by month observations of enough size so as to form reasonable proxies for cigarette consumption. Fortunately, there is a data set that meets this condition: the Vital Statistics Detailed Natality Data Files [National Center for Health Statistics, various years]. Since 1989, this database has recorded, for every birth in America, whether the mother smokes and how much, as well as the state of residence of the mother. As a result, there are roughly 4 million observations per year on smoking behavior, providing sufficient sample size to measure state by month smoking rates; in our final database, the typical state month cell has 5320 observations. This is clearly not a representative population, but it is a population of particular interest, since maternal smoking and poor subsequent infant health are perhaps the leading externality associated with smoking behavior [Evans, Ringel, and Stech 1999].

We use the full set of 1989–1996 Natality files to measure monthly smoking rates for every state for which the smoking
information was collected. The key question of interest asks women about smoking during pregnancy; we assume that these women are answering with reference to the month of birth. To the extent that they are answering with reference to smoking anytime during pregnancy, we will underestimate the responsiveness to future increases.

Our dependent variable is the number of cigarettes smoked each day per woman in each state/month cell, which is formed by dividing total cigarettes smoked per day among smoking women by the total number of women in the cell. As Table II shows, the weighted (by number of births in a cell) mean of this variable over the 1989–1996 period is slightly less than 2. This implies that per capita monthly consumption of (20-cigarette) packs of cigarettes is about 3. This compares with per capita annual packs of cigarettes sold in each state are from authors’ tabulations of state-reported tax paid cigarette sales; population is annual population by state from the Census Bureau, and is available at http://www.census.gov/population/www/estimates/st_stts.html Data for Natality sample are tabulated by the authors from natality data described in text. Standard deviations are in parentheses.

rettes sold, from our monthly sales data, of 9. The fact that this figure is lower is not surprising, as the women in this sample are less likely to smoke than the typical person, and they smoke less intensively when they do smoke. Overall, as we show in Table II, the smoking rate for our sample of women is 16.2 percent, and those who smoke consume on average only about two-thirds of a pack per day. Averaging over all smokers over age 18 for 1989–1996, using data from the Behavioral Risk Factor Surveillance System data, the average smoking rate is 23 percent, and the average cigarettes smoked per day per smoker is 18.7.

One concern with these data is that mothers may underreport smoking while pregnant. While we cannot definitively address this concern, it is noticeable that the smoking participation rate in these data is almost exactly the same as that from a National Health Interview Survey supplement in 1991 which provides a retrospective survey of women on their smoking while pregnant. So there does not appear to be any systematic underreporting on birth certificates relative to these NHIS data. Moreover, underreporting would not lead to a systematic bias to the estimates unless it is somehow correlated with price changes, which seems unlikely.

II.1. Empirical Strategy

Our empirical strategy is straightforward. We run regressions of the form,

\[ SMOK_{sm} = \alpha + \beta \times EFFECT_{sm} + \gamma \times ENACT_{sm} \]
\[ + \delta \times M_{m} + \phi \times S_{s} + \epsilon, \]

where \( SMOK \) is the measure of smoking in state \( s \) in month \( m \); \( EFFECT \) is the effective tax rate in that state and month; \( ENACT \) is the enacted tax rate in that state and month; and \( M \) and \( S \) are full sets of month (we include dummies for each calendar month in our sample period) and state dummies, respectively. \( ENACT \) is the same as \( EFFECT \) except when a change has been enacted and not yet effective, so this is our future price variable. For these regressions we exclude both the months in

6. The NHIS supplement data indicate that 20.6 percent of women smoked at some time during their pregnancy, and 16.6 percent smoked throughout the pregnancy. The fact that the latter figure so closely matches our data provides further suggestive evidence that women are responding to this question with reference to the month of birth.
which tax changes are enacted and they are effective, since both of these events can happen at any time during the month, so that the response in that month may be quite muted. In some specifications, we also include a lagged value of the effective rate; we use a twice lagged tax rate, since we are excluding the month of the tax change. All natality data regressions are weighted by the size of the cell to reflect sampling variability in our aggregation strategy.

Note that we use everywhere taxes, and not prices, to test for forward-looking behavior, because price data are not available on such a high frequency basis by state. A legitimate question is then whether a finding of anticipatory behavior reflects anticipation by consumers or producers; if cigarette prices in a state are increased in anticipation of tax changes, then demand may be falling through the standard law of demand. This issue is raised by Showalter [1999], who finds no evidence of anticipatory pricing at an annual frequency.

While monthly data on cigarette prices are not available, there are quarterly data from the American Chamber of Commerce Research Association [ACCRA] on prices for selected cities of a carton of Winston cigarettes. We have used these data to investigate anticipatory pricing for tax changes where the announcement of the price increase is in a different quarter than the implementation. We found no evidence of sizable price increases before the tax was actually implemented. For example, the state of Alaska enacted a 70 cent tax increase at the end of May 1997, which was to be implemented in October. Yet the price of cigarettes rose by only 4 cents in the third quarter before rising by 88 cents in the fourth. Similarly, Michigan at the end of 1993 enacted a 50 cent tax increase which was to be implemented in May. There was only a 5 cent per pack price increase from the end of 1993 through the second quarter of 1994, and then a 50 cent increase in the third quarter. These findings suggest that producers are not increasing prices in anticipation of state-specific excise tax increases. This supposition is confirmed below by the fact that we find sales dramatically increasing in anticipation of tax increases, which would not occur if prices had already risen.

Finally, as mentioned above, a central issue for interpretation of this test is the potential for omitted variables which are correlated with both taxes and cigarette consumption, even in a state fixed effects specification. But a key difference between our analysis and that of BGM is that we are focusing on a very high
frequency (monthly) relationship between changes in smoking and changes in taxes. We strongly suspect that any omitted factors operate over a longer time frame; for example, it seems very unlikely that state excise tax decisions are responding to smoking rates in the very recent past. But we will nevertheless subject our finding to the same set of tests that we applied to the BGM results to demonstrate that it is robust.

Thus, to summarize, our test remedies the deficiencies of earlier work in several ways. First we rely only on tax increases that have already been announced to identify our anticipatory effect. Second, we use data on actual cigarette consumption, rather than sales. Third, we use information on tax changes, not price changes. Finally, we examine very high frequency changes which are unlikely to suffer from the type of bias that hinders testing for forward-looking behavior in annual data.

### III. A New Test—Results

The results of using these two data sources to examine the impact of future tax increases are presented in Table III. We begin with the packs/capita aggregate sales data. We find from the data a strong negative effect of the current effective tax rate,
with a price elasticity of $-0.8$. This elasticity is much higher than that found by BGM. The difference appears to arise from the fact that the tax-induced movement in prices causes larger consumption declines than does the price-induced movement in prices, and from higher elasticities estimated with more recent data.

In the next row, we include the future tax change term, as well as lagged effective taxes. In fact, the coefficient on the announced rate is actually \textit{positive} and highly significant. At a monthly frequency, such a positive reaction to future price increases is sensible, as consumers hoard cigarettes at lower prices for future use. This hoarding effect is consistent with the evidence in Keeler et al. [1993], who find cigarette sales rising in the months before a 1989 excise tax increase in California. Indeed, we see this response in our data, for the large increase in the excise tax from 10 to 35 cents in California that was announced in November 1988 and effective in January 1989. In November, cigarette sales were just slightly down from what they had been the previous November, at 6.68 packs per capita. Then, in December, sales jumped to 8.71 packs per capita, before falling back to around 6 over the next few months.

This sizable hoarding effect could be taken as one type of forward-looking behavior by consumers, in that they are stocking up in anticipation of a tax increase. It is not clear how much of the hoarding effect we find is due to consumer versus retailer behavior, as some state excise tax increases exempt floorstocks held by retailers when the tax changes. For a sample of the twelve largest tax changes in recent years, we found in the legislation or through contact with state taxation officials that in ten of twelve cases floorstocks were included, so that any hoarding effect would be due to consumers. In either case, this sizable hoarding effect casts doubt on the usefulness of sales data for testing for anticipatory consumption behavior.

We also find that including the enacted and lagged rate significantly increases the term on the current price, which now implies an elasticity of $-1.5$, with a sizable and significant positive elasticity on the lag as well as the lead. The positive impact of the lagged rate, and the large value of the current tax rate, no

\footnote{The elasticities presented for the tax coefficients are price elasticities, which are evaluated by first estimating models of price as a function of tax, and using the resulting coefficients to estimate pass-through; the coefficients imply pass-through of taxes to prices of roughly 110 percent.}
doubt reflects monthly timing of purchases in our data: if individuals are hoarding in the months before a tax change, then sales will fall most sharply in the month of the change, before rising again somewhat thereafter. This is what we see in the second column of Table III: a rise in sales in the months before a change, a sharp decline in the month of the change, and then an offsetting increase thereafter.

The next two columns of Table III consider the impact of effective and enacted taxes in our natality data. Here, when we just include the effective tax rate term, we find a much smaller impact of taxes, with a price elasticity of just $-0.35$. This is consistent with the notion that women who are still smoking at the time that they are giving birth may be less sensitive to economic factors such as prices. Nevertheless, the coefficient is highly significant, so that if there is an anticipatory response we should be able to estimate it with these data.

When we include the enacted tax rate as well in the next row of Table III, the coefficient on the enacted rate is in fact negative and significant. Thus, using this more refined test, we find strong evidence of forward-looking behavior, for this population at least. Including the enacted and lagged rate leads in this case to a sizable fall in the current tax coefficient, with a relatively (but insignificant) lag term. The small coefficient on the effective rate presumably reflects some lag in adjusting to price changes, as is reflected in the large lag term. Thus, the results indicate that the response to a tax next period is equal to the sum of the current and lagged response to a current tax.\(^8\)

### III.1. Specification Testing

It is of course important to subject our estimates to the same scrutiny to which we subjected the BGM results. Table IV therefore includes fixed trends in our model. For the sales data, doing so lowers the impact of current effective rates, with an implied

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8. Over the time period on which we focus, in addition to the numerous tax changes there were also other public policy interventions to reduce smoking, most notably a series of state “clean air” laws that restricted smoking in particular public places. If the passage of these clean air laws is correlated with tax changes, it could confound our results. However, controlling for the presence of various categories of clean air laws (using data described in Gruber [2000]) makes little difference to our results. The impact of these laws themselves is mixed: for total packs sold, we find that restrictions on smoking in government workplaces and restaurants significantly lower smoking; but for cigarette consumption by mothers, we find instead that restrictions in private workplaces and other sites (such as buses, supermarkets, etc.) significantly lower smoking.
elasticity of $-0.62$ from the first column of Table IV. Once again, we find that when the enacted and lagged tax rates are entered, the current tax effect rises, and the enacted and lagged tax rates are both significant and positive.

For the natality data, the results are also similar when fixed trends are added. We find that the impact of the effective tax rate when entered alone (the third column of Table IV) is weakened, with an elasticity now of $-0.27$. And we once again find that there is a significant negative impact of enacted but not yet effective taxes; the effect is roughly 80 percent as large as in the fixed effects model. We find here as well that there is virtually no instantaneous impact of effective tax changes on consumption, with most of the effect showing up with a lag. Once again, the current plus lagged effective tax coefficients are roughly equal to the enacted tax coefficient. Thus, unlike the earlier results from
BGM (and presumably from other tests using annual data), our findings are relatively robust to the inclusion of fixed trends.

We have also considered differenced models. Here, we cannot simply difference our monthly panel, since it would imply that for tax changes with several months between enactment and effective dates, the difference in the enacted rate would be zero after the first month. Therefore, we have pursued an alternative approach of taking the average smoking level over all months between the enactment and effective dates, and taking the difference between this average and the smoking level in the month before the tax change was enacted. We also include in these differenced regressions, as controls, any months where there were no tax changes enacted in that month or in the two months before; we then take the difference between smoking in that month and smoking two months earlier as a control observation. Our true enactment differences are matched to the change in the enacted rate (the change from the old effective rate to the new enacted rate), and our control observations are matched to a tax change of zero. The regression includes a full set of month dummies.

The results of this exercise, for both packs per capita and cigarette consumption from the natality data, are presented in the final row of Table IV. We find very similar results to the fixed trends specification, albeit with slightly larger standard errors.

One additional concern with our finding is that it reflects not anticipatory responses to future price changes, but rather changes in reporting in the wake of announced tax increases. That is, it is possible that when future price increases are announced, women become more exposed to antismoking sentiment and are thus less likely to report that they smoke. While this concern is impossible to address precisely, we have investigated it in two casual ways. First, for the two announced increases in Massachusetts in 1992 and 1996, both of which had roughly two months between the enacted and effective dates, we examined the major newspapers in the Boston area for any evidence of increased antismoking counteradvertising. We found no such advertising in the intervening months.

Second, for the differenced models presented in Table IV that consider how smoking changes when a tax rate is announced, we have included along with the differenced tax rate a dummy for the presence of a tax change. If it is the announcement of the tax change per se that matters, and not the price change, then the inclusion of this dummy should significantly weaken our differ-
ences relationship. In fact, however, there is no impact on our coefficient when the dummy is included (although the standard error does rise by about 50 percent), and the dummy itself is not significant. This test is not definitive, of course, because anti-smoking rhetoric could be proportional to the size of the tax change. But it certainly suggests that it is not the presence of a tax change per se, but rather the future rise in price, that is causing women to reduce their smoking.

Thus, to summarize, we have provided a more robust framework for testing for anticipatory responses by consumers to future changes in the taxation of cigarettes. Even in this more robust framework, we continue to find evidence of adjacent complementarity. This does not, however, necessarily provide support for Becker and Murphy's formulation of the smoking decision, as we document in the remainder of the paper.

IV. TIME-INCONSISTENT PREFERENCES AND ADDICTION

The term “rational addiction” obscures the fact that the Becker-Murphy model imposes two assumptions on consumer behavior. The first is that of forward-looking decision-making, which is supported by the evidence above, and which will be a key feature of our alternative models as well. But the second is the assumption that consumers are time consistent. Psychological evidence documents overwhelmingly that consumers are time inconsistent [Ainslie 1992]. In experimental settings, consumers consistently reveal a lower discount rate when making decisions over time intervals farther away than for ones closer to the present, raising the specter of intrapersonal conflict over decisions that have implications for the future. In this section we will develop a time-inconsistent alternative to the Becker-Murphy model. We will first show that our alternative also yields forward-looking behavior in smoking decisions; indeed, it has been impossible to derive sharp empirical tests to distinguish our model from Becker and Murphy’s. But, as we show in the final section, this time-inconsistent alternative implies radically different government policy prescriptions.

9. The above type of time inconsistency has been recently applied in the context of savings decisions [Laibson 1997; Laibson, Repetto, and Tobacman 1998; O'Donoghue and Rabin 1999b], retirement decisions [Diamond and Köszegi 1998], and even growth [Barro 1999].
IV.1. Motivation

There is, to date, little nonexperimental evidence for time inconsistency in decision-making. But it is important to note that there is no evidence, psychological or other, that supports time-consistent preferences over these time-inconsistent ones. And since smoking is a short-term pleasure, and the psychological evidence indicates that time inconsistency is most prevalent with short horizons, this formulation should be especially fruitful in the context of addictive bads such as smoking.

There is also indirect evidence that people’s preferences for smoking are time inconsistent. Two key features distinguish time-consistent and time-inconsistent agents. The first is the use of commitment devices or self-control techniques. We distinguish a self-control device from an alternative technology for smoking cessation, quitting aids: whereas quitting aids decrease the disutility from not smoking, self-control devices lower the utility from smoking. Time-consistent decision-makers might use a quitting aid, but in general they will not use a self-control device— with time consistency, lowering the utility of an undesired alternative is irrelevant for decision-making. But for some types of time-inconsistent agents (what we label below sophisticated agents, who recognize their own time inconsistency), self-control devices are valued as a means of combating one’s own time-inconsistent tendencies.

In the relatively small medical literature on self-initiated attempts at quitting smoking, the voluntary use of self-control devices figures prominently. People regularly set up socially managed incentives to refrain from smoking by betting with others, telling them about the decision, and otherwise making it embarrassing to smoke [Prochaska et al. 1982]. Various punishment and self-control strategies for quitting are also widely studied in controlled experiments on smoking cessation [Miller 1978; Murray and Hobbs 1981] (see Bernstein [1970] for a variety of “aversive stimulus” techniques), and they are recommended by both academic publications [Grabowski and Hall 1985] and self-help books [CDC]. In one study, for example, subjects tore up a dollar bill for every cigarette they smoked above their given daily limit, and reduced that limit gradually. Presumably, these experiments are incorporating self-control devices because they are seen as the best option for helping individuals quit smoking, as could be the case if individuals were time inconsistent.
A second feature that distinguishes time-consistent agents from time-inconsistent agents is an inability to actualize predicted or desired future levels of smoking. The former phenomenon is specific to a class of hyperbolic discounters whom we label naive below, in that they do not understand that they cannot make consistent plans through time.

In fact, unrealized intentions to quit at some future date are a common feature of stated smoker preferences. According to Burns [1992], eight of ten smokers in America express a desire to quit their habit. Unfortunately, these desires can be interpreted in a number of ways, and we are not aware of any evidence for adults on their specific predictions or intentions about future smoking behavior. For youths, however, there is clear evidence that they underestimate the future likelihood of smoking. For example, among high school seniors who smoke, 56 percent say that they will not be smoking five years later, but only 31 percent of them have in fact quit five years hence. Moreover, among those who smoke more than one pack per day, the smoking rate five years later among those who stated that they would be smoking (72 percent) is actually lower than the smoking rate among those who stated that they would not be smoking (74 percent) [U. S. Department of Health and Human Resources 1994].

Less forceful, but still suggestive, evidence for naive time inconsistency comes from attempted quits. According to Harris [1993], 38 of the 46 million smokers in America in 1993 have tried to stop at one point or another, with an average smoker trying to quit once every eight and a half months. Most have tried several times. Fifty-four percent of serious attempts at quitting fail within one week. These facts do not necessarily contradict a time-consistent model that incorporates learning or uncertainty, since smokers might experiment with quitting to find out how hard it is or simply “gamble” in the hope of stumbling on an instance when it is easy. But it seems implausible that smokers learn so slowly or that the situations in which they try quitting are so variable.

IV.2. The Model

To carry out the above task, we take the important insights about forward-looking behavior captured in the Becker-Murphy model, and integrate them with a potentially more realistic description of intertemporal choice in this context. The crucial question we are dealing with is the shape of time discounting. Suppose
that we are in a \( T \)-period decision model. For a time-consistent agent, discounted utility at time \( t \) takes the familiar form,

\[
(2) \quad \sum_{i=0}^{T-t} \delta^i U_{t+i},
\]

where the \( U_{t+i} \) denote the instantaneous utilities. We will contrast this type of discounting with the alternative developed by Laibson [1997], quasi-hyperbolic discounting. For quasi-hyperbolic discounters, discounted utility becomes

\[
(3) \quad U_t + \beta \sum_{i=1}^{T-t} \delta^i U_{t+i}.
\]

\( \beta \) and \( \delta \) are usually assumed to be between zero and one. The extra discount parameter \( \beta \) is intended to capture the essence of hyperbolic discounting; namely, that the discount factor between consecutive future periods (\( \delta \)) is larger than between the current period and the next one (\( \beta \delta \)). At the same time, this formulation still allows one to take advantage of some of the analytical simplicity of the time-consistent model. For a more thorough introduction see Laibson [1997]. Our model marries this intertemporal preference structure with the instantaneous preferences in Becker and Murphy’s [1988] rational addiction model.\(^\text{10}\)

Let \( a_t \) and \( c_t \) denote, respectively, the consumption of the addictive and the “ordinary” (nonaddictive) goods in period \( t \). Both can take any value on the real line. Furthermore, we denote the period \( t \) stock of past consumption by \( S_t \). \( S_t \) evolves according to

\[
(4) \quad S_{t+1} = (1 - d)(S_t + a_t).
\]

\( d \) is the depreciation rate of the stock; the higher is \( d \), the less does past behavior influence the stock of accumulated consumption, and thus, indirectly, utility.\(^\text{11}\)

10. We deviate from both Becker and Murphy and Laibson, however, by assuming no savings—some exogenously given income is consumed in each period. In practice, there is no role for savings in the execution of the Becker-Murphy model either: they hold the marginal utility of wealth constant when analyzing price changes, which serves the same role as our quasi-linear utility function and no-savings assumption. In addition, low savings among the low income population that is most likely to smoke renders this assumption relatively innocuous.

11. For notational and arithmetic simplicity, this differs from Becker and
We assume, as in Becker and Murphy [1988], that instantaneous utility is additively separable in these two goods; that is,
\begin{equation}
U_t = U(a_t, c_t, S_t) = v(a_t, S_t) + u(c_t).
\end{equation}
\(v_{as}(a_t, S_t)\) is positive, because consumption of addictive goods generally increases their future marginal utility. Let \(I_t\) be period \(t\) income and \(p_t\) the period \(t\) price. We normalize the price of the nonaddictive good to be 1.

We consider both agents who discount exponentially (equation (2)) and who discount quasi-hyperbolically (equation (3)). One can distinguish between two extreme kinds of hyperbolic discounter agents. Naive agents, although they are impatient in the sense that they attach extra value to the current period relative to the future ones, are unaware of their future self-control problem: self \(t\) does not realize that self \(t + 1\) will in turn overvalue period \(t + 1\). Thus, a naive agent maximizes her intertemporal utility in expression (3), unconscious of the fact that her future selves will change her plans. Sophisticated agents, on the other hand, realize their self-control problem: self \(t\) knows that self \(t + 1\) will want to do something other than what self \(t\) would have her do. Therefore, the best thing self \(t\) can achieve is to make a plan that she will actually follow. Formally, this is modeled as a subgame-perfect equilibrium in a game played by the successive intertemporal selves, the action spaces in our case being the vectors of consumption \((a_t, c_t)\). In our setting, the two kinds of hyperbolic discounters behave quite similarly, so we will only discuss sophisticates in detail. In general, however, sophisticates and naifs behave differently in a number of important ways. See O'Donoghue and Rabin [1999a] for an excellent discussion of sophistication and naiveté, as well for a few basic behavioral contrasts between the two.

IV.3. Time-Consistent Agents

Standard methods reveal the following Euler-equation for time-consistent agents. The most natural way to think about it is that a small perturbation in consumption in period \(t\) that is undone in period \(t + 1\) does not change utility. In contrast to a simple savings problem, however, we also have a \(v_s(a_{t+1}, S_{t+1})\)

Murphy [1988], who have \(S_{t+1} = (1 - d)S_t + a_t\). As long as depreciation is not full \((d < 1)\), their model and our time-consistent case are isomorphic through a simple change of variables.
term in the Euler equation, because a change in \( S_{t+1} \) affects utility directly, whereas in a savings problem wealth does not.

**Lemma 1.** Suppose that \( u(c_t) \) and \( v(a_t, S_t) \) are differentiable. Then, for a time-consistent agent the following Euler-equation holds:

\[
(6) \quad v_a(a_t, S_t) - p_t u'(c_t) = (1 - d) \delta [v_a(a_{t+1}, S_{t+1}) - p_{t+1} u'(c_{t+1}) - v_s(a_{t+1}, S_{t+1})].
\]

**IV.4. Sophisticated Hyperbolic Discounters**

Now we move on to the more difficult problem, the problem for sophisticated agents.

**Lemma 2.** Suppose that \( u(c_t) \) and \( v(a_t, S_t) \) are differentiable and that a Markov-perfect subgame-perfect equilibrium with differentiable strategy profiles exists. Then, for each \( t \in \{0, \ldots, T - 1\} \) we have

\[
(7) \quad v_a(a_t, S_t) - p_t u'(c_t) = (1 - d) \delta \left[ \left( 1 + (1 - \beta) \frac{\partial a_{t+1}}{\partial S_{t+1}} \right) \times (v_a(a_{t+1}, S_{t+1}) - p_{t+1} u'(c_{t+1})) - \beta v_s(a_{t+1}, S_{t+1}) \right].
\]

**Proof of Lemma 2.** See Appendix 1.

**V. Solving the Models**

Following Becker and Murphy [1988], we take \( v(a_t, S_t) \) and \( u(c_t) \) to be quadratic:

\[
(8) \quad v(a_t, S_t) = \alpha_a a_t + \alpha_s s_t + \frac{\alpha_{aa}}{2} a_t^2 + \alpha_{as} a_t s_t + \frac{\alpha_{ss}}{2} s_t^2,
\]

\[
(9) \quad u(c_t) = \alpha_c c_t + \frac{\alpha_{cc}}{2} c_t^2,
\]

where \( \alpha_a, \alpha_{as}, \) and \( \alpha_c \) are positive and \( \alpha_s, \alpha_{aa}, \alpha_{ss}, \) and \( \alpha_{cc} \) are negative. The key parameter is \( \alpha_{as} \), which measures the effect of past consumption on the marginal utility of current consumption. \( \alpha_{as} > 0 \) means that if you had done more drugs in the past, you will crave them more in the present. This is what can give rise to addictive behavior. The physiological evidence that \( \alpha_{as} \) is positive for many goods is overwhelming. For most of this paper, we will
take $U(a_t, c_t, S_t)$ to be strictly concave; that is, we suppose its Hessian is negative definite.

In this case, for both the exponential and hyperbolic discounting models, it is very easy to prove by backward induction that $a_t$ is linear in $S_t$, that is, $a_t = \lambda_t S_t + \mu_t$, where $\lambda_t$ and $\mu_t$ are constants. The following theorem helps establish that for a general class of parameter values, marginal propensities to addiction are stationary for both types far from the end of the horizon.

**Theorem 1.** Suppose that $\beta \geq 1/2$, $U(a_t, c_t, S_t)$ is strictly concave, and $p_t = p$, a constant. Then, $\lim_{j \to \infty} \lambda_{T-j} = \lambda^{*s}$, where $\lambda^{*s}$ is given as the unique solution on the interval $(-1, (\alpha_{as}/(-\alpha_{aa} - p^2\alpha_{cc})))$ of

$$
\lambda^{*s} = -1 + \frac{\alpha_{as} - \alpha_{aa} - p^2\alpha_{cc}}{-\alpha_{aa} - p^2\alpha_{cc} + \delta(1-d)^2[(1 + (1 - \beta)\lambda^{*s})}
\times (\alpha_{ao}\lambda^{*s} + \alpha_{as} + p^2\alpha_{cc}\lambda^{*s}) - \beta\alpha_{as}\lambda^{*s} - \beta\alpha_{ss}] .
$$

Furthermore, $\lambda^{*s} > 0$ (that is, there is adjacent complementarity) if and only if

$$
\alpha_{as} > \frac{\beta\delta(1-d)^2}{1 - \delta(1-d)^2} (-\alpha_{ss}).
$$

*Proof of Theorem 1.* See Appendix 1.12

Setting $\beta = 1$ in the above expression gives the implicit expression for the marginal propensity to respond to the stock for time-consistent agents, $\lambda^{*TC}$.

We are interested in the responses of different kinds of agents to price responses occurring at different dates. For this, we will assume that $\alpha_{cc} = 0$, thereby eliminating income effects, which are probably very small for small price changes in many addictive goods. We also assume constant income, $I_t = I$. We will assume for much of what follows that both models exhibit adja-

12. Notice that one of the assumptions of the theorem is $\beta \geq 1/2$. If this is not the case, it seems possible (although we conjecture it will not usually be the case) that the agent exhibits wild cyclical behavior characterized by periodic binges and brutal cuts. However, most of the psychological literature points to a $\beta$ above one-half, at least for the time period we consider most relevant for time inconsistency in smoking decisions, a few weeks or few months. For small rewards, a weekly discount rate of 10 to 30 percent seems reasonable [Kirby and Herrnstein 1995]; this implies that $\beta$ is at least 0.7 (and even higher if $\delta$ is less than one over this period as well). Thaler [1981] finds monthly discount rates on the order of 20 to 30 percent, and three-monthly discount rates of up to 50 percent. The evidence reviewed by Ainslie [1992] indicates that yearly discount rates are about 40 percent, and $\beta = 0.6$ is the estimate used by Laibson [1997].
cent complementarity. We want the problem to be well-behaved, that is, for \( \mu_T - j \) to converge for both types as \( j \to \infty \), so that consumption rules are approximately stationary far from the end of the horizon. For time-consistent agents, this is simple, and a precise proof is contained in Appendix 2.

The strategic nature of sophisticates’ consumption decisions complicates the analysis in that case. For small \( \beta \), the model can exhibit some “violent” characteristics with respect to price changes. For example, when the price increases permanently, the current self knows that this will act as a deterrent for future selves, decreasing her need to control her addiction now. This could lead her to increase consumption drastically. Since a drastic increase in consumption in response to a current price increase sounds implausible, we make sufficient assumptions in Appendix 2 to rule out this possibility.13

Under these conditions, it is easy to derive responses to different price changes for the two types. We do so in Appendix 2 and summarize some responses to permanent price increases in Appendix 3. From this appendix it is clear that as long as the good is sufficiently addictive, both types respond to a future price increase by decreasing consumption. In particular, for both types, the knowledge (or expectation) that future selves will decrease their consumption decreases the marginal utility of consumption today due to the complementarity of intertemporal consumption levels (the “make quitting easier” effect).14 Therefore, Becker, Grossman, and Murphy’s [1994] test cannot distinguish the rational addiction model from alternatives such as ours.

In principle, the price responses to changes at different points in the future can be used to back out the parameters \( \beta \) and \( \delta \) using the formulas in Appendix 3, thereby allowing us to assess the degree of time inconsistency in smoking decisions. Even more straightforward comparisons of this nature can be used to test time inconsistency per se, as we describe in Gruber and Köszegi [2000]. In practice, however, we were unable to carry out these

13 After doing this, there are still phenomena of this nature that look more reasonable. For example, we might observe “yuppie binges:” that before a big project or a new job, many normally restrained people get wasted on alcohol or high on drugs, only because they know that their job is important enough for them not to keep up with the habit permanently.

14 An effect going the other way is a substitution effect: one wants to shift consumption toward times when it is cheaper, holding the cumulative effect on the stock constant. For time-consistent agents, the former effect dominates iff the good is addictive, \( \lambda^{*T_C} > 0 \). For quasi-hyperbolic discounters, one needs stronger addictivity for the former to dominate.
tests. Even grouping together all of the months between enactment and effective dates, we obtain an estimate of the price response which is only 2.3 times its standard error. It is therefore impossible to break down this period into the smaller windows required to carry out these tests; the estimates for windows of different lengths (e.g., price change in one versus two periods ahead) are simply too imprecise to permit comparison to each other. Future work with more precise data can perhaps implement these suggested tests to assess the shape of discounting.

In any case, the purpose of this section is once again to emphasize that this alternative formulation of the addiction model yields predictions for forward-looking behavior that are virtually identical to those of the rational addiction model. Thus, past “tests” of the rational addiction model are not robustly testing that model versus our own. Yet, as we demonstrate next, these models have radically different implications for government decision making.15

VI. OPTIMAL GOVERNMENT POLICY

A key implication of the rational addiction framework for modeling addiction is that government regulatory policy toward addictive goods should depend only on their interpersonal externalities. Just as the government has no cause, absent market failures, for interfering with revealed preference in the realm of nonaddictive goods, there is no reason to take addictiveness per se as a call to government action, if individuals are pursuing these activities “rationally.” It is this framework that underlies the well-known efforts of Manning et al. [1991] and others to formulate optimal taxation of cigarettes and alcohol as a function of the size of their external costs. These estimates, which are frequently cited and influential in debates over excise taxation,

15. O’Donoghue and Rabin [1997] formulate a related model of the consumption decision of time-inconsistent agents for addictive goods. In contrast to ours, their setup allows for two consumption choices, hit or not hit; the level of addiction, in turn, can also take on two values: hooked or not hooked, and the agent is hooked if she hit last period. This discreteness assumption generates important differences from our model. If the discrete model is extended to include prices, it turns out that neither of the types will ever quit in response to a future price change. This is because quitting is a one-time decision (not a smooth decline in consumption as in the continuous model), so that one might as well wait until the price change to quit—quitting earlier would not be any easier. Discreteness also makes it difficult to use the model for optimal tax analysis, our next agenda.
suggest that the optimal tax rate for cigarettes in particular is fairly low, since the net external costs of smoking are small.

But models with time-inconsistent agents extend the role of government policy by breaking down revealed preference concepts of consumer choice. The argument that people act in their best interests, so—barring well-known qualifications—the government should leave them alone, is immediately invalidated in our setting. Therefore, although our models are explicitly of the no-externality type, a benevolent social planner would want to intervene in this economy.

Of course, the question arises why we consider only government interventions to combat self-control problems. If a sophisticated agent had access to an effective private self-control device, she would take advantage of it, reducing the value of a government intervention. However, we find it unlikely that fully effective self-control devices can be found in this context. Market-provided self-control mechanisms are probably undercut by the market mechanism itself: although firms have a financial incentive to provide self-control to agents, other firms have a financial incentive to break it down. For example, if a firm developed a self-control shot that causes pain when the consumer smokes, another firm has an incentive to develop a drug that relieves these effects for agents who temporarily want to get rid of their commitment. Other problems arise in contracting setups. If there are ex post gains to be made, the future self might want to renegotiate today’s contract. But even if there are none, there is an ex post incentive to cheat on the contract: smoking is hard to verify in court. This leaves us with privately provided self-control mechanisms like betting with others or becoming involved in situations where it is very difficult to smoke, but these mechanisms are likely to run into enforcement problems similar to those discussed above.

VI.1. Setup

As in any model where different socially relevant actors have different tastes, a discussion of optimal government policy must start with the setup of the social welfare function. In the context of hyperbolic discounting, these actors are not separate individuals, but different intertemporal incarnations of the same indi-

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16. At least in the interpersonal sense. One might look at the intrapersonal conflicts that are generated by a hyperbolic model as intrapersonal externalities.
17. For example, the agent could post a bond, which she loses if she smokes.
individual. The question of social welfare maximization in such a situation has largely been ignored, so we face the difficult problem of specifying the social preferences to be used for our purposes. For this application, we take the agent’s long-run preferences as those relevant for social welfare maximization. Clearly, if the representative agent were to vote in a tax change today that is instituted starting tomorrow, these are the preferences she would use in choosing the new tax rate.

We consider the case of a representative consumer with a very long life and a social planner restricted to a tax on the addictive good that is invariant over time. With the income effect absent, a tax shifts consumption in the same direction for all levels of addiction, so it cannot affect $\lambda^{*s}$ or $\lambda^{*TC}$, which measure the slope of consumption with respect to the stock. However, since the constant in the consumption function ($\mu^{*s}$ or $\mu^{*TC}$) depends linearly on the price, the social planner can essentially choose these constants by setting the appropriate tax. We assume that any tax is fully passed through to consumers, and that revenues are lump-sum redistributed to the representative consumer in each period. This implies that in the social planner’s optimization problem, the relevant price is the pretax price $p$—the consumer ends up spending the tax receipts on the nonaddictive good. Thus, the social planner solves

$$\max_{\mu} \sum_{t=0}^{\infty} \delta^t [v(\lambda^{*s}S_t + \mu, S_t) + \alpha(I_t - p(\lambda^{*s}S_t + \mu))]
$$

Subject to

$$S_0, S_{t+1} = (1 - d)(S_t + \lambda^{*s}S_t + \mu),$$

Although the posttax price $p + \tau$ does not explicitly appear in the maximization, it is implicitly there, as $\mu$ depends on it.

The above is a quadratic in $\mu$ with a negative prime coefficient, so the first-order condition gives the optimal tax. One can think of the derivative of (12) as a sum of the derivatives with respect to each period’s consumption. Then the first-order condition for the optimal choice of $\mu$ is

18. In the language used to describe the policies, we will assume throughout that the addiction in question is harmful in the no-tax setting; that is, consuming more today reduces future discounted utility. Because of the quadraticity of the utility functions, this is not a necessary consequence of our model: for any $a_t$, there is a region where utility is increasing in $S_t$. The results for these beneficial addictions should be symmetric to those below.

19. Otherwise the maximum utility would be infinite, a nonstarter.
Combining this equation with the first-order condition for sophisticates, and rearranging, we obtain

\[
(14) \quad (1 - \beta) \sum_{t=0}^{\infty} \delta^t(v_a(a_t, S_t) - (p + \tau)\alpha_c) = \beta \frac{1}{1 - \delta} \frac{1 - \delta(1 - d)}{1 - \delta(1 - d)(1 + \lambda^{**})} \tau \alpha_c.
\]

Since the consumer’s optimization problem, which was used to arrive at the above expression, depends on the posttax price \( p \), this price now explicitly appears in the equation.

It is easy to show that the optimal tax is positive: the derivative of (12) at \( m = m^* \) can be written in the form,

\[
(15) \quad \sum_{t=0}^{\infty} \delta^t(v_a(a_t, S_t) - p\alpha_c + \delta(1 - d)V_s(S_{t+1})),
\]

where \( V_s(S_t) \) stands for the exponentially discounted utility from leaving stock \( S_t \) and consuming according to the sophisticated consumption function from then on. A hyperbolic discounter agent solves \( v_a(a_t, S_t) - p\alpha_c + \beta \delta(1 - d)V_s(S_{t+1}) = 0 \), and since by assumption \( V_s(S_{t+1}) \) is negative, the above derivative is negative for \( \beta < 1 \). Therefore, the optimal \( \mu \) is lower than \( \mu^{**} \), and consequently the optimal tax is greater than zero.\(^{20}\)

Note that, since the optimal tax is positive for \( \beta < 1 \), the left-hand side of the equation (14) is positive. But \( v_a(a_t, S_t) - (p + \tau)\alpha_c > 0 \) means that the addiction is harmful—higher consumption lowers utility from future periods. Therefore, at

\(^{20}\) The exact form of the social welfare function is not crucial to demonstrating that the optimal tax should be positive. It is easy to prove that a small positive tax is Pareto-improving—it increases the discounted utility of each intertemporal incarnation of the agent. To see this, note first that a small decrease in a self’s consumption causes a second-order loss to her discounted utility, while a decrease in future selves’ consumption gives a first-order gain. In addition, future selves gain through the fact that they receive a lower stock of consumption \( S \). But the form is obviously critical for calibration; we return to this issue in the calibration section.
least in an average sense, the optimal tax is not so large as to make the addiction harmless on the margin. The reason is that the tax is there to correct a marginal self-control problem. If there was no self-control problem (on average), there would be nothing to correct—the agent’s different intertemporal selves would not disagree, so the losses to consuming more would be second-order. But then, the selves would be consuming too little, since their private costs are higher than the social costs due to the tax.

In Gruber and Köszegi [2000] we note that this is not true for the case of naive hyperbolic discounters; in this case, the good could appear beneficial on average. The reason is that here the tax not only corrects a self-control problem, but also a misperception problem—the agent is wrong in predicting her future behavior. This is a very important qualitative difference in terms of optimal taxation. Whereas in the sophisticated case taxation that eliminates all harmful consumption can never be justified, even if the good is very addictive and people have severe self-control problems (low β’s), such extreme taxes may be the best policy for the naive case.21 To put it in more plain terms, a sort of “cautious” paternalism is recommended for parts of the population that realize they have a self-control problem, while a more “short-leashed” policy should apply to those who do not.

In that paper we also develop a host of additional implications of our model for government policy. Most interestingly, we note that, since smoking in different periods is complementary, taxes in different periods are substitutes. Thus, if we think the taxes are too low in certain periods of life, due to addictivity taxes should be higher than otherwise in earlier periods. Similarly, if the model is written over space, if we cannot regulate smoking in the home, we should over-regulate it in other settings such as restaurants or bars. This provides a novel rationale for both overregulation of youth smoking (if there are political constraints against regulating adult smoking) and banning smoking in public places.

VI.2.A Calibration Exercise

In this subsection we attempt to calibrate our model and calculate an actual optimal tax for sophisticated agents. To do so,

21. To be more precise, naifs will think that consuming the good is beneficial, whereas in reality it is not. Thus, naifs might say, “There is no harm in smoking this one cigarette, so why don’t you let me?,” and they would be right—if they really consumed according to their plans. Ultimately, they are not right because they do not think they’ll get addicted, and they will.
we will assume that the disutility associated with smoking is linear. Let $h_s$ denote the money equivalent of the per-period future marginal utility of an extra cigarette (so it should be negative). This accounts for the pure disutility effect of the stock, but not the impact of current consumption on future smoking decisions.

Starting from equation (14) for the optimal tax, substituting on the left-hand side using the sophisticates’ first-order condition expressed in terms of stock, and then setting $h_s = v_s(a_t,S_t)$ for each $t$ gives

$$(16) \quad (1 - \beta) \frac{\delta(1 - d)}{1 - \delta(1 - d)(1 + (1 - \beta)\lambda^{*s})} (-h_s)$$

$$= \frac{1 - \delta(1 - d)}{1 - \delta(1 - d)(1 + \lambda^{*s})} \tau.$$  

In our calibration, we will use the combined discounted damage of a cigarette in all future periods, $H_S = ((1 - d)\delta/(1 - (1 - d)\delta))h_s$. Rewriting the above expression,

$$(17) \quad \tau = \frac{1 - \delta(1 - d)(1 + \lambda^{*s})}{1 - \delta(1 - d)(1 + (1 - \beta)\lambda^{*s})} (1 - \beta)(-H_S).$$

Notice that—contrary to a Pigouvian intuition—the optimal tax is smaller than $1 - \beta$ times the marginal internality of the stock. The reason is that a sophisticated agent tries to influence her future behavior through her current consumption of cigarettes. Even under optimal taxes, a sophisticated agent feels a need to exert control on the future selves by consuming less. This effect helps the government, and therefore it is not necessary to tax the full marginal externality.

One difficulty with estimating the optimal tax is parameterizing $H_s$. Clearly, there is a lot of disutility associated with smoking that is hard to quantify, such as that from constant coughing and increased vulnerability to various illnesses. We will ignore all these, and assume that the only disutility from smoking is in the increased chance of early death. Viscusi [1993] reviews the literature on life valuation and suggests a consensus range of 3–7 million 1990 dollars for the value of a worker’s life; choosing the midpoint value and expressing it in current dollars gives a figure of $6.4 million. Presumably, this is a present discounted value for all remaining years. Making reasonable assumptions
about an average worker’s age and life expectancy, and using a
discount rate of 4 percent, we find that an extra year at the end
of a smoker’s life is worth $99,110. Finally, we use the fact that
smokers die on average 6.1 years early [Cutler et al. 2000]; this is
a mean difference in life spans between typical smokers and
nonsmokers. At these figures, the cost in terms of life years lost
per pack of cigarettes is $30.45. This is an enormous figure which
is on the order of 100 times as large as estimates of the interper-
sonal externalities from smoking.

Another serious difficulty lies in choosing the right period
length for our purposes. The quasi-hyperbolic discounting model is
only a theoretical device meant to capture the essence of hyperbolic
discounting, and is not designed for actual policy simulations. Since
our empirical analysis was done in terms of a monthly time period,
we will continue to work with this time frame, and assume that $\beta = 0.9$ and $\delta = 1$ over this time period. Our choice of $\beta$ is intended to
parameterize a modest time inconsistency problem—most of the
psychological evidence indicates that monthly discount rates are
substantially higher than 10 percent. Physiological and empirical
evidence suggests that $\lambda^{**}$ is fairly high for smoking. Evidence is less
clear on the depreciation rate.

Since the tax is quite sensitive to $d$ and $\lambda^{**}$, Table V shows
the optimal tax (in dollars) for a few values. The implied tax levels
are still very high except for a combination of low $d$ and high $\lambda^{**}$.
We have not been able to pin down $d$ and $\lambda^{**}$ empirically, so we
cannot identify the relevant cell in Table V. However, there is
information in our data that can be exploited to rule out at least
some combinations of $d$ and $\lambda^{**}$: namely, the speed of conver-
gence to a new steady state after an enacted and immediately
effective price change. A combination of low $d$ and high $\lambda^{**}$ would
imply that this convergence is very slow—even long after the

<table>
<thead>
<tr>
<th>$d$ = 0.5</th>
<th>$d$ = 0.6</th>
<th>$d$ = 0.7</th>
<th>$d$ = 0.8</th>
<th>$d$ = 0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda^{*}$ = 0.9</td>
<td>0.33</td>
<td>1.30</td>
<td>1.95</td>
<td>2.41</td>
</tr>
<tr>
<td>$\lambda^{**}$ = 0.7</td>
<td>0.98</td>
<td>1.70</td>
<td>2.19</td>
<td>2.56</td>
</tr>
<tr>
<td>$\lambda^{***}$ = 0.5</td>
<td>1.60</td>
<td>2.10</td>
<td>2.44</td>
<td>2.70</td>
</tr>
</tbody>
</table>

This table presents optimal internality taxes (in dollars) for various combinations of the rate of depreciation ($d$) and the impact of the past stock on current smoking for sophisticated hyperbolic agents ($\lambda^{**}$), based on authors’ calculations from equation (17) in the text.
price change, agents should be reducing their consumption from period to period. In particular, the change in smoking from month $N + 1$ to month $2N + 1$ over the change in smoking from month 1 to month $N + 1$ will equal $(1 - d)^N (1 + \lambda^{ss})^N$.

In our sample the convergence to a new steady state seems relatively fast, ruling out a combination of high $\lambda^{ss}$ and low $d$. For example, the consumption change from months 7 to 13 is only about 0.126 of the consumption change from months 1 to 7. With $d = 0.5$ and $\lambda^{ss} = 0.9$, this figure should be 0.74; even with $d = 0.5$ and $\lambda^{ss} = 0.7$, the figure should be 0.38. This drop is compatible with $d = 0.6$ and $\lambda^{ss} = 0.7$. Most exercises that we tried in this vein ruled out $d = 0.5$ and $\lambda^{ss} = 0.9$. We can therefore safely put a lower bound on the optimal tax at roughly a dollar per pack.

This is a sizable estimate. The widely cited analysis by the Congressional Research Service concluded that the interpersonal externalities associated with smoking are on the order of 33 cents per pack. These costs ignore the costs of second-hand smoke (19–70 cents/pack, according to Chaloupka and Warner [2000]) and the long-run costs of low birth weight (42 to 72 cents/pack, according to Evans, Ringel, and Stech [1999]). The existing average excise tax on cigarettes in the United States adding the state and federal tax, is 65 cents. So this internality tax from smoking is 300 percent of the externality tax if we ignore second-hand smoke and low birth weight costs, and it remains 60 percent of that tax even if we include the upper bounds of those other costs. Moreover, even in the absence of externalities, these internalities would justify a 50 percent rise in cigarette excise taxation.

Moreover, this estimate is likely to be a lower bound; if we took into account the full internality, not only the fatal health consequences of smoking, the tax would likely be much higher. And, as noted earlier, we have chosen a degree of time inconsistency which is considerably lower than that used in the previous literature.22

We will not attempt to calculate the optimal tax for naifs, because that would involve making assumptions about the degree of underestimation of the self-control problem. However, if there

22. On the other hand, the optimal tax would be lower for a sensible alternative social welfare function, which maximizes utility according to the preferences of the self when the tax change is instituted, thus using a quasi-hyperbolically discounted welfare function. With the exponentially discounted social welfare function that we use, the social planner wants to correct every self’s self-control problem, including self 0’s, whereas with hyperbolic discounting self 0 wants to respect her own preferences and just correct future selves’ behavior. But the difference is not large so long as $\delta$ is close to one and $\beta$ is far from zero.
is such underestimation, the tax for naifs is going to be higher, and much higher if the underestimation is serious.

VII. CONCLUSIONS

The theoretical and empirical insights of Becker and Murphy [1988] and of Becker, Grossman, and Murphy [1994], as well as subsequent work in the vein of their pioneering efforts, have greatly advanced the modeling of addictive processes by economists. These are important and timely advances, as policy-makers are becoming increasingly interested in regulating addictive behaviors. In the case of cigarettes, recent years have seen increased state taxation, regulation of smoking in public places, and a spate of court cases brought by the states and now the Justice Department against the industry.

We have attempted in this paper to make two contributions to the literature on addiction. First, we have suggested a more convincing framework for testing a central hypothesis of the rational addiction model, that individuals are forward-looking with respect to their decisions to consume addictive goods. We find that announced but not yet effective tax increases lead to both increased sales and decreased consumption of cigarettes, which is very consistent with forward-looking behavior by consumers. The use of announced price changes, and the robustness of our finding to specification checks, provides the strongest evidence to date for adjacent complementarity, and clearly rules out myopic formulations of addiction.

Second, we have noted that the rational addiction framework embeds another important assumption besides forward-looking consumption behavior: time consistency. This assumption is at odds with virtually all laboratory experiments and with a variety of casual real-world evidence on smoking decisions. When we change the Becker and Murphy model to incorporate time-inconsistent preferences, we obtain predictions for price changes that are very similar to what are delivered by their model. But we obtain radically different implications for policy. Instead of the standard result that the optimal tax on cigarettes depends only on their associated externalities, we find that there are substantial "internalities" as well which justify government intervention. For very modest parameterization of these internalities, and ignoring any costs other than those associated with the excess
mortality of smoking, we find that there are sizable optimal “internality” taxes on the order of $1 per pack or more.

This result should not be surprising. The key feature of smoking, particularly in contrast to other “addictive bads” such as drinking, is that its internal effects dwarf its external costs: the vast majority of harm done by a smoker is to him or herself. At standard values of the value of a life/year, we estimate above that a pack of cigarettes costs $30.45 in terms of lost life expectancy. If even a small share of these internal costs are to be considered by government policy-makers, the resulting justification for intervention easily outweighs any externalities associated with smoking.

Of course, we have not proved time inconsistency in smoking decisions; we were unable to design and implement a test that could effectively distinguish quasi-hyperbolic and exponential preferences in this context. At the same time, the fact that there is no empirical support, or even laboratory support, for exponential discounting in this or related contexts suggests that alternative models of the type that we have derived be taken seriously. The important general point for thinking about government policy in this context is that, when standard public finance analyses suggest that the tax on addictive bads is simply equal to their external costs, those analyses are implicitly embracing a rational addiction model. Given the enormous magnitude of the internal costs to smoking, however, alternative models such as ours must be considered in designing regulatory policy toward addictive goods.

APPENDIX 1: PROOFS OF THEOREMS

We start with a proof of Lemma 2.

**LEMMA 2.** Suppose that \( u(c_t) \) and \( v(a_t, S_t) \) are differentiable and that a Markov-perfect subgame-perfect equilibrium with differentiable strategy profiles exists. Then, for each \( t \in \{0, \ldots, T - 1\} \) we have

\[
(18) \quad v_a(a_t, S_t) - p_t u'(c_t) = (1 - d) \delta \left[ \left( 1 + (1 - \beta) \frac{\partial a_{t+1}}{\partial S_{t+1}} \right) \times (v_a(a_{t+1}, S_{t+1}) - p_{t+1} u'(c_{t+1})) - \beta v_a(a_{t+1}, S_{t+1}) \right].
\]

**Proof of Lemma 2.** Since instantaneous utilities and future selves’ strategies are differentiable, self \( t \)'s discounted utility is
differentiable in \( a_t \). Furthermore, since an equilibrium exists, self \( t \)'s maximization problem must have a solution. Then, as self \( t \)'s consumption of the addictive product is unrestricted, the derivative of her discounted utility at \( a_t \) is zero. Therefore,

\[
0 = v_a(a_t, S_t) - p_t u'(c_t) + \beta \delta \left[ (v_a(a_{t+1}, S_{t+1})
- p_{t+1} u'(c_{t+1}))(1 - d) \frac{\partial a_{t+1}}{\partial S_{t+1}} + v_s(a_{t+1}, S_{t+1})(1 - d) \right]
+ \beta \delta(1 - d)^2 \left[ (v_a(a_{t+2}, S_{t+2}) - p_{t+2} u'(c_{t+2})) \frac{\partial a_{t+2}}{\partial S_{t+2}} + v_s(a_{t+2}, S_{t+2}) \right] + \cdots
\]

\[
= v_a(a_t, S_t) - p_t u'(c_t) + \beta \sum_{i=1}^{T-t} \delta^i(1 - d)^i \prod_{j=1}^{i-1} \left( 1 + \frac{\partial a_{t+j}}{\partial S_{t+j}} \right)
\left[ (v_a(a_{t+i}, S_{t+i}) - p_{t+i} u'(c_{t+i})) \frac{\partial a_{t+i}}{\partial S_{t+i}} + v_s(a_{t+i}, S_{t+i}) \right].
\]

The complicated second term on the right-hand side comes from the following consideration. It is trivial to prove by induction that the derivative of \( S_{t+i} \) with respect to \( a_t \) is \((1 - d)^i \prod_{j=1}^{i-1} (1 + (\partial a_{t+j}/\partial S_{t+j})).\) Now this has two effects on future instantaneous utility. First, it affects utility directly—the stock of past consumption is assumed to affect current utility. That is the \( v_s(a_{t+i}, S_{t+i}) \) term. Second, it affects utility through changing the consumption of self \( t + i \). That is the \((v_a(a_{t+i}, S_{t+i}) - p_{t+i} u'(c_{t+i}))/(\partial a_{t+j}/\partial S_{t+i}) \) term.

We can write the same optimality condition for self \( t + 1 \):

\[
0 = v_a(a_{t+1}, S_{t+1}) - p_{t+1} u'(c_{t+1})
+ \beta \sum_{i=1}^{T-t-1} \delta^i(1 - d)^i \prod_{j=1}^{i-1} \left( 1 + \frac{\partial a_{t+1+j}}{\partial S_{t+1+j}} \right)
\]

23. Notice that self \( t \) really only has one choice variable, because a choice of \( a_t \) ties down \( c_t \) due to the no-saving assumption.
\[
\times \left[ (v_a(a_{t+1}, S_{t+1}) - p_{t+1}u'(c_{t+1})) \frac{\partial a_{t+1}}{\partial S_{t+1}} \right] \\
+ \beta \sum_{i=1}^{T-t-1} \delta^i (1 - d^i) \prod_{j=1}^{i-1} \left( 1 + \frac{\partial a_{t+j}}{\partial S_{t+j}} \right) v_s(a_{t+1}, S_{t+1}).
\]

Multiplying equation (2) by \(\delta(1 - d)(1 + (\partial a_{t+1}/\partial S_{t+1}))\) and subtracting it from equation (19), we get
\[
0 = v_a(a_t, S_t) - p_t u'(c_t) \\
+ \beta \delta \left[ (v_a(a_{t+1}, S_{t+1}) - p_{t+1}u'(c_{t+1}))(1 - d) \frac{\partial a_{t+1}}{\partial S_{t+1}} \\
+ v_s(a_{t+1}, S_{t+1})(1 - d) \right] \\
- \delta(1 - d) \left( 1 + \frac{\partial a_{t+1}}{\partial S_{t+1}} \right) (v_a(a_{t+1}, S_{t+1}) - p_{t+1}u'(c_{t+1})).
\]

Rearranging this gives the desired Euler equation.

We pick up the discussion from the observation that for each \(t\) and each type of agent, \(a_t = \lambda_t S_t + \mu_t\) for some constants \(\lambda_t\) and \(\mu_t\). Then
\[
S_{t+1} = (1 - d)(S_t + a_t) = (1 - d)(S_t + \lambda_t S_t + \mu_t)
\]
(22)
\[
a_{t+1} = \lambda_{t+1} S_{t+1} + \mu_{t+1} = \lambda_{t+1} (1 - d)(S_t + \lambda_t S_t + \mu_t) + \mu_{t+1}.
\]
(23)

Plugging this into the sophisticates’ first-order condition, equation (18), and assuming \(p_t = p\) in each period:
\[
\alpha_a + \alpha_{aa}(\lambda_t S_t + \mu_t) + \alpha_{as} S_t - p[\alpha_c + \alpha_{cc}(I_t - p(\lambda_t S_t + \mu_t))] \\
= (1 - d) \delta[(1 + (1 - \beta)\lambda_{t+1})[\alpha_a + \alpha_{aa}(\lambda_{t+1}(1 - d) \\
\times (S_t + \lambda_t s_t + \mu_t) + \mu_{t+1}) \\
+ \alpha_{as}(1 - d)(S_t + \lambda_t s_t + \mu_s) - p(\alpha_c + \alpha_{cc})
\]

24. To be more precise, for the solution to the first-order condition to give a maximum, we need strict concavity at each stage. But we know that if a function \(C(a_t, S_t)\) is strictly concave and continuously differentiable, then \(C_a(a_t, S_t) = 0\) gives the global maximum for a fixed \(S_t\), and this maximum is strictly concave in \(S_t\). This consideration implies for time-consistent agents that the previous period’s problem is also strictly concave. Then for the sophisticated problem to be strictly concave, notice that her value function starting in the next period is quadratic, and dominated by the time-consistent agent’s quadratic value function.
The above has to be true for all \(S_t\), so the coefficient of \(S_t\) in the expression has to be zero. After “some” manipulation, this implies that

\[
\lambda_t = -1 + \frac{\alpha_{as} - \alpha_{aa} - p^2 \alpha_{cc}}{- \alpha_{aa} - p^2 \alpha_{cc} + \delta(1-d)[(1 - (1 - \beta)\lambda_{t+1})]
\times (\alpha_{aa}\lambda_{t+1} + \alpha_{as} + p^2 \alpha_{cc}\lambda_{t+1}) - \beta \alpha_{as}\lambda_{t+1} - \beta \alpha_{ss}].
\]

This is a backward recursion for the \(\lambda\)’s in the different periods. It looks quite scary, but can be understood with some effort. That is what Theorem 1 does.

**Theorem 1.** Suppose that \(\beta \geq \frac{1}{2}\) and that \(U(a_t, c_t, S_t)\) is strictly concave. Then the backward recursion (25) converges, that is, \(\lim_{j \to \infty} \lambda_{T-j} = \lambda^{*s}\), where \(\lambda^{*s}\) is given as the unique solution on the interval \((-1, (\alpha_{as}/(-\alpha_{aa} - p^2 \alpha_{cc})))\) of

\[
\lambda^{*s} = -1 + \frac{\alpha_{as} - \alpha_{aa} - p^2 \alpha_{cc}}{- \alpha_{aa} - p^2 \alpha_{cc} + \delta(1-d)[(1 - (1 - \beta)\lambda^{*s})]
\times (\alpha_{aa}\lambda^{*s} + \alpha_{as} + p^2 \alpha_{cc}\lambda^{*s}) - \beta \alpha_{as}\lambda^{*s} - \beta \alpha_{ss}].
\]

Furthermore, \(\lambda^{*s} > 0\) if and only if

\[
\alpha_{as} > \frac{\beta \delta(1-d)^2}{1 - \delta(1-d)^2}(-\alpha_{ss}).
\]

**Proof of Theorem 1.** Define the function \(f_s(\lambda)\) according to equation (25). We will prove that \(f_s(\lambda_T) < \lambda_T = \alpha_{as}/(-\alpha_{aa} - p^2 \alpha_{cc}), f(-1) > -1, and that \(f_s\) is continuous and increasing on \((-1, \lambda_T)\). This is sufficient to establish that \(\lambda_{T-i}\) converges. Then clearly \(\lambda^{*s} > 0\) if and only if \(f_s(0) > 0\), which is equivalent to (11).

First, notice that the second term of \(f_s(\lambda)\) is the reciprocal of a quadratic with a negative coefficient on \(\lambda^2\). Then if this term is positive for two points on the real line, it is also positive in-between these two points. Moreover, it is easy to show that on the interval where this term is positive, \(f_s\) is strictly convex.\(^{25}\)

\(^{25}\) The second derivative of the reciprocal of a quadratic \(q\) is \(-(q^2q'' - q(q')^2)/q^4\), which is positive as long as \(q\) is positive and concave.
Therefore, it is sufficient to show that $f_s(-1) > -1$, $\lambda_T > f_s'(\lambda_T) > -1$, and that $f_s'(-1) \geq 0$. The first two ensure that we are on the continuous and strictly convex section of $f_s$, and the last one (together with convexity) ensures that $f_s$ is increasing on $(-1, \lambda_T)$.

The rest is just carrying out the above. We have

$$f_s(-1) = -1$$

$$+ \frac{\alpha_{as} - \alpha_{aa} - p^2\alpha_{cc}}{-\alpha_{aa} - p^2\alpha_{cc} + \beta\delta(1-d)^2[\alpha_{aa} + 2\alpha_{as} - p^2\alpha_{cc} - \alpha_{ss}]} > -1$$

as both the numerator and the denominator are positive in the second term. Proceeding,

$$f_s(\lambda_T) = f_s\left(\frac{\alpha_{as}}{-\alpha_{aa} - p^2\alpha_{cc}}\right)$$

$$= -1 + \frac{\alpha_{as} - \alpha_{aa} - p^2\alpha_{cc}}{-\alpha_{aa} - p^2\alpha_{cc} + \delta(1-d)^2[-\beta\alpha_{as}\lambda_T - \beta\alpha_{ss}]}$$

$$= \frac{\alpha_{as} - \delta(1-d)^2[-\beta\alpha_{as}\lambda_T - \beta\alpha_{ss}]}{-\alpha_{aa} - p^2\alpha_{cc} + \delta(1-d)^2[-\beta\alpha_{as}\lambda_T - \beta\alpha_{ss}]}.$$

This being $< \lambda_T = (\alpha_{as}/(-\alpha_{aa} - p^2\alpha_{cc}))$ is equivalent to $-\alpha_{as}\lambda_T - \alpha_{ss} > 0$. But the latter can be rewritten as $\alpha_{as}^2 < \alpha_{ss} (\alpha_{aa} + p^2\alpha_{cc})$, and since owing to the concavity of $U(a_t, c_t, S_t)$ we have $\alpha_{as}^2 < \alpha_{ss}\alpha_{aa}$, this inequality holds. $-\alpha_{as}\lambda_T - \alpha_{ss} > 0$ also implies that the second term is positive, so that $f(\lambda_T) > 1$.

Finally,

$$f_s'(\lambda) = -\frac{(\alpha_{aa} + p^2\alpha_{cc} + (1-2\beta)\alpha_{as})}{[\alpha_{as} - \alpha_{aa} - p^2\alpha_{cc} + \delta(1-d)^2[(1+1-\beta)\lambda] + (\alpha_{aa} + \alpha_{as} + p^2\alpha_{cc}\lambda) - \beta\alpha_{as}\lambda - \beta\alpha_{ss}]^2}$$

which gives

$$f_s'(-1) = -(1-2\beta)$$

$$\times \left[\frac{\alpha_{as} - \alpha_{aa} - p^2\alpha_{cc}}{-\alpha_{aa} - p^2\alpha_{cc} + \delta(1-d)^2[(1+1-\beta)\lambda] + (\alpha_{aa} + \alpha_{as} + p^2\alpha_{cc}\lambda) - \beta\alpha_{as}\lambda - \beta\alpha_{ss}]^2}\right] \geq 0.$$

This completes the proof.
Note that the above proof does not work if $U(a_t,c_t,S_t)$ is not concave, possibly leading to “wild” behavior on the part of the agent. In particular, in that case one cannot prove, and it is not in general true, that $f_s(\lambda_T) < \lambda_T$. Also, it is not the case that the backward program that just looks at first-order conditions at each stage finds a maximum for every period. But inasmuch as it does, we can say the following. Carefully looking at the graph of $f_s(l)$, it seems possible that $l_{T-2}$ first increases, then jumps to around 2, and then starts increasing again, restarting the cycle. Behaviorally, this means that the agent goes through periods of addiction followed by brutal “cold turkey” types of quits, a phenomenon described by Becker and Murphy [1988].

**APPENDIX 2: THE RESPONSIVENESS TO PRICE AND ITS IMPLICATIONS**

Condition (24) implies for sophisticates

\[
\begin{align*}
\mu^s_t &= \text{constant} - \frac{p_t \alpha_c - p_{t+1} \alpha_c (1 - d) \delta (1 + (1 - \beta) \lambda_{t+1})}{-\alpha_{aa} + \delta (1 - d)^2 [1 + (1 - \beta) \lambda_{t+1}]} \\
&\quad \times [\alpha_{aa} \lambda_{t+1} + \alpha_{as}] - \beta \alpha_{as} \lambda_{t+1} - \beta \alpha_{ss}] \\
&\quad + \left\{ (1 - d) \delta [\beta \alpha_{as} - (1 + (1 - \beta) \lambda_{t+1}) \alpha_{aa}] \\
&\quad - \frac{\alpha_{aa} + \delta (1 - d)^2 [1 + (1 - \beta) \lambda_{t+1}]}{\mu^s_{t+1}} \right\} [\alpha_{aa} \lambda_{t+1} + \alpha_{as}] - \beta \alpha_{as} \lambda_{t+1} - \beta \alpha_{ss}].
\end{align*}
\]

The limit of the coefficient of $\mu^s_{t+1}$ in the expression exists and is equal to

\[
\begin{align*}
(1 - d) \delta [\beta \alpha_{as} - (1 + (1 - \beta) \lambda^{**}) \alpha_{aa}] \\
&\quad - \frac{\alpha_{aa} + \delta (1 - d)^2 [1 + (1 - \beta) \lambda^{**}]}{[\alpha_{aa} \lambda^{**} + \alpha_{as}] - \beta \alpha_{as} \lambda^{**} - \beta \alpha_{ss}],}
\end{align*}
\]

which is clearly positive. It is easy to see that in general $\mu^i_T$ converges if and only if the above limit is less than 1.

**Lemma 3.** $\mu^{TC}_{T-j}$ converges.

*Proof of Lemma 3.* Suppose not. Then one can easily choose parameters so that $\mu^{TC}_{T-j}$ diverges; i.e., $|\mu^{TC}_{T-j}| \rightarrow \infty$. Now consider any $S$. There are real numbers $M$ and $N$ such that for a small enough $t$, $M < V^{t,TC}(S) < N$. The lower bound comes from the consideration that one can just consume $d/(1 - d)$ $S$ in each period (the steady-state consumption corresponding to $S$), giving
a discounted utility \( (1/(1 - \delta) v(d/(1 - d))) S, S) + \alpha_c(I - p (d/(1 - d))) S \) in the limit. The upper bound comes from the fact that \( U(a_t, c_t, S_t) \) is strictly concave quadratic, and thus has a global maximum. (Note that this also implies that \( N \) can be chosen independently of \( S \).)

Now since \( |\mu_{Tj}^{TC}| \to \infty \), also \( |V_{S}^{T-j, TC} (S)| = |v_s(a_{T-j}, S)| \to \infty \). But since \( \lim_{j \to \infty} V_{S}^{T-j, TC} (S) = \alpha_{ss} + \alpha_{as} \lambda^{*TC} \), this contradicts that the value function is bounded from above.

Call the limit \( \mu_{*TC} \). Unfortunately, for sophisticated hyperbolic discounters we do not necessarily get convergence. To see this, rewrite (33) as

\[
\frac{(1 - d)\delta[\alpha_{as} - \alpha_{aa}]}{-\alpha_{aa} + \delta(1 - d)^2[(1 + (\beta)\lambda^{**})]}
\times [\alpha_{aa} \lambda^{**} + \alpha_{as}] - \beta \alpha_{as} \lambda^{**} + \beta \alpha_{ss}] - \frac{(1 - d)\delta(1 - \beta)(\alpha_{aa} \lambda^{**} + \alpha_{as})}{-\alpha_{aa} + \delta(1 - d)^2[(1 + (\beta)\lambda^{**})]}
\times [\alpha_{aa} \lambda^{**} + \alpha_{as}] - \beta \alpha_{as} \lambda^{**} - \beta \alpha_{ss}].
\]

As \( \beta \to 0 \), this approaches \( (1 - d)\delta(1 + \lambda_T) \), which can easily be greater than 1.\(^{26}\) If \( \mu_{*TC}^{T-j} \) diverges, the model exhibits certain “violent” characteristics (for example, there would be huge consumption reactions to even minimal price changes, possibly even in the “wrong” way), which we do not want to deal with. Therefore, we make a sufficient assumption for (33) to be less than 1. We assume that \( (1 - d)\delta(1 + \lambda^{**}) < 1 \). By the continuity of \( \lambda^{**} \) with respect to \( \beta \), this is true if \( \beta \) is sufficiently close to 1.

We will use the following lemma extensively to study consumption responses to price changes.

**Lemma 4.** For a recursion of the form \( x_{j+1} = l + kx_j \) with \( 0 < k < 1 \), we have

\[
\frac{d}{dl} \lim_{j \to \infty} x_j = \frac{1}{1 - k}.
\]

**Proof of Lemma 4.** An easy geometric argument.

We will consider permanent price decreases of \( \Delta p \) that start either last period, this period, next period, or two periods into the
A direct application of Lemma 4 yields a consumption response to a present permanent decrease in price for sophisticates of

\[
\frac{\Delta p \alpha_c}{m^s} \frac{1 - (1 - d) \delta (1 + (1 - \beta) \lambda^{*s})}{1 - k},
\]

where

\[
(37) \quad m_s = -\alpha_{aa} + \delta(1 - d)^2[(1 + (1 - \beta)\lambda_{t+1})[\alpha_{aa}\lambda_{t+1} + \alpha_{as}] - \beta\alpha_{as}\lambda_{t+1} - \beta\alpha_{ss}],
\]

\[
k = \frac{(1 - d)\delta[\beta\alpha_{as} - (1 + (1 - \beta)\lambda_{t+1})\alpha_{aa}]}{-\alpha_{aa} + \delta(1 - d)^2[(1 + (1 - \beta)\lambda_{t+1})[\alpha_{aa}\lambda_{t+1} + \alpha_{as}] - \beta\alpha_{as}\lambda_{t+1} - \beta\alpha_{ss}]}.
\]

If the price change is next period, \(\mu_{t+1}\) changes by the same amount as if it was this period, so the change in \(\mu_t\) is also the same except for a term \(\Delta p\alpha_c / m^2\).

\[
(38) \quad \frac{\Delta p \alpha_c}{m^s} \frac{1 - (1 - d) \delta (1 + (1 - \beta) \lambda^{*s})}{1 - k} = \frac{\Delta p \alpha_c}{m^s}.
\]

For time-consistent agents, the expressions are similar, but they reduce nicely because in that case \(k = (1 - d)\delta(1 + \lambda^{*TC})\). The most convenient forms are given below.

**APPENDIX 3: SUMMARY OF PRICE RESPONSES**

<table>
<thead>
<tr>
<th></th>
<th>This period</th>
<th>Next period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-consistent</td>
<td>[-\frac{(1 - (1 - d)\delta)}{1 - (1 - d)} \frac{\Delta p \alpha_c}{m^{TC}} \times \delta(1 + \lambda^{*TC})]</td>
<td>[-\frac{\Delta p \alpha_c}{m^{TC}} \frac{(1 - d)\delta \lambda^{*TC}}{1 - (1 - d)} \times \delta(1 + \lambda^{*TC})]</td>
</tr>
<tr>
<td>Sophisticated</td>
<td>[-\frac{\Delta p \alpha_c}{m^s} \times \delta(1 + (1 - \beta)\lambda^{*s})}{1 - k}]</td>
<td>[-\frac{\Delta p \alpha_c}{m^s} \times \delta(1 + (1 - \beta)\lambda^{*s})}{1 - k}]</td>
</tr>
</tbody>
</table>

**Two periods ahead**

| Sophisticated          | \[-k \left[ \frac{\Delta p \alpha_c}{m^s} \frac{1 - (1 - d) \delta (1 + (1 - \beta) \lambda^{*s})}{1 - k} - \frac{\Delta p \alpha_c}{m^s} \right]\] |

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